

Economics 11: Midterm

30th October, 2008

Instructions: The test is closed book. Calculators are allowed. Please write your answers on this sheet.

Question	Score
Questions 1–5	
Question 6	
Question 7	
Question 8	
Total	

Short Questions (25 points)*Question 1*

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$$

- (a) Derive the MRS.
- (b) Show that preferences are convex.

Question 2

Given any two cars, I always prefer the one that is bigger and faster. Given this information, are my preferences transitive? Are they complete?

Question 3

Assume that preferences satisfy the usual axioms (transitivity, completeness and continuity). Also assume they satisfy monotonicity. Can an indifference curve cross itself? Explain your answer.

Question 4

- (a) Define the expenditure function (either mathematically or in words).
- (b) Intuitively explain why the expenditure function is concave in prices.

Question 5

- (a) Define an inferior good.
- (b) An agent consumes two goods and always spends all her income. Can both goods be inferior?

6. Basic Consumer Choice (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^{1/3} + x_2^{2/3}$$

The prices of the goods are $p_1 = 1$ and $p_2 = 1$. The consumer has income $m = 9$. Calculate the consumer's optimal demand.

Note: For this question, you may find the quadratic formula useful. If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Space for Question 6.

7. Hicksian Demand (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1 x_2^2$$

The prices of the goods are (p_1, p_2) .

- (a) Set up the expenditure minimisation problem.
- (b) Derive the agent's Hicksian demands.
- (c) Derive the agent's expenditure function.

Space for Question 7.

8. Choice with Satiation (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = -\frac{1}{2}(x_1 - 10)^2 - \frac{1}{2}(x_2 - 10)^2$$

The agent faces prices $p_1 = 1$ and $p_2 = 1$ and has income m . [Hint: you may find it useful to plot the indifference curves].

- (a) Suppose $m = 10$. Solve for the agent's optimal choice.
- (b) Suppose $m = 30$. Solve for the agent's optimal choice.
- (c) Given the above prices, plot the agent's income offer curve (income expansion path) and the Engel curve for good 1.¹

¹Reminder: The Engel curve shows how demand for a good varies with income.

Space for Question 8.

Space for Rough Work (Do not write your answers here)

Space for Rough Work (Do not write your answers here)

Economics 11: Solutions to Midterm

30th October, 2008

Instructions: The test is closed book. Calculators are allowed.

Short Questions (25 points)

Question 1

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$$

- (a) Derive the MRS.
- (b) Show that preferences are convex.

Solution

- (a) The MRS is

$$\frac{MU_1}{MU_2} = \frac{\frac{1}{2}x_1^{-1/2}x_2^{1/2}}{\frac{1}{2}x_1^{1/2}x_2^{-1/2}} = \frac{x_2}{x_1}$$

- (b) We can show preferences are convex by showing the MRS is decreasing in x_1 along the indifference curve. Along the indifference curve, $x_1^{1/2}x_2^{1/2} = k$. Rearranging, we get $x_2 = k^2/x_1$. Plugging into MRS,

$$MRS = \frac{k^2}{x_1^2}$$

This is clearly decreasing in x_1 .

Question 2

Given two cars, I always prefer the one that is bigger and faster. Given this information, are my preferences transitive? Are they complete?

Solution

They are transitive, but not complete. It's unclear how I rank two cars A and B , where A is faster and b is bigger.

Question 3

Assume that preferences satisfy the usual axioms (transitivity, completeness and continuity). Also assume they satisfy monotonicity. Can an indifference curve cross itself? Explain your answer.

Solution

No. This violates monotonicity.

Question 4

- (a) Define the expenditure function.
- (b) Intuitively explain why the expenditure function is concave in prices.

Solution

- (a) The expenditure function is the minimal expenditure needed to attain a target utility level.
- (b) Suppose p_1 rises. If I keep my demands constant then I attain the same utility level and my expenditure rises linearly. I can do better than this by rebalancing my demands, introducing concavity.

More formally, fix p_2 and any allocations (x_1, x_2) that attain the target utility \bar{u} . Define the pseudo-expenditure function by

$$\eta_{x_1, x_2}(p_1) = p_1 x_1 + p_2 x_2$$

which is linear in p_1 . The expenditure function is given by the lower envelope of

$$\{\eta_{x_1, x_2}(p_1) : u(x_1, x_2) = \bar{u}\}$$

Since the minimum of linear functions is concave, the expenditure function is therefore concave.

Question 5

- (a) Define an inferior good.
- (b) An agent consumes two goods and always spends all her income. Can both goods be inferior?

Solution

- (a) Demand decreases as income increases.
- (b) No: if the agent's income rises, her expenditure on at least one good must rise.

Basic Consumer Choice (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^{1/3} + x_2^{2/3}$$

The prices of the goods are $p_1 = 1$ and $p_2 = 1$. The consumer has income $m = 9$. Calculate the consumer's optimal demand.

Note: For this question, you may find the quadratic formula useful. If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution

The tangency condition states that

$$\frac{\frac{1}{3}x_1^{-2/3}}{\frac{2}{3}x_2^{-1/3}} = 1$$

Rearranging, $x_2 = 8x_1^2$. Using the budget constraint,

$$x_1^* = 1 \quad \text{and} \quad x_2^* = 8$$

Hicksian Demand (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1 x_2^2$$

The prices of the goods are (p_1, p_2) .

- (a) Set up the expenditure minimisation problem.
- (b) Derive the agent's Hicksian demands.
- (c) Derive the agent's expenditure function.

Solution

- (a) The agent minimises

$$\mathcal{L} = p_1 x_1 + p_2 x_2 + \lambda[\bar{u} - x_1 x_2^2]$$

- (b) The FOCs are:

$$\begin{aligned} p_1 &= \lambda x_2^2 \\ p_2 &= 2\lambda x_1 x_2 \end{aligned}$$

From these we find $2p_1 x_1 = p_2 x_2$. The constraint states that

$$x_1 x_2^2 = \bar{u}$$

Solving for the Hicksian demands

$$h_1 = \frac{1}{4^{1/3}} \bar{u}^{1/3} \left(\frac{p_2}{p_1} \right)^{2/3}$$

and

$$h_2 = 2^{1/3} \bar{u}^{1/3} \left(\frac{p_1}{p_2} \right)^{1/3} = \frac{2}{4^{1/3}} \bar{u}^{1/3} \left(\frac{p_1}{p_2} \right)^{1/3}$$

(c) The expenditure function is

$$e = p_1 h_1 + p_2 h_2 = \frac{3}{4^{1/3}} \bar{u}^{1/3} p_2^{2/3} p_1^{1/3}$$

Choice with Satiation (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = -\frac{1}{2}(x_1 - 10)^2 - \frac{1}{2}(x_2 - 10)^2$$

The agent faces prices $p_1 = 1$ and $p_2 = 1$ and has income m . [Hint: you may find it useful to plot the indifference curves].

- (a) Suppose $m = 10$. Solve for the agent's optimal choice.
- (b) Suppose $m = 30$. Solve for the agent's optimal choice.
- (c) Given the above prices, plot the agent's income offer curve (income expansion path) and the Engel curve for good 1.¹

Solution

The key point here is to observe that $(10, 10)$ is a bliss point.

(a) The tangency condition says

$$\frac{x_1 - 10}{x_2 - 10} = 1$$

Rearranging, $x_1 = x_2$. Using the budget constraint,

$$x_1^* = 5 \quad \text{and} \quad x_2^* = 5$$

(b) The agent can afford the bliss point $(10, 10)$, and so will consume this.

(c) Demands are given by

$$x_1^* = \min\{m/2, 10\} \quad \text{and} \quad x_2^* = \min\{m/2, 10\}$$

¹Reminder: The Engel curve shows how demand for a good varies with income.

The income offer curve is a 45° slope, and then stops at $(10, 10)$. The Engel curve rises at slope $1/2$ and stops at 10.

Economics 11: Practice Midterm

October 22, 2008

Short Questions

Question 1

A consumer spends his entire budget on two goods: X and Y.

- (i) True or false: An increase in the price of X will lead the consumer to purchase less X.
- (ii) True or false: An increase in the price of X will always lead a consumer to purchase more Y.

Question 2

Graphically explain the effect in the budget constraint of an increase in an individual's income without changing relative prices. Explain the impact on the quantity demanded of both goods.

Question 3

Explain the monotonicity and convexity axioms and their implications for the shape of the indifference curves.

Question 4

Suppose that the consumers demand for x_1 , as a function of its own price p_1 , the price of the other good p_2 and income m is given by:

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + 2p_2}$$

Suppose one unit of x_1 costs twice as much as one unit of x_2 . What is the income elasticity of demand? What is the own-price elasticity of demand? What is the cross-price elasticity of demand?

Question 5

David likes only Peanut Butter (B) and Toast (T), and he always eats each piece of toast with two ounces of peanut butter.

- a) Find David's demand for peanut butter and toast.
- b) How much of each good will David consume, if he has \$30 of income, the price of peanut butter is \$2, and the price of a toast is \$6?
- c) Suppose that the price of a toast rises to \$11. How will his consumption change?
- d) How much should David's income be to compensate for the rise in the price of the toast?

Question 6

Thomas likes Hamburgers and Pizza and views them as perfect substitutes. In particular, he is indifferent between two Hamburgers and three slices of Pizza.

- a) Define Thomas's utility function over Hamburgers (H) and Pizza (P)
- b) For which prices will Thomas eat only Hamburgers?
- c) Thomas has \$30 to spend on Hamburgers and Pizza. Each Hamburger costs \$3 and each slice of Pizza costs \$2. What does Thomas consume?
- d) This week there is a promotion in Northern Lights and you can buy each slice of Pizza for \$1.5. What does Thomas consume?
- e) During the promotion in Northern Lights, the Hamburger store at Lu Valle wants to attract clients like Thomas. What is the maximum price they can charge?

Question 7

A consumer has utility $U(x_1, x_2) = \ln(x_1) + 2 \ln(x_2)$ and income m .

- a) Find the uncompensated demand for x_1 and x_2 , and find the indirect utility function
- b) Use the own price Slutsky equation for x_1 to determine the substitution effect.
- c) Find the compensated demand for x_1 and x_2 and the expenditure function $e(p_1, p_2, \bar{u})$.

Economics 11: Solutions to Practice Midterm

October 28, 2008

Short Questions

Question 1

A consumer spends his entire budget on two goods: X and Y.

- (i) True or false: An increase in the price of X will lead the consumer to purchase less X.
- (ii) True or false: An increase in the price of X will always lead a consumer to purchase more Y.

Solution

- (i) False: It will depend on good X. If X is a Giffen good, the negative substitution effect of the price increase is more than outweighed by the positive income effect, thus the demand for X is increasing in the price.
- (ii) False: X and Y could be either gross complements or gross substitutes. If they are gross complements, the negative income effect always outweighs the substitution effect, and the overall demand for Y is decreasing in the price of X. If they are gross substitutes, the substitution effect outweighs the income effect, and the demand for Y is increasing in the price of X.

Question 2

Graphically explain the effect in the budget constraint of an increase in an individual's income without changing relative prices. Explain the impact on the quantity demanded of both goods.

Solution

The budget line shifts parallel when the income changes and the price ratio remains the same. The impact on the quantity demanded will depend on the type of good. If X is inferior the quantity demanded decreases when the income of the consumer increases: $dX/dm < 0$. If X is normal the quantity demanded increases when the income of the consumer increases: $dX/dm > 0$.

Question 3

Explain the monotonicity and convexity axioms and their implications for the shape of the indifference curves.

Solution

Monotonicity: Consumers prefer to consume more rather than less of each good. Implies that indifference curves between two goods are thin, decreasing and cannot cross.

Convexity: Consumers prefer averages to extremes. Implies that indifference curves are convex.

Question 4

Suppose that the consumers demand for x_1 , as a function of its own price p_1 , the price of the other good p_2 and income m is given by:

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + 2p_2}$$

Suppose one unit of x_1 costs twice as much as one unit of x_2 . What is the income elasticity of demand? What is the own-price elasticity of demand? What is the cross-price elasticity of demand?

Solution

Income elasticity:

$$\frac{\partial x_1^*}{\partial m} \frac{m}{x_1^*} = 1$$

Since $p_1 = 2p_2$, the own price elasticity is:

$$\frac{\partial x_1^*}{\partial p_1} \frac{p_1}{x_1^*} = -\frac{p_1}{p_1 + 2p_2} = -\frac{1}{2}$$

Since $p_1 = 2p_2$, the cross price elasticity is:

$$\frac{\partial x_1^*}{\partial p_2} \frac{p_2}{x_1^*} = -\frac{2p_2}{p_1 + 2p_2} = -\frac{1}{2}$$

Question 5

David likes only Peanut Butter (B) and Toast (T), and he always eats each piece of toast with two ounces of peanut butter.

- a) Find David's demand for peanut butter and toast.
- b) How much of each good will David consume, if he has \$30 of income, the price of peanut butter is \$2, and the price of a toast is \$6?
- c) Suppose that the price of a toast rises to \$11. How will his consumption change?
- d) How much should David's income be to compensate for the rise in the price of the toast?

Solution

(a) It is a case of perfect complements. The utility function $U(T, B) = \min\{2T, B\}$ represents David's preferences. Indifference curves are L shaped with corners located on the line $B = 2T$. Using the budget line, $p_B B + p_T T = m$, we have

$$T^* = I/(2p_B + p_T) \quad \text{and} \quad B^* = 2I/(2p_B + p_T).$$

(b) $T = 30/(2 * 2 + 6) = 3$ and $B = 2T = 6$. Utility is $2T = B = 6$.

(c) $T = 30/(2 * 2 + 11) = 2$ and $B = 2T = 4$. Utility is $2T = B = 4$.

(d) David must enjoy a utility of 6 to be back on his indifference curve. This means that he must consume $T = 3$ with the new prices. Therefore m' must solve $3 = m'/(2 * 2 + 11)$. Therefore, $m' = 45$.

Question 6

Thomas likes Hamburgers and Pizza and views them as perfect substitutes. In particular, he is indifferent between two Hamburgers and three slices of Pizza.

- a) Define Thomas's utility function over Hamburgers (H) and Pizza (P)

- b) For which prices will Thomas eat only Hamburgers?
- c) Thomas has \$30 to spend on Hamburgers and Pizza. Each Hamburger costs \$3 and each slice of Pizza costs \$2. What does Thomas consume?
- d) This week there is a promotion in Northern Lights and you can buy each slice of Pizza for \$1.5. What does Thomas consume?
- e) During the promotion in Northern Lights, the Hamburger store at Lu Valle wants to attract clients like Thomas. What is the maximum price they can charge?

Solution

- (a) Preferences are represented by $U(H, P) = 3H + 2P$.
- (b) When $p_h/p_p < 3/2$.
- (c) $p_h/p_p = 3/2$. As a result, he will demand any non-negative value as long as $m = 3H + 2P$
- (d) $p_h/p_p = 3/1.5 = 2 > 3/2$, then he will demand only Pizza: $P^* = m/p_p = 30/1.5 = 20$.
- (e) We need $p_h/p_p < 3/2$, then $p_h < 3/2 * p_p = 3/2 * 3/2 = 9/4$.

Question 7

A consumer has utility $U(x_1, x_2) = \ln(x_1) + 2\ln(x_2)$ and income m .

- a) Find the uncompensated demand for x_1 and x_2 , and find the indirect utility function
- b) Use the own price Slutsky equation for x_1 to determine the substitution effect.
- c) Find the compensated demand for x_1 and x_2 and the expenditure function $e(p_1, p_2, \bar{u})$.

Solution

(a) The tangency condition says

$$\frac{1/x_1}{2/x_2} = \frac{p_1}{p_2}$$

Rearranging, $2x_1p_1 = x_2p_2$. Using the budget constraint,

$$x_1^* = \frac{m}{3p_1} \quad \text{and} \quad x_2^* = \frac{2m}{3p_2}$$

Indirect utility:

$$v = 3 \ln(m) - \ln(27/4) - \ln(p_1) - 2 \ln(p_2)$$

(b) The total effect of a price change is:

$$\frac{\partial x_1^*}{\partial p_1} = -\frac{1}{3p_1^2}$$

The income effect is

$$-x_1^* \frac{\partial x_1^*}{\partial m} = -\frac{1}{9p_1^2}$$

The substitution effect is therefore

$$\frac{\partial h_1}{\partial p_1} = \frac{\partial x_1^*}{\partial p_1} + x_1^* \frac{\partial x_1^*}{\partial m} = -\frac{2}{9p_1^2}$$

Hence 2/3 of the change in demand is due to the substitution effect; 1/3 is due to the income effect.

(c) Inverting the indirect utility function,

$$e(p_1, p_2, \bar{u}) = \frac{3}{4^{1/3}} e^{\bar{u}/3} p_1^{1/3} p_2^{2/3}$$

Using Sheppard's Lemma, we can find Hicksian demands:

$$h_1 = \frac{1}{4^{1/3}} e^{\bar{u}/3} \left(\frac{p_2}{p_1} \right)^{2/3} \quad \text{and} \quad h_2 = \frac{2}{4^{1/3}} e^{\bar{u}/3} \left(\frac{p_1}{p_2} \right)^{1/3}$$

Alternatively, you could have solve the expenditure minimisation problem.

Economics 11: Final

11:30am–2:30pm, 8th December, 2008

Instructions: The test is closed book. Regular calculators are allowed; graphic calculators are not. Please write your answers on this sheet. There are 200 points.

Name:

UCLA ID:

TA:

Question	Score
Questions 1–6	
Question 7	
Question 8	
Question 9	
Question 10	
Question 11	
Question 12	
Total	

Short Questions (40 points)**Question 1 (6 points)**

A firm faces linear input prices r_1 and r_2 and output price p .

- (a) Define the profit function, $\pi^*(p, r_1, r_2)$, either in words or mathematically.
- (b) Argue that $\pi^*(p, r_1, r_2)$ is convex in p .

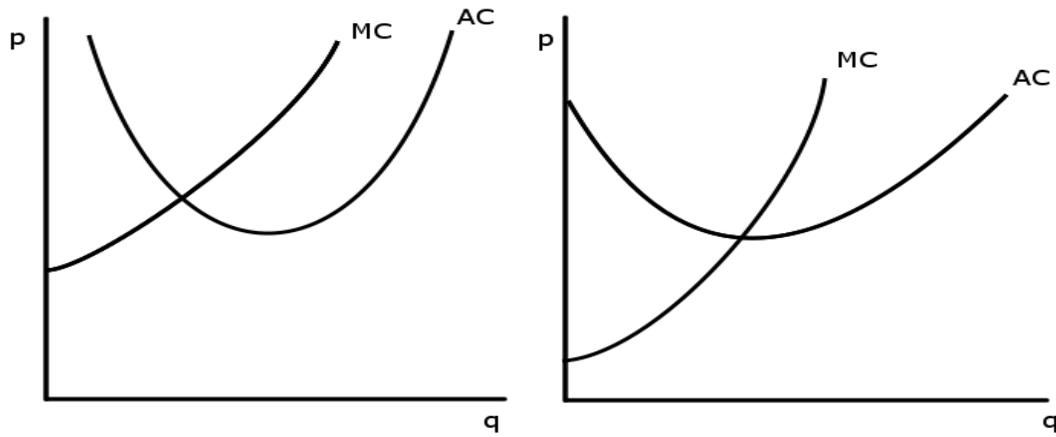


Figure 1: Cost Curves.

Question 2 (6 points)

Figure 1 illustrates the marginal costs and average costs for two cases. Does a cost function exist that generates these pictures, or is something wrong?

Question 3 (7 points)

An agent has preferences over two goods $(x_1, x_2) \in \mathbb{R}_+^2$. Suppose preferences are complete, transitive and continuous. Suppose they are also monotone and *strictly concave*, so that whenever $x \succ y$ then

$$x \succ tx + (1 - t)y \quad \text{for } t \in (0, 1).$$

That is, the MRS is strictly increasing in x_1 along the indifference curve.

The agent has income m and faces positive prices p_1 and p_2 . Will the agent ever choose a bundle containing positive amount of both goods? Explain your answer.

Question 4 (7 points)

A good has market demand curve $X = 100 - p$. There are an unlimited number of potential firms with costs curves $c(q) = q + q^2$.

What is the long-run equilibrium price, allowing for the free entry of firms? How many firms will there be?

Question 5 (7 points)

A firm has two inputs (z_1 and z_2) and a production function $f(z_1, z_2)$ that is monotone.

- (a) Define an isoquant, either in words or mathematically.
- (b) Explain why two isoquants can never cross.

Question 6 (7 points)

- (a) Define “decreasing returns to scale”, either in words or mathematically.
- (b) Suppose a firm has one input and one output. The firm also has a convex cost function. Does this firm’s production function necessarily exhibit decreasing returns to scale?

Exercises (160 points)**7. Choice with Satiation (25 points)**

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = -\frac{1}{2}(10 - x_1)^2 - \frac{1}{2}(10 - x_2)^2$$

The agent faces prices $p_1 = 1$ and $p_2 = 1$ and has target utility \bar{u} . [Hint: you may find it useful to plot the indifference curves].

(a) What is the maximum utility the agent can attain? What is the minimum utility the agent can attain?

(a) Solve for the agent's Hicksian demand, $h(p_1, p_2, \bar{u})$, for attainable \bar{u} .

(b) Solve for the agent's expenditure function.

Space for Question 7.

8. Consumer Surplus (25 points)

An agent has utility $u(x_1, x_2) = (x_1^{-1} + x_2^{-1})^{-1}$ for goods x_1 and x_2 . The prices of the goods are p_1 and p_2 and that the agent has income m . One can show the agent's indirect utility function is given by

$$v = \frac{m}{(p_1^{1/2} + p_2^{1/2})^2}$$

- (a) Derive the agent's expenditure function.
- (b) Suppose that $p_1 = 1$, $p_2 = 1$ and $m = 40$. Calculate the utility of the agent.
- (c) Suppose the price of x_1 rises to $p_1 = 4$. Calculate the money needed to compensate the agent for this price increase.

Space for Question 8.

9. Slutsky Equation (30 points)

An agent has Cobb–Douglas utility

$$u(x_1, x_2) = x_1^{1/3} x_2^{1/3}$$

for goods x_1 and x_2 .

- (a) Derive the agent's Marshallian demand.
- (b) Derive the indirect utility of the agent, $v(p_1, p_2, m)$.
- (c) Derive the agent's Hicksian demand.
- (d) Verify the Slutsky equation, which states that:

$$\frac{\partial}{\partial p_1} x_1^*(p_1, p_2, m) = \frac{\partial}{\partial p_1} h_1(p_1, p_2, \bar{u}) - x_1^*(p_1, p_2, m) \frac{\partial}{\partial m} x_1^*(p_1, p_2, m)$$

Space for Question 9.

10. Partial Equilibrium (30 points)

There is an economy with 20 agents. Of these agents, ten have income $m = 10$ and ten have $m = 20$. Each agent has utility function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

over goods x_1 and x_2 . The price of good 2 equals 1. The price of good 1 is to be determined.

- (a) Derive each agent's demand curve for good 1.
- (b) Derive the market demand for good 1.

There are $J = 25$ firms who produce good 1. Each has one input (z) and a production function given by

$$q = (z - 1)^{1/2}$$

The cost of the input is $r = 1$.

- (c) Derive each firm's short run supply curve (assuming the firm cannot shut down).
- (d) Derive the market short run supply curve (assuming firms cannot shut down).
- (e) Find the market price for good 1. Show that, at this price, new entrants will wish to enter.
- (f) Find the long-run free-entry equilibrium price for good 1, assuming all potential entrants have the same production technology. In addition, find the output of each firm (q), the number of firms in the industry (J) and the total industry output (Q).

Space for Question 10.

11. Taxation (30 points)

There are two goods (x and y) and 5 agents. Each agent has utility function

$$u = 30x - \frac{1}{2}x^2 + y$$

and has income m . The price of good y is 1. The price of good x , denoted by p , is to be determined.

(a) Ignoring any boundary problems, calculate the demand of each agent for good x and the market demand for good x .

Suppose there are 10 firms that produce good x . Each has cost $c(q) = q^2/2$.

(b) Calculate the market supply function.

(c) Calculate the equilibrium price of x and the market output.

Suppose there is a 100% value added tax put on good x . This means that if, for example, the firm charges \$5 then the customer pays \$10.

(d) Draw a picture illustrating the tax change. Identify the change in consumer surplus, producer surplus, the tax revenue and the deadweight loss.

(e) What is the new equilibrium price? What price does the firm receive after the tax?

(f) What is the change in producer and consumer surplus? What is government revenue? What is the deadweight loss?

Space for Question 11.

12. General Equilibrium (30 points)

There are two consumers (A and B) and two goods (x and y). The agents both have utility functions

$$u^i(x, y) = \ln(x) + y$$

The prices of x and y are p_x and p_y , and are to be determined. The endowments of the two agents are $(\omega_x^A, \omega_y^A) = (5, 10)$ and $(\omega_x^B, \omega_y^B) = (15, 10)$

- (a) Solve for the competitive equilibrium price and allocations.
- (b) Solve for the contract curve (the set of Pareto efficient allocations).
- (c) Show the equilibrium allocation is on the contract curve.

Space for Question 12.

Space for Rough Work

Space for Rough Work

Space for Rough Work

Economics 11: Solutions to Final

11:30am–2:30pm, 8th December, 2008

Instructions: The test is closed book. Regular calculators are allowed; graphic calculators are not.

Short Questions (40 points)

Question 1 (6 points)

A firm faces linear input prices r_1 and r_2 and output price p .

- (a) Define the profit function, $\pi^*(p, r_1, r_2)$, either in words or mathematically.
- (b) Argue that $\pi^*(p, r_1, r_2)$ is convex in p .

Solution

- (a) The profit function is the maximal profit when faced with output price p and input prices r_1 and r_2 .
- (b) Fix $p = p'$ and solve for the optimal output $q' = q^*(p')$. Now suppose we fix the output and change p , yielding a pseudo-profit function $pq' - c(q')$ which is linear in p . Of course, as p rises the firm can increase her output, so the real cost function lies above this straight line and is therefore convex.

Question 2 (6 points)

Figure 1 illustrates the marginal costs and average costs for two cases. Does a cost function exist that generates these pictures, or is something wrong?

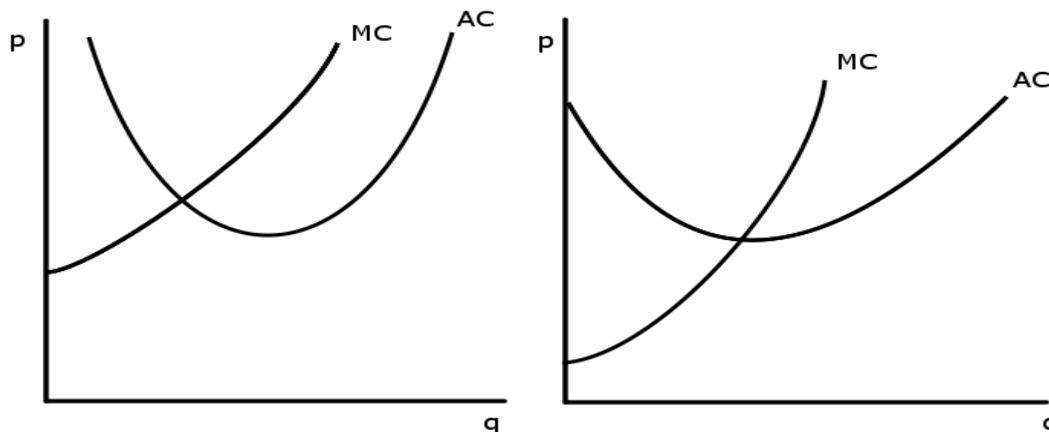


Figure 1: Cost Curves.

Solution

The first picture is impossible: the MC must intersect the AC at the lowest point.

The second picture is impossible: if there is a fixed cost then the AC for the first unit is infinity; if there is no fixed cost then $AC = MC$ for the first unit.

Question 3 (7 points)

An agent has preferences over two goods $(x_1, x_2) \in \mathbb{R}_+^2$. Suppose preferences are complete, transitive and continuous. Suppose they are also monotone and *strictly concave*, so that whenever $x \succ y$ then

$$x \succ tx + (1-t)y \quad \text{for } t \in (0, 1).$$

That is, the MRS is strictly increasing in x_1 along the indifference curve.

The agent has income m and faces positive prices p_1 and p_2 . Will the agent ever choose a bundle containing positive amount of both goods? Explain your answer.

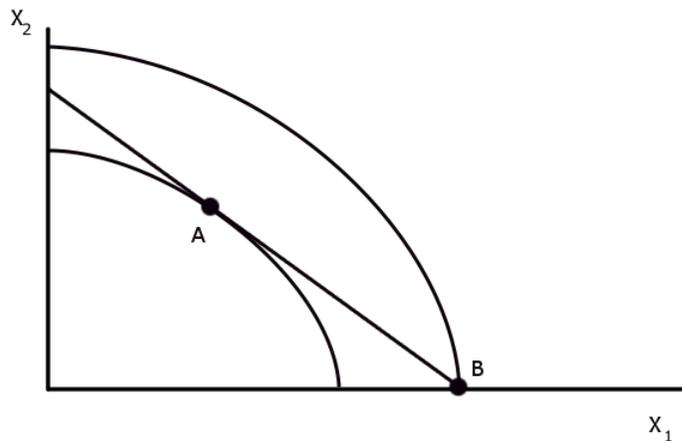


Figure 2: Concave Preferences.

Solution

No. When her preferences are strictly concave the agent prefers extremes to averages and will only consume one good. See figure 2. The tangency condition is satisfied at point A, but the agent prefers point B.

A formal proof is as follows. Suppose, by contradiction, that the agent chooses an interior bundle, z . Define x and y as the bundles that the agent can attain if she spent all her income on x_1 and x_2 , respectively. Without loss of generality, suppose $x \succcurlyeq y$. By strict concavity $x \succ z$, contradicting the assumption that the agent chooses z .

Note that we need preferences to strictly concave: Perfect substitutes are concave, but can lead to agents choosing bundles containing both goods.

Question 4 (7 points)

A good has market demand curve $X = 100 - p$. There are an unlimited number of potential firms with costs curves $c(q) = q + q^2$.

What is the long-run equilibrium price, allowing for the free entry of firms? How many firms will there be?

Solution

The average cost curve is $1 + q$. This is minimised at 1. Hence the long-run price will be 1. There will be infinitely many firms, each with a market share close to zero.

Question 5 (7 points)

A firm has two inputs (z_1 and z_2) and a production function $f(z_1, z_2)$ that is monotone.

- (a) Define an isoquant, either in words or mathematically.
- (b) Explain why two isoquants can never cross.

Solution

- (a) See Firm notes.
- (b) See argument for why indifference curves do not cross.

Question 6 (7 points)

- (a) Define “decreasing returns to scale”, either in words or mathematically.
- (b) Suppose a firm has one input and one output. The firm also has a convex cost function. Does this firm’s production function necessarily exhibit decreasing returns to scale?

Solution

- (a) A firm has decreasing returns to scale if doubling the inputs less than doubles the outputs.
- (a) Yes, the firm has decreasing returns. The cost function is convex so doubling the output more than doubles the cost. Since there is only one input, this means the inputs used must more than double.

Exercises (160 points)

7. Choice with Satiation (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = -\frac{1}{2}(10 - x_1)^2 - \frac{1}{2}(10 - x_2)^2$$

The agent faces prices $p_1 = 1$ and $p_2 = 1$ and has target utility \bar{u} . [Hint: you may find it useful to plot the indifference curves].

- What is the maximum utility the agent can attain?
- Solve for the agent's Hicksian demand, $h(p_1, p_2, \bar{u})$.
- Solve for the agent's expenditure function.

Solution

- The highest attainable utility is $\bar{u} = 0$.
- For $-100 \leq \bar{u} \leq 0$ we can minimise the Lagrangian

$$\mathcal{L} = x_1 + x_2 + \lambda[\bar{u} + \frac{1}{2}(10 - x_1)^2 + \frac{1}{2}(10 - x_2)^2]$$

The constraint binds at the optimum so the Lagrange multiplier will be positive. The FOCs are

$$\begin{aligned} 1 &= \lambda(10 - x_1) \\ 1 &= \lambda(10 - x_2) \end{aligned}$$

This yields $x_1 = x_2$. Substituting into the constraint, we have $-\bar{u} = (10 - x_1)^2$. This equation has two roots: $x_1 = 10 + (-\bar{u})^{1/2}$ and $x_1 = 10 - (-\bar{u})^{1/2}$. While these deliver the same utility,

the latter is clearly cheaper.¹ Hence the Hicksian demands are

$$h_1 = 10 - (-\bar{u})^{1/2} \quad \text{and} \quad h_2 = 10 - (-\bar{u})^{1/2}$$

(c) The expenditure function is given by $e(\bar{u}) = h_1 + h_2$. Substituting in,

$$\begin{aligned} e(\bar{u}) &= 0 && \text{for } \bar{u} < -100 \\ &= 20 - 2(-\bar{u})^{1/2} && \text{for } \bar{u} \in [-100, 0] \\ &= \infty && \text{for } \bar{u} > 0 \end{aligned}$$

8. Consumer Surplus (25 points)

An agent has utility $u(x_1, x_2) = (x_1^{-1} + x_2^{-1})^{-1}$ for goods x_1 and x_2 . The prices of the goods are p_1 and p_2 and that the agent has income m . One can show the agent's indirect utility function is given by

$$v = \frac{m}{(p_1^{1/2} + p_2^{1/2})^2}$$

- (a) Derive the agent's expenditure function.
- (b) Suppose that $p_1 = 1$, $p_2 = 1$ and $m = 40$. Calculate the utility of the agent.
- (c) Suppose the price of x_1 rises to $p_1 = 4$. Calculate the money needed to compensate the agent for this price increase.

Solution

(a) Inverting the indirect utility function, the agent's expenditure function is given by

$$e(p_1, p_2, \bar{u}) = \bar{u}(p_1^{1/2} + p_2^{1/2})^2$$

(b) Plugging in, the utility is $u = 10$.

¹In addition, under the first root, the Lagrange multiplier is negative which contradicts the Kuhn-Tucker conditions.

(c) The compensation is given by

$$CV = e(p'_1, p_2, \bar{u}) - e(p_1, p_2, \bar{u}) = 90 - 40 = 50$$

9. Slutsky Equation (30 points)

An agent has Cobb–Douglas utility

$$u(x_1, x_2) = x_1^{1/3} x_2^{1/3}$$

for goods x_1 and x_2 .

- (a) Derive the agent's Marshallian demand.
- (b) Derive the indirect utility of the agent, $v(p_1, p_2, m)$.
- (c) Derive the agent's Hicksian demand.
- (d) Verify the Slutsky equation, which states that:

$$\frac{\partial}{\partial p_1} x_1^*(p_1, p_2, m) = \frac{\partial}{\partial p_1} h_1(p_1, p_2, \bar{u}) - x_1^*(p_1, p_2, m) \frac{\partial}{\partial m} x_1^*(p_1, p_2, m)$$

Solution

(a) The agent maximises

$$\mathcal{L} = x_1^{1/3} x_2^{1/3} + \lambda[m - p_1 x_1 - p_2 x_2]$$

This yields FOCs

$$\begin{aligned} \frac{1}{3} x_1^{-2/3} x_2^{1/3} &= \lambda p_1 \\ \frac{1}{3} x_1^{1/3} x_2^{-2/3} &= \lambda p_2 \end{aligned}$$

Hence $x_1 p_1 = x_2 p_2$. Using the budget constraint we find

$$x_1^* = \frac{m}{2p_1} \quad \text{and} \quad x_2^* = \frac{m}{2p_2}$$

(b) The indirect utility of the agent is

$$v(p_1, p_2, m) = x_1^{1/3} x_2^{1/3} = \left(\frac{m}{2p_1}\right)^{1/3} \left(\frac{m}{2p_2}\right)^{1/3}$$

(b) The agent minimises

$$\mathcal{L} = p_1 x_1 + p_2 x_2 + \lambda[\bar{u} - x_1^{1/3} x_2^{1/3}]$$

This yields FOCs

$$\begin{aligned} \lambda \frac{1}{3} x_1^{-2/3} x_2^{1/3} &= p_1 \\ \lambda \frac{1}{3} x_1^{1/3} x_2^{-2/3} &= p_2 \end{aligned}$$

Hence $x_1 p_1 = x_2 p_2$. Using the constraint we find

$$h_1 = \left(\frac{p_2}{p_1}\right)^{1/2} (\bar{u})^{3/2} \quad \text{and} \quad h_2 = \left(\frac{p_1}{p_2}\right)^{1/2} (\bar{u})^{3/2}$$

(c) The LHS of the Slutsky equation is

$$\frac{\partial}{\partial p_1} x_1^*(p_1, p_2, m) = -\frac{m}{2} p_1^{-2}$$

The RHS of the Slutsky equation is

$$\begin{aligned} \frac{\partial}{\partial p_1} h_1(p_1, p_2, \bar{u}) - x_1^*(p_1, p_2, m) \frac{\partial}{\partial m} x_1^*(p_1, p_2, m) &= -\frac{1}{2} p_1^{-3/2} p_2^{1/2} (\bar{u})^{3/2} - \frac{1}{2} m p_1^{-1} \times \frac{1}{2} p_1^{-1} \\ &= -\frac{1}{2} p_1^{-3/2} p_2^{1/2} \times \frac{m}{2} p_1^{-1/2} p_2^{-1/2} - \frac{1}{4} m p_1^{-2} \\ &= -\frac{1}{4} m p_1^{-2} - \frac{1}{4} m p_1^{-2} \\ &= -\frac{1}{2} m p_1^{-2} \end{aligned}$$

as required.

10. Partial Equilibrium (30 points)

There is an economy with 20 agents. Of these agents, ten have income $m = 10$ and ten have $m = 20$. Each agent has utility function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

over goods x_1 and x_2 . The price of good 2 equals 1. The price of good 1 is to be determined.

(a) Derive each agent's demand curve for good 1.

(b) Derive the market demand for good 1.

There are $J = 25$ firms who produce good 1. Each has one input (z) and a production function given by

$$q = (z - 1)^{1/2}$$

The cost of the input is $r = 1$.

(c) Derive each firm's short run supply curve (assuming the firm cannot shut down).

(d) Derive the market short run supply curve (assuming firms cannot shut down).

(e) Find the market price for good 1. Show that, at this price, new entrants will wish to enter.

(f) Find the long-run free-entry equilibrium price for good 1, assuming all potential entrants have the same production technology. In addition, find the output of each firm (q), the number of firms in the industry (J) and the total industry output (Q).

Solution

(a) The agent maximises

$$\mathcal{L} = x_1^{2/3} x_2^{1/3} + \lambda[m - p_1 x_1 - p_2 x_2]$$

This yields FOCs

$$\begin{aligned} \frac{2}{3} x_1^{-1/3} x_2^{1/3} &= \lambda p_1 \\ \frac{1}{3} x_1^{2/3} x_2^{-2/3} &= \lambda p_2 \end{aligned}$$

Hence $x_1 p_1 = 2x_2 p_2$. Using the budget constraint we find

$$x_1^* = \frac{2m}{3p_1} \quad \text{and} \quad x_2^* = \frac{m}{3p_1}$$

(b) The market demand for good 1 is

$$X = 10 \frac{20}{3p_1} + 10 \frac{40}{3p_1} = \frac{200}{p_1}$$

(c) In order to produce q the firm needs inputs $z = q^2 + 1$. The cost function is

$$c(q) = rz = q^2 + 1$$

The short run supply curve is given by $p_1 = 2q$ or $q^* = p_1/2$.

(d) There are 25 firms, so the market supply is $Q = 25p_1/2$.

(e) Equating supply and demand,

$$\frac{200}{p_1} = \frac{25p_1}{2}$$

Rearranging, $p_1^2 = 16$ or $p_1 = 4$. Each firm produces $q = 2$. The profit is

$$\pi = pq - c(q) = 8 - 5 = 3$$

Hence entry is profitable.

(f) The firm's average cost is $AC = q + q^{-1}$. Differentiating, this is minimised at $q = 1$ when $AC = MC = 2$. Hence the long-run equilibrium price is $p = 2$, where each firm produces $q = p/2 = 1$. Market demand is $X = 100$, so there are $J = 100$ firms.

11. Taxation (30 points)

There are two goods (x and y) and 5 agents. Each agent has utility function

$$u = 30x - \frac{1}{2}x^2 + y$$

and has income m . The price of good y is 1. The price of good x , denoted by p , is to be determined.

(a) Ignoring any boundary problems, calculate the demand of each agent for good x , and the market demand for good x .

Suppose there are 10 firms that produce good x . Each has cost $c(q) = q^2/2$.

(b) Calculate the market supply function.

(c) Calculate the equilibrium price of x and the market output.

Suppose there is a 100% value added tax put on good x . This means that if, for example, the firm charges \$5 then the customer pays \$10.

(d) Draw a picture illustrating the tax change. Identify the change in consumer surplus, producer surplus, the tax revenue and the deadweight loss.

(e) What is the new equilibrium price? What price does the firm receive after the tax?

(f) What is the change in producer and consumer surplus? What is government revenue? What is the deadweight loss?

Solution

(a) Substituting the agent's budget constraint into her utility function, the agent maximises

$$30x - \frac{1}{2}x^2 + (m - px)$$

The FOC is $30 - x = p$. Hence the agent's demand is $x^* = 30 - p$. The market demand is $X = 150 - 5p$.

(b) Each firm maximises

$$pq - \frac{1}{2}q^2$$

Differentiating, this yields the supply function $q^* = p$. Market supply is $Q = 10p$.

(c) Equating supply and demand,

$$150 - 5p = 10p$$

Hence the market price is $p = 10$. The output is $Q = 100$.

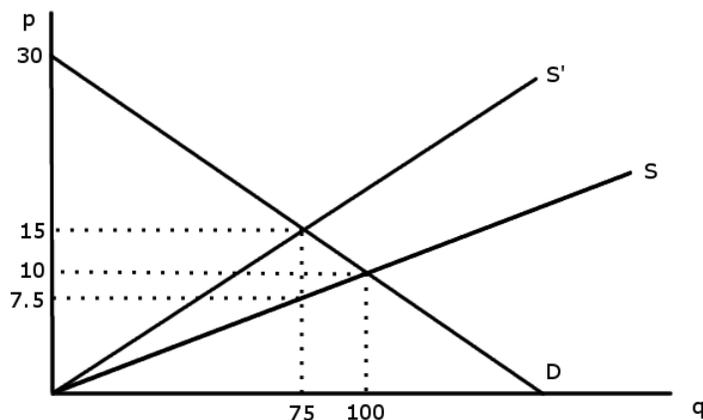


Figure 3: The effect of VAT.

(d) See figure 3.

(e) There is a 100% VAT. Hence at a price of p the firm receives $p_f = p/2$ and supplies $q^* = p_f = p/2$. The market supply is $Q = 5p$. Equating supply and demand,

$$150 - 5p = 5p$$

Hence the new market price is $p = 15$. The firm receives $p_f = 7.5$. The output is $Q = 75$.

(f) The old CS is $100 \times (30 - 10)/2 = 1000$. The new CS is $75 \times (30 - 15)/2 = 562.5$, yielding a difference of 437.5. Producer surplus falls by $75 \times 2.5 + 2.5 \times (100 - 75)/2 = 218.75$. Government revenue is $7.5 \times 75 = 562.5$. Hence the deadweight loss is 218.75.

12. General Equilibrium (30 points)

There are two consumers (A and B) and two goods (x and y). The agents both have utility functions

$$u^i(x, y) = \ln(x) + y$$

The prices of x and y are p_x and p_y , and are to be determined. The endowments of the two agents are $(\omega_x^A, \omega_y^A) = (5, 10)$ and $(\omega_x^B, \omega_y^B) = (15, 10)$

(a) Solve for the competitive equilibrium price and allocations.

(b) Solve for the contract curve (the set of Pareto efficient allocations).

(c) Show the equilibrium allocation is on the contract curve.

Solution

(a) An agent's budget constraint is $m = p_x x + p_y y$. Rearranging,

$$y = \frac{m}{p_y} - \frac{p_x}{p_y} x$$

The agent maximises

$$u(x, y) = \ln(x) + y = \ln(x) + \frac{m}{p_y} - \frac{p_x}{p_y} x$$

The FOC is

$$\frac{1}{x} = \frac{p_x}{p_y}$$

Hence we have

$$x^* = \frac{p_y}{p_x} \quad \text{and} \quad y^* = \frac{m}{p_y} - 1$$

where $m^A = 5p_x + 10p_y$ and $m^B = 15p_x + 10p_y$.

To find the competitive price, let's solve for market 1.

$$\frac{p_y}{p_x} + \frac{p_y}{p_x} = 20$$

Which implies that $p_y = 10p_x$. Normalising $p_x = 1$, we have $p_y = 10$. The agent's incomes are $m^A = 105$ and $m^B = 115$. The equilibrium allocations are $(x^A, y^A, x^B, y^B) = (10, 9.5, 10, 10.5)$.

(b) With quasilinear preferences, we have $MRS = 1/x$. The contract curve is thus given by

$$\frac{1}{x^A} = \frac{1}{x^B}$$

which implies $x^A = x^B$. This means that the contract curve is a vertical line. Intuitively preferences are quasilinear, so the indifference curves are vertical shifts of each other; the points of tangency thus occur along one vertical line.

(c) Clearly the competitive allocation satisfies $x^A = x^B$. It is therefore Pareto efficient.

Economics 11: Practice Final

December 4, 2008

Note: In order to give you extra practice on production and equilibrium, this practice final is skewed towards topics covered after the midterm. The actual final will test all the material.

Part I: Short Questions

Question 1

True or false? The allocation of tax burden between consumers and producers depends on whether the tax is imposed on consumers or producers. If true, explain why. If false, what does it depend on?

Question 2

Using a graph, discuss the welfare effects of an import tariff. Who gains, who loses from the tariff, and what are the net welfare effects?

Question 3

What is the supply function for a perfectly competitive industry with constant returns to scale? What will be the equilibrium price and equilibrium profits for this industry?

Question 4

State the First Welfare Theorem, and illustrate the result using a graph for a simple two-agent, two-good endowment economy.

Question 5

Assume utility is quasilinear in x and the demand is given by $x = 10 - p$. Find the consumer surplus when $p = 5$.

Question 6

Show graphically the income and substitution effects, when the price of X decreases (assume X is a normal good).

Part II: Exercises

Question 1

An agent lives for 2 periods. In period 1 her income is 500. In period 2 her income is 0. The interest rate is 100%. The agent's utility is given by

$$u(x_1, x_2) = x_1 x_2$$

where x_1 is consumption in period 1 and x_2 is consumption in period 2.

Solve for the agent's optimal consumption.

Question 2

A firm has production function $f(z_1, z_2) = z_1^{1/4} z_2^{1/4}$. The prices of the inputs are r_1 and r_2 .

- Find the firm's demand for inputs (as a function of output and the input prices).
- Find the cost function of the firm.
- For $r_1 = 4$ and $r_2 = 1$, find the firm's supply function.

Question 3

Wheat is produced under perfectly competitive conditions. Individual wheat farmers have U-shaped, long-run average cost curves that reach a minimum average cost of \$3 per bushel when 100 bushels are produced.

- If market demand curve for wheat is given by $Q = 2,600 - 200p$. What is the long-run equilibrium price of wheat? How much total wheat will be demanded and how many wheat firms will there be?

b) Suppose demand curve shifts outward to $Q = 3,200 - 200p$. If farmers cannot adjust their output in the short run, what will market price be with this new demand curve? What will the profit of typical firm be?

c) Given the new demand curve described in part (b), what is the new long-run equilibrium price, quantity produced and equilibrium number of farmers?

Question 4

Consider a 2×2 exchange economy with two individuals (A and B) and two goods (x and y). A's preferences are given by

$$u_A = x_A^{1/5} y_A^{4/5}$$

B's preferences are given by

$$u_B = x_B^{4/5} y_B^{1/5}$$

The endowments are $\omega^A = (8, 12)$ and $\omega^B = (12, 8)$.

a) Find the equilibrium prices.

b) Find the equilibrium allocation.

c) Derive the equation of the contract curve.

d) Sketch an Edgeworth box showing endowments, competitive equilibrium prices and consumption choices, and indifference curves through the endowments and through the equilibrium consumption choices

Question 5

Consider a 2×2 exchange economy with two individuals (A and B) and two goods (x and y). Each agent has utility function

$$u = x^{1/2} + y^{1/2}$$

The endowments of the agents are $\omega_A = (40, 0)$ and $\omega_B = (0, 10)$.

Find the equilibrium prices and allocations.

Economics 11: Solutions to Practice Final

December 4, 2008

Note: In order to give you extra practice on production and equilibrium, this practice final is skewed towards topics covered after the midterm. The actual final will test all the material.

Part I: Short Questions

Question 1

True or false? The allocation of tax burden between consumers and producers depends on whether the tax is imposed on consumers or producers. If true, explain why. If false, what does it depend on?

Solution

False. Who bears most of the tax burden, as well as the size of the social loss, depends on the elasticities of demand and supply. The analysis is independent of whether tax is levied on producers or consumers.

Question 2

Using a graph, discuss the welfare effects of an import tariff. Who gains, who loses from the tariff, and what are the net welfare effects?

Solution

See lecture on Tuesday 2nd Dec.

Question 3

What is the supply function for a perfectly competitive industry with constant returns to scale? What will be the equilibrium price and equilibrium profits for this industry?

Solution

With constant returns to scale, the marginal cost is constant, hence the firm, and industry, supply curve is given by a flat horizontal line at $P = MC$. The equilibrium price will equal the marginal cost of production, and firm and industry profits are zero.

Question 4

State the First Welfare Theorem, and illustrate the result using a graph for a simple two-agent, two-good endowment economy.

Solution

The First Welfare Theorem states that every competitive equilibrium is Pareto Efficient. Graphically, this implies that in an Edgeworth box, the equilibrium allocation must be on the contract curve.

Question 5

Assume utility is quasilinear in x and the demand is given by $x = 10 - p$. Find the consumer surplus when $p = 5$.

Solution

The consumer surplus equals the area under the demand curve. The consumer's willingness to pay for the first unit is $p = 10$. Hence we have

$$CS = \int_5^{10} [10 - p]dp = [10p - p^2/2]_5^{10} = 50 - 37\frac{1}{2} = 12\frac{1}{2}$$

Alternatively, we can use the geometry of the triangle. At $p = 5$ the agent buys $q = 5$. Hence The area of the triangle is

$$CS = \frac{1}{2} \text{base} \times \text{height} = \frac{1}{2} 5 \times 5 = 12\frac{1}{2}$$

Question 6

Show graphically the income and substitution effects, when the price of X decreases (assume X is a normal good).

Solution

See EMP notes.

Part II: Exercises

Question 1

An agent lives for 2 periods. In period 1 her income is 500. In period 2 her income is 0. The interest rate is 100%. The agent's utility is given by

$$u(x_1, x_2) = x_1 x_2$$

where x_1 is consumption in period 1 and x_2 is consumption in period 2.

Solve for the agent's optimal consumption.

Solution

In general, the agent's budget constraint is

$$x_1 + \frac{1}{1+r}x_2 = m_1 + \frac{1}{1+r}m_2$$

This becomes,

$$x_1 + \frac{1}{2}x_2 = 500$$

The agent therefore maximises

$$\mathcal{L} = x_1 x_2 + \lambda [500 - x_1 - \frac{1}{2}x_2]$$

The FOCs are

$$\begin{aligned}x_2 &= \lambda \\x_1 &= \lambda/2\end{aligned}$$

Hence we have $2x_1 = x_2$. Using the budget constraint,

$$x_1^* = 250 \quad \text{and} \quad x_2^* = 500$$

Question 2

A firm has production function $f(z_1, z_2) = z_1^{1/4} z_2^{1/4}$. The prices of the inputs are r_1 and r_2 .

- Find the firm's demand for inputs (as a function of output and the input prices).
- Find the cost function of the firm.
- For $r_1 = 4$ and $r_2 = 1$, find the firm's supply function.

Solution

- (a) The cost minimisation problem is

$$\min_{z_1, z_2} \mathcal{L} = r_1 z_1 + r_2 z_2 + \lambda [q - z_1^{1/4} z_2^{1/4}]$$

The FOCs yields $r_1 z_1 = r_2 z_2$. Substituting into the constraint,

$$z_1^* = q^2 \left(\frac{r_2}{r_1} \right)^{1/2} \quad \text{and} \quad z_2^* = q^2 \left(\frac{r_1}{r_2} \right)^{1/2}$$

- (b) The cost function is

$$c(q; r_1, r_2) = r_1 z_1^* + r_2 z_2^* = 2q^2 (r_1 r_2)^{1/2}$$

- (c) For $r_1 = 4$ and $r_2 = 1$ the cost function is $c(q) = 4q^2$. The marginal cost is $MC(q) = 8q$. The firm's profit maximisation problem implies $p = MC$, so that $p = 8q$. Inverting, the supply function is $q^*(p) = p/8$.

Question 3

Wheat is produced under perfectly competitive conditions. Individual wheat farmers have U-shaped, long-run average cost curves that reach a minimum average cost of \$3 per bushel when 100 bushels are produced.

- a) If market demand curve for wheat is given by $Q = 2,600 - 200p$. What is the long-run equilibrium price of wheat? How much total wheat will be demanded and how many wheat firms will there be?
- b) Suppose demand curve shifts outward to $Q = 3,200 - 200p$. If farmers cannot adjust their output in the short run, what will market price be with this new demand curve? What will the profit of typical firm be?
- c) Given the new demand curve described in part (b), what is the new long-run equilibrium price, quantity produced and equilibrium number of farmers?

Solution

(a) In the long-run free-entry equilibrium, $p = 3$. Demand is $Q = 2000$. Hence there are $J = 20$ firms.

(b) Suppose $Q = 2000$. The price is $p = 6$. Hence firm's make profits

$$\pi = 6 \times 100 - 3 \times 100 = 300$$

(c) The new long-run price is $p = 3$. Demand is $Q = 2600$. Hence there are $J = 26$ firms.

Question 4

Consider a 2×2 exchange economy with two individuals (A and B) and two goods (x and y). A's preferences are given by

$$u_A = x_A^{1/5} y_A^{4/5}$$

B's preferences are given by

$$u_B = x_B^{4/5} y_B^{1/5}$$

The endowments are $\omega^A = (8, 12)$ and $\omega^B = (12, 8)$.

- a) Find the equilibrium prices.
- b) Find the equilibrium allocation.

c) Derive the equation of the contract curve.

d) Sketch an Edgeworth box showing endowments, competitive equilibrium prices and consumption choices, and indifference curves through the endowments and through the equilibrium consumption choices

Solution

(a) Under Cobb–Douglas demands, A's demand are

$$x_A = \frac{m_A}{5p_x} \quad \text{and} \quad y_A = \frac{4m_A}{5p_x}$$

where $m_A = p_x\omega_{x,A} + p_y\omega_{y,A}$. B's demands are

$$x_B = \frac{4m_A}{5p_x} \quad \text{and} \quad y_B = \frac{m_A}{5p_x}$$

where $m_B = p_x\omega_x^B + p_y\omega_y^B$.

Net demand for x is

$$x_A + x_B - \omega_{x,A} - \omega_{x,B} = \frac{m_A}{5p_x} + \frac{4m_A}{5p_x} - \omega_x^A - \omega_x^B$$

Setting this equal to zero yields the price ratio

$$\frac{p_x}{p_y} = \frac{\omega_y^A + 4\omega_x^B}{4\omega_x^A + \omega_y^B} = 1$$

(b) The equilibrium allocation for A is

$$x_A = \frac{1}{5} \left[\omega_x^A + \frac{p_y}{p_x} \omega_y^A \right] \quad \text{and} \quad y_A = \frac{4}{5} \left[\frac{p_x}{p_y} \omega_x^A + \omega_y^A \right]$$

The equilibrium allocation for B is

$$x_B = \frac{4}{5} \left[\omega_x^B + \frac{p_y}{p_x} \omega_y^B \right] \quad \text{and} \quad y_B = \frac{1}{5} \left[\frac{p_x}{p_y} \omega_x^B + \omega_y^B \right]$$

Plugging in, $(x_A, y_A, x_B, y_B) = (4, 16, 16, 4)$.

(c) The contract curve is

$$\frac{y_A}{4x_A} = \frac{4y_B}{x_A} = \frac{4(20 - y_A)}{20 - x_A}$$

(d) See the lecture notes.

Question 5

Consider a 2×2 exchange economy with two individuals (A and B) and two goods (x and y). Each agent has utility function

$$u = x^{1/2} + y^{1/2}$$

The endowments of the agents are $\omega_A = (40, 0)$ and $\omega_B = (0, 10)$.

Find the equilibrium prices and allocations.

Solution

Preferences are CES. As in Practice Problems 4, the demands are given by

$$x_i = \frac{m^i}{p_x} \frac{p_y}{p_x + p_y} \quad \text{and} \quad y_i = \frac{m^i}{p_y} \frac{p_x}{p_x + p_y}$$

The incomes of the two agents are $m^A = 40p_x$ and $m^B = 10p_y$. Market clearing for good x implies

$$\frac{40p_x}{p_x} \frac{p_y}{p_x + p_y} + \frac{10p_y}{p_x} \frac{p_y}{p_x + p_y} = 40$$

Multiplying by $p_x(p_x + p_y)$ and dividing by 10,

$$4p_x p_y + p_y^2 = 4p_x(p_x + p_y)$$

Simplifying, $p_y = 2p_x$. Intuitively, good y is more expensive since it is rarer.

Normalise $p_x = 1$, so that $p_y = 2$. Then $m^A = 40$ and $m^B = 20$. Using the demand functions, allocations are $(x_A, y_A, x_B, y_B) = (80/3, 20/3, 40/3, 10/3)$.

Economics 11: Midterm

Instructions: The test is closed book/notes. Calculators are allowed. Please write your answers on this sheet. There are 100 points.

Name:

UCLA ID:

TA:

Question	Score
Questions 1–5	
Question 6	
Question 7	
Question 8	
Total	

Short Questions (25 points)

Question 1

There are two goods, x_1 and x_2 . Preferences are *convex* and *monotone*. Are the following statements true or false? Explain your answer.

- (a) Indifference curves cannot cross.
- (b) There can be a bliss point, where utility is maximised.
- (c) Indifference curves have no kinks.

Question 2

Marshallian demand $x_i^*(p_1, p_2, m)$ for good i is homogenous of degree 0 in (p_1, p_2, m) . Explain.

Question 3

What is an inferior good? What is a Giffen good? What is the relationship between them?

Question 4

The expenditure function $e(p_1, p_2, \bar{u})$ and indirect utility function $v(p_1, p_2, m)$ are related by the following equation:

$$e(p_1, p_2, v(p_1, p_2, m)) = m.$$

What is the intuition behind this equation?

Question 5

There is one good, $x \geq 0$. Two agents, A and B , have utility functions $u_A(x) = x$ and $u_B(x) = 2x$. Can we say that one unit of the good makes B happier than A ?

6. Basic Consumer Choice (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^3 x_2$$

The prices of the goods are p_1 and p_2 . The consumer has income m .

(a) Calculate the agent's optimal demand. [Note: this problem is well behaved in that the FOCs are sufficient.]

(b) Calculate the agent's indirect utility function.

(c) Roy's identity for good 1 states that

$$\frac{\partial v(p_1, p_2, m)}{\partial p_1} = -x_1^*(p_1, p_2, m) \frac{\partial v(p_1, p_2, m)}{\partial m}$$

Verify this equation (you need not verify it for good 2).

(d) Provide an intuition for Roy's identity.

Space for Question 6.

7. Hicksian Demand (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = -x_1^{-1/2} - x_2^{-1/2}$$

The prices of the goods are $p_1 = 1$ and $p_2 = 1$. The target utility is \bar{u} (note, this will be a negative number).

- (a) Derive the agent's Hicksian demands, as a function of \bar{u} . [Note: this problem is well behaved in that the FOCs are sufficient.]
- (b) Derive the agent's expenditure function, as a function of \bar{u} .

Space for Question 7.

8. New Utility Function (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u = \min\{x_1 + 2x_2, 2x_1 + x_2\}$$

- (a) Draw a typical indifference curve for the agent.
- (b) What is the MRS when $x_1 > x_2$? What is the MRS when $x_1 < x_2$?
- (c) The agent has income $m = 10$. Prices are $(p_1, p_2) = (1, 1)$. Derive Marshallian demand (x_1^*, x_2^*) .
- (d) The agent has income $m = 10$. Prices are $(p_1, p_2) = (1, 3)$. Derive Marshallian demand (x_1^*, x_2^*) .

Space for Question 8.

Space for Rough Work (Do not write your answers here)

Space for Rough Work (Do not write your answers here)

Economics 11: Solutions to Midterm

Instructions: The test is closed book. Calculators are allowed.

Short Questions (25 points)

Question 1

There are two goods, x_1 and x_2 . Preferences are *convex* and *monotone*. Are the following statements true or false? Explain your answer.

- (a) Indifference curves cannot cross.
- (b) There can be a bliss point, where utility is maximised.
- (c) Indifference curves have no kinks.

Solution

- (a) True. If they crossed then what happens at the crossing point?
- (b) False. This violates monotonicity.
- (c) False. For example, suppose $u = \min\{x_1 + 2x_2, 2x_1 + x_2\}$. [Perfect complements doesn't quite satisfy monotonicity, but it's a good enough answer].

Question 2

Marshallian demand $x_i^*(p_1, p_2, m)$ for good i is homogenous of degree 0 in (p_1, p_2, m) . Explain.

Solution

If we double all prices and income then the budget set is unaffected. Hence the optimal demand is unaffected.

Question 3

What is an inferior good? What is a Giffen good? What is the relationship between them?

Solution

With an inferior good, an increase in income lowers demand.

With a Giffen good, an increase in price raises demand.

A Giffen good is a special case of an inferior good. It is a super-inferior good, where the income effect outweighs the substitution effect.

Question 4

The expenditure function $e(p_1, p_2, \bar{u})$ and indirect utility function $v(p_1, p_2, m)$ are related by the following equation:

$$e(p_1, p_2, v(p_1, p_2, m)) = m.$$

What is the intuition behind this equation?

Solution

Fix prices (p_1, p_2) . If we start with income m , then $v(p_1, p_2, m)$ is the maximum utility. The equation says that the cheapest way to attain utility $v(p_1, p_2, m)$ is by spending m dollars.

Intuitively, if the agent could achieve $v(p_1, p_2, m)$ with less than m dollars, then she could have obtained more utility than $v(p_1, p_2, m)$ in the initial utility maximisation problem.

Question 5

There is one good, $x \geq 0$. Two agents, A and B , have utility functions $u_A(x) = x$ and $u_B(x) = 2x$. Can we say that one unit of the good makes B happier than A?

Solution

You cannot say. Utility is an ordinal measure, and is therefore incomparable across agents.

6. Basic Consumer Choice (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^3 x_2$$

The prices of the goods are p_1 and p_2 . The consumer has income m .

(a) Calculate the agent's optimal demand. [Note: this problem is well behaved in that the FOCs are sufficient.]

(b) Calculate the agent's indirect utility function.

(c) Roy's identity for good 1 states that

$$\frac{\partial v(p_1, p_2, m)}{\partial p_1} = -x_1^*(p_1, p_2, m) \frac{\partial v(p_1, p_2, m)}{\partial m}$$

Verify this equation (you need not verify it for good 2).

(d) Provide an intuition for Roy's identity.

Solution

(a) The Lagrangian is

$$\mathcal{L} = x_1^3 x_2 + \lambda[m - p_1 x_1 - p_2 x_2]$$

The FOCs are

$$\begin{aligned} 3x_1^2 x_2 &= \lambda p_1 \\ x_1^3 &= \lambda p_2 \end{aligned}$$

Rearranging, $3p_2 x_2 = p_1 x_1$. Using the budget constraint, we find that

$$x_1^* = \frac{3m}{4p_1} \quad \text{and} \quad x_2^* = \frac{1m}{4p_2}$$

(b) The indirect utility function is

$$v(p_1, p_2, m) = (x_1^*)^3 (x_2^*) = \frac{27}{256} \frac{m^4}{p_1^3 p_2}$$

(c) The LHS of Roy's identity is

$$\frac{\partial v(p_1, p_2, m)}{\partial p_i} = -\frac{81}{256} \frac{m^4}{p_2^3 p_1^4}$$

The RHS of Roy's identity is

$$-x_i^*(p_1, p_2, m) \frac{\partial v(p_1, p_2, m)}{\partial m} = -\frac{3}{4} \frac{m}{p_1} \times \frac{108}{256} \frac{m^3}{p_1^3 p_2}$$

which is the same as the LHS.

(d) Suppose p_1 rises by Δp_1 . Holding consumption fixed, income falls by $x_1^* \Delta p_1$, so utility falls by

$$x_1^*(p_1, p_2, m) \frac{\partial v(p_1, p_2, m)}{\partial m} \Delta p_1.$$

as on the RHS of Roy's identity. There is a second effect: the change in prices leads to a rebalancing of consumption. However, since the original choice was optimal, this rebalancing effect is of second order and can be ignored.

7. Hicksian Demand (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = -x_1^{-1/2} - x_2^{-1/2}$$

The prices of the goods are $p_1 = 1$ and $p_2 = 1$. The target utility is \bar{u} (note, this will be a negative number).

(a) Derive the agent's Hicksian demands, as a function of \bar{u} . [Note: this problem is well behaved in that the FOCs are sufficient.]

(b) Derive the agent's expenditure function, as a function of \bar{u} .

Solution

(a) The Lagrangian is

$$\mathcal{L} = x_1 + x_2 + \lambda[\bar{u} + x_1^{-1/2} + x_2^{-1/2}]$$

The FOCs are

$$1 = \frac{\lambda}{2}x_1^{-3/2} \quad (1)$$

$$1 = \frac{\lambda}{2}x_2^{-3/2} \quad (2)$$

Rearranging, we find that $x_1 = x_2$.

At the optimum, we know that $-\bar{u} = x_1^{-1/2} + x_2^{-1/2}$. Using the fact that $x_1 = x_2$, we obtain

$$h_1 = h_2 = \frac{4}{\bar{u}^2}$$

(b) The expenditure function is

$$e = h_1 + h_2 = \frac{8}{\bar{u}^2}$$

8. New Utility Function (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u = \min\{x_1 + 2x_2, 2x_1 + x_2\}$$

- (a) Draw a typical indifference curve for the agent.
- (b) What is the MRS when $x_1 > x_2$? What is the MRS when $x_1 < x_2$?
- (c) The agent has income $m = 10$. Prices are $(p_1, p_2) = (1, 1)$. Derive Marshallian demand (x_1^*, x_2^*) .
- (d) The agent has income $m = 10$. Prices are $(p_1, p_2) = (1, 3)$. Derive Marshallian demand (x_1^*, x_2^*) .

Solution

- (a) This is a mix of perfect substitutes and perfect complements. See Figure 1.
- (b) When $x_1 > x_2$ then $2x_1 + x_2 > x_1 + 2x_2$. Hence utility equals $x_1 + 2x_2$ and the MRS is $1/2$.

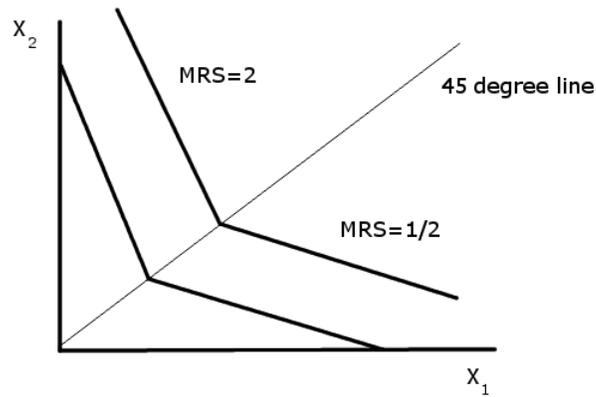


Figure 1: **Typical Indifference Curves.**

When $x_1 < x_2$ then $2x_1 + x_2 < x_1 + 2x_2$. Hence utility equals $2x_1 + x_2$ and the MRS is 2.

(c) When $(p_1, p_2) = (1, 1)$ the price ratio is 1 and the consumer chooses to purchase at the kink. Hence she buys the same amount of both goods and $(x_1^*, x_2^*) = (5, 5)$.

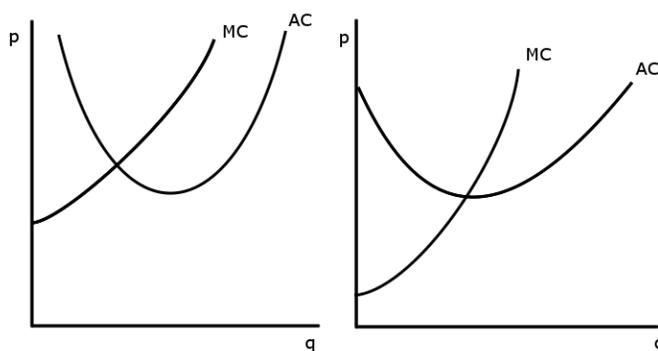
(d) When $(p_1, p_2) = (1, 3)$ the price ratio is $1/3$ and the consumer chooses to purchase only good 1. Hence she buys $(x_1^*, x_2^*) = (10, 0)$.

Economics 11: Practice Second Midterm

Short Questions

Question 1

The following figure illustrates the marginal costs and average costs for two cases. Does a cost function exist that generates these pictures, or is something wrong?



Question 2

An agent has preferences over two goods $(x_1, x_2) \in \mathbb{R}_+^2$. Suppose preferences are complete, transitive and continuous. Suppose they are also monotone and *strictly concave*, so that whenever $x \succ y$ then

$$x \succ tx + (1 - t)y \quad \text{for } t \in (0, 1).$$

That is, the MRS is strictly increasing in x_1 along the indifference curve.

The agent has income m and faces positive prices p_1 and p_2 . Will the agent ever choose a bundle containing positive amount of both goods? Explain your answer.

Question 3

(a) Define “decreasing returns to scale”, either in words or mathematically.

(b) Suppose a firm has one input and one output. The firm also has a convex cost function. Does this firm’s production function necessarily exhibit decreasing returns to scale?

Question 4

A good has market demand curve $X = 100 - p$. There are an unlimited number of potential firms with costs curves $c(q) = q + q^2$.

What is the long-run equilibrium price, allowing for the free entry of firms? How many firms will there be?

5. Consumer Surplus with CES utility

An agent has utility $u(x_1, x_2) = x_1^{1/2} + x_2^{1/2}$, income $m = 4$ and faces prices $p_1 = 2$ and $p_2 = 2$. The indirect utility function is given by

$$v(p_1, p_2, m) = \left[m \frac{p_1 + p_2}{p_1 p_2} \right]^{1/2}.$$

Suppose the price of good 1 rises to $p'_1 = 3$. Calculate the increase in the agent's income required to compensate her for this price rise.

6. Firm's Problem with CES Production

A firm has production function

$$f(z_1, z_2) = z_1^{1/2} + z_2^{1/2}$$

The firm's inputs cost r_1 and r_2 . The output price is p .

(a) Suppose $z_2 = 1$, and assume $q \geq 1$ to avoid boundary problems. Derive the firm's short-run cost and short-run marginal cost.

(b) Calculate the long-run cost function, when both factors are flexible.

7. Profit Maximisation

A firm has cost curve

$$c(q) = 100 + q^2$$

The firm can also shut down and make profits $\pi = 0$.

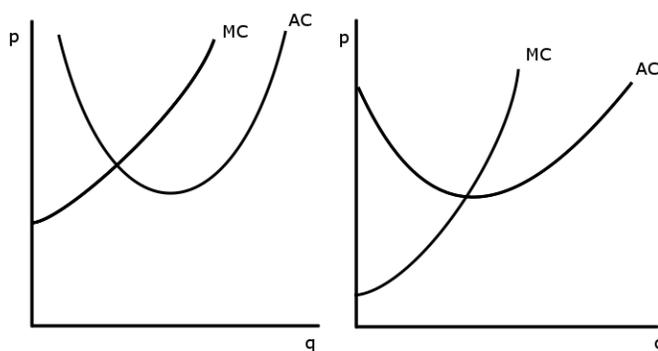
- (a) Suppose the firm faces price $p = 30$. What is the profit maximising quantity? What is the maximal profit? Will the firm shut down?
- (b) Suppose $p = 10$. What is the profit maximising quantity? What is the maximal profit? Will the firm shut down?
- (c) For which price levels will the firm shut down?
- (d) Draw the supply function $q^*(p)$ as a function of the output price p .

Economics 11: Solutions to Practice Second Midterm

Short Questions

Question 1

The following figure illustrates the marginal costs and average costs for two cases. Does a cost function exist that generates these pictures, or is something wrong?



Solution

The first picture is impossible: the MC must intersect the AC at the lowest point.

The second picture is impossible: if there is a fixed cost then the AC for the first unit is infinity; if there is no fixed cost then $AC = MC$ for the first unit.

Question 2

An agent has preferences over two goods $(x_1, x_2) \in \mathbb{R}_+^2$. Suppose preferences are complete, transitive and continuous. Suppose they are also monotone and *strictly concave*, so that whenever $x \succ y$ then

$$x \succ tx + (1 - t)y \quad \text{for } t \in (0, 1).$$

That is, the MRS is strictly increasing in x_1 along the indifference curve.

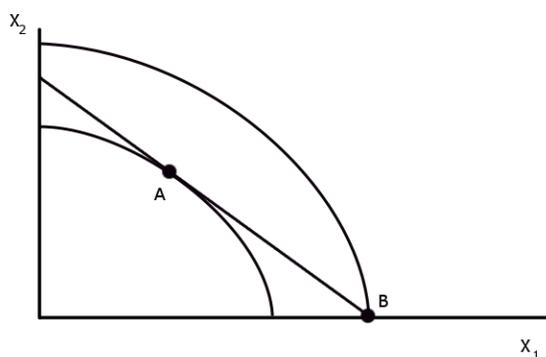
The agent has income m and faces positive prices p_1 and p_2 . Will the agent ever choose a bundle containing positive amount of both goods? Explain your answer.

Solution

No. When her preferences are strictly concave the agent prefers extremes to averages and will only consume one good. See the figure below. The tangency condition is satisfied at point A, but the agent prefers point B.

A formal proof is as follows. Suppose, by contradiction, that the agent chooses an interior bundle, z . Define x and y as the bundles that the agent can attain if she spent all her income on x_1 and x_2 , respectively. Without loss of generality, suppose $x \succcurlyeq y$. By strict concavity $x \succ z$, contradicting the assumption that the agent chooses z .

Note that we need preferences to strictly concave: Perfect substitutes are concave, but can lead to agents choosing bundles containing both goods.



Question 3

- (a) Define “decreasing returns to scale”, either in words or mathematically.
- (b) Suppose a firm has one input and one output. The firm also has a convex cost function. Does this firm’s production function necessarily exhibit decreasing returns to scale?

Solution

- (a) A firm has decreasing returns to scale if doubling the inputs less than doubles the outputs.
- (a) Yes, the firm has decreasing returns. The cost function is convex so doubling the output more than doubles the cost. Since there is only one input, this means the inputs used must more than double.

Question 4

A good has market demand curve $X = 100 - p$. There are an unlimited number of potential firms with costs curves $c(q) = q + q^2$.

What is the long-run equilibrium price, allowing for the free entry of firms? How many firms will there be?

Solution

The average cost curve is $1 + q$. This is minimised at 1. Hence the long-run price will be 1. There will be infinitely many firms, each with a market share close to zero.

5. Consumer Surplus with CES utility

An agent has utility $u(x_1, x_2) = x_1^{1/2} + x_2^{1/2}$, income $m = 4$ and faces prices $p_1 = 2$ and $p_2 = 2$. The indirect utility function is given by

$$v(p_1, p_2, m) = \left[m \frac{p_1 + p_2}{p_1 p_2} \right]^{1/2}.$$

Suppose the price of good 1 rises to $p'_1 = 3$. Calculate the increase in the agent's income required to compensate her for this price rise.

Solution

Plugging in the parameters, we see $v(p_1, p_2, m) = 2$. Inverting the indirect utility function we obtain the expenditure function,

$$e(p_1, p_2, \bar{u}) = \bar{u}^2 \frac{p_1 p_2}{p_1 + p_2}$$

Letting, $\bar{u} = 2$, the compensating variation is

$$CV = e(p'_1, p_2, \bar{u}) - e(p_1, p_2, \bar{u}) = \bar{u}^2 \frac{p'_1 p_2}{p'_1 + p_2} - \bar{u}^2 \frac{p_1 p_2}{p_1 + p_2} = \frac{4}{5}$$

6. Firm's Problem with CES Production

A firm has production function

$$f(z_1, z_2) = z_1^{1/2} + z_2^{1/2}$$

The firm's inputs cost r_1 and r_2 . The output price is p .

(a) Suppose $z_2 = 1$, and assume $q \geq 1$ to avoid boundary problems. Derive the firm's short-run cost and short-run marginal cost.

(b) Calculate the long-run cost function, when both factors are flexible.

Solution

(a) The firm's short run production function is

$$f(z_1, z_2) = z_1^{1/2} + 1$$

Hence the inputs needed to produce q are given by

$$z_1^* = (q - 1)^2$$

where we assumed that $q \geq 1$. The cost function is given by

$$SRTC(q) = r_1 z_1 + r_2 z_2 = r_1 (q - 1)^2 + r_2$$

The marginal cost is

$$SRMC(q) = 2r_1(q - 1)$$

(b) The firm minimises the Lagrangian

$$\mathcal{L} = r_1 z_1 + r_2 z_2 + \lambda [q - z_1^{1/2} - z_2^{1/2}]$$

The FOCs are

$$r_1 = \lambda \frac{1}{2} z_1^{-1/2}$$
$$r_2 = \lambda \frac{1}{2} z_2^{-1/2}$$

Rearranging, we have

$$\frac{r_1^2}{r_2^2} = \frac{z_2}{z_1}$$

Substituting into the production function, we obtain

$$z_1^* = \left(q \frac{r_2}{r_1 + r_2} \right)^2 \quad \text{and} \quad z_2^* = \left(q \frac{r_1}{r_1 + r_2} \right)^2$$

The cost function is given by

$$c(q) = r_1 z_1^* + r_2 z_2^* = \frac{r_1 r_2}{r_1 + r_2} q^2$$

[Note the similarity between this and the expenditure function used in the first question.]

7. Profit Maximisation

A firm has cost curve

$$c(q) = 100 + q^2$$

The firm can also shut down and make profits $\pi = 0$.

(a) Suppose the firm faces price $p = 30$. What is the profit maximising quantity? What is the maximal profit? Will the firm shut down?

(b) Suppose $p = 10$. What is the profit maximising quantity? What is the maximal profit? Will the firm shut down?

(c) For which price levels will the firm shut down?

(d) Draw the supply function $q^*(p)$ as a function of the output price p .

Solution

The firm maximises

$$\pi = pq - 100 - q^2$$

Assuming an internal optimum, the FOC implies

$$q^* = \frac{p}{2}$$

Maximal profits are

$$\pi^* = pq^* - 100 - (q^*)^2 = \frac{p^2}{4} - 100$$

Recall, the firm can also shut down and make $\pi = 0$.

- (a) If $p = 30$, then $q^* = 15$ and $\pi^* = 125$. This is positive, so the firm should not shut down.
- (b) If $p = 10$, then the internal optimum yields $q^* = 5$ and $\pi^* = -75$. Hence the firm is better off by shutting down.
- (c) Setting $\pi^* = 0$, we find that $p = 20$. Hence the firm shuts down if $p < 20$, operates if $p > 20$, and is indifferent at $p = 20$.
- (d) The supply function is $q^*(p) = 0$ for $p \leq 20$ and $q^*(p) = p/2$ for $p \geq 20$.

Economics 11: Second Midterm

Instructions: The test is closed book/notes. Calculators are allowed. Please write your answers on this sheet. There are 100 points.

Name:

UCLA ID:

TA:

Question	Score
Questions 1–4	
Question 5	
Question 6	
Question 7	
Total	

Short Questions (25 points)

Question 1

The substitution effect is negative. Assuming there are two goods, this means that the Hicksian demand $h_1(p_1, p_2, \bar{u})$ is decreasing in p_1 , and similarly for good 2. Show this is true, either graphically or mathematically.

[Hint: For the graphical proof, you should sketch the expenditure minimising choices, and consider the effect of a rise in p_1 . For the mathematical proof, you should use Shephard's Lemma, which states that $\partial e / \partial p_1 = h_1$, and the concavity of the expenditure function.]

Question 2

An agent chooses to consume in periods $t \in \{1, 2\}$. Her income is m_1 and m_2 , while her consumption is x_1 and x_2 . The consumer faces interest rate r .

In class we claimed that the agents budget is given by the equation

$$m_1 + \frac{1}{1+r}m_2 \geq x_1 + \frac{1}{1+r}x_2$$

Explain this equation.

Question 3

The cost function $c(r_1, r_2, q)$ is concave in the input price of good 1, r_1 . Explain the idea behind this result, either in words or pictures (or both).

Question 4

A consumer's demand for good 1 is given by

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

She has income $m = 8$, and faces prices $p_1 = 1$ and $p_2 = 1$.

Calculate (a) the income elasticity of demand, (b) the own-price elasticity of demand, and (c) the cross-price elasticity of demand.

5. Consumer Surplus (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = (1 + x_1)(1 + x_2)$$

The prices of the goods are p_1 and p_2 . The consumer has income m . [Assume m is sufficiently large so we obtain an interior solution.]

- (a) Calculate the agent's Marshallian demand. [Note: this problem is well behaved in that the FOCs are sufficient.]
- (b) Are the goods (gross) complements or substitutes?
- (c) Calculate the agent's indirect utility function.
- (d) Calculate the agent's expenditure function.
- (e) Suppose the consumer has income $m = 8$ and faces prices $p_1 = 1$ and $p_2 = 1$. What is her utility?
- (f) Suppose the price of good 1 increases to $p'_1 = 4$. How much extra income does the agent need to compensate her for the price rise?

Space for Question 5.

6. Labour Supply (25 points)

An agent chooses leisure x_1 , and consumption x_2 to maximise her utility. Her utility function is given by

$$u(x_1, x_2) = x_1^2 x_2$$

The agent is endowed with T hours that she divides into leisure and work. The agent's wage is given by w , while we normalise the price of good 2 to 1. As a result, the agent's budget constraint is given by

$$w(T - x_1) \geq x_2$$

- (a) Derive the agent's Marshallian demand for x_1 and x_2 .
- (b) How does a change in the wage affect the demand for x_1 ? Interpret this result in terms of income and substitution effects.
- (c) How does a change in the wage affect the demand for x_2 ? Interpret this result in terms of income and substitution effects.

Space for Question 6.

7. Cost Minimisation (25 points)

A firm has two inputs, z_1 and z_2 . It has production function

$$f(z_1, z_2) = (z_1 + z_2)^{1/2}$$

The firm faces input price r_1 and r_2 , where we assume $r_1 < r_2$.

- (a) Does the production technology satisfy increasing/constant/decreasing returns to scale, or none of the above.
- (b) The firm wishes to attain target output q . Derive the cost minimising inputs $z_i^*(r_1, r_2, q)$.
- (c) Derive the cost function $c(r_1, r_2, q)$.
- (d) Derive the average cost and the marginal cost functions.
- (e) Does the average cost increase or decrease in q ? Intuitively, how does this result relate to your finding in part (a)?

Space for Question 7.

Space for Rough Work (Do not write your answers here)

Space for Rough Work (Do not write your answers here)

Economics 11: Solutions to Second Midterm

Instructions: The test is closed book. Calculators are allowed.

Short Questions (25 points)

Question 1

The substitution effect is negative. Assuming there are two goods, this means that the Hicksian demand $h_1(p_1, p_2, \bar{u})$ is decreasing in p_1 , and similarly for good 2. Show this is true, either graphically or mathematically.

[Hint: For the graphical proof, you should sketch the expenditure minimising choices, and consider the effect of a rise in p_1 . For the mathematical proof, you should use Shephard's Lemma, which states that $\partial e / \partial p_1 = h_1$, and the concavity of the expenditure function.]

Solution

For the mathematical proof, Shephard's lemma and the convexity of the expenditure function implies that

$$\frac{\partial}{\partial p_1} h_1 = \frac{\partial}{\partial p_1} \left[\frac{\partial e}{\partial p_1} \right] \leq 0$$

For the graphical proof, fix a indifference curve. When p_1 rises the budget line gets steeper, causing h_1 to fall (and h_2 to rise).

Question 2

An agent chooses to consume in periods $t \in \{1, 2\}$. Her income is m_1 and m_2 , while her consumption is x_1 and x_2 . The consumer faces interest rate r .

In class we claimed that the agents budget is given by the equation

$$m_1 + \frac{1}{1+r} m_2 \geq x_1 + \frac{1}{1+r} x_2$$

Explain this equation.

Solution

Interpretation 1: \$1 in period 1 is worth $\$(1 + r)$ in period 2. Hence \$1 in period 2 is worth $\$1/(1 + r)$ in period 1. The LHS of the equation is thus the agent's lifetime income, while the RHS is her lifetime consumption.

Interpretation 2: In the first period the agent saves $m_1 - x_1$. This is worth $(1 + r)(m_1 - x_1)$ in period 2. Hence the agent can spend

$$x_2 \leq m_2 + (1 + r)(m_1 - x_1)$$

Rearranging yields the above equation.

Question 3

The cost function $c(r_1, r_2, q)$ is concave in the input price of good 1, r_1 . Explain the idea behind this result, either in words or pictures (or both).

Solution

Suppose r_1 increases. If input demands are held constant, then costs rise linearly. However, the firm can do better by rebalancing its demands, implying that actual costs rise less than linearly. For a picture, see the slides.

Question 4

A consumer's demand for good 1 is given by

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

She has income $m = 8$, and faces prices $p_1 = 1$ and $p_2 = 1$.

Calculate (a) the income elasticity of demand, (b) the own-price elasticity of demand, and (c) the cross-price elasticity of demand.

Solution

This demand function comes from a utility function exhibiting perfect complements. Given the parameters, $x_1 = 4$. The income elasticity is

$$\frac{m}{x_1} \frac{\partial x_1}{\partial m} = 1$$

The own-price is

$$\frac{p_1}{x_1} \frac{\partial x_1}{\partial p_1} = -\frac{p_1}{p_1 + p_2} = -\frac{1}{2}$$

The cross-price elasticity is

$$\frac{p_2}{x_1} \frac{\partial x_1}{\partial p_2} = -\frac{p_2}{p_1 + p_2} = -\frac{1}{2}$$

5. Consumer Surplus (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = (1 + x_1)(1 + x_2)$$

The prices of the goods are p_1 and p_2 . The consumer has income m . [Assume m is sufficiently large so we obtain an interior solution.]

- (a) Calculate the agent's Marshallian demand. [Note: this problem is well behaved in that the FOCs are sufficient.]
- (b) Are the goods (gross) complements or substitutes?
- (c) Calculate the agent's indirect utility function.
- (d) Calculate the agent's expenditure function.
- (e) Suppose the consumer has income $m = 8$ and faces prices $p_1 = 1$ and $p_2 = 1$. What is her utility?
- (f) Suppose the price of good 1 increases to $p'_1 = 4$. How much extra income does the agent need to compensate her for the price rise?

Solution

- (a) The Lagrangian is

$$\mathcal{L} = (1 + x_1)(1 + x_2) + \lambda[m - p_1x_1 - p_2x_2]$$

The FOCs are

$$(1 + x_2) = \lambda p_1$$

$$(1 + x_1) = \lambda p_2$$

Rearranging, $(1 + x_1)p_1 = (1 + x_2)p_2$. Using the budget constraint, we find that

$$x_1^* = \frac{1}{2} \frac{m - p_1 + p_2}{p_1} \quad \text{and} \quad x_2^* = \frac{1}{2} \frac{m - p_2 + p_1}{p_2}$$

(b) An increase in p_2 increases demand for x_1 . Similarly, an increase in p_1 increases demand for x_2 . Hence these are substitutes.

(c) Substituting,

$$v(p_1, p_2, m) = (1 + x_1^*)(1 + x_2^*) = \frac{1}{4} \frac{(m + p_1 + p_2)^2}{p_1 p_2}$$

(d) Inverting,

$$e(p_1, p_2, \bar{u}) = 2(p_1 p_2 \bar{u})^{1/2} - p_1 - p_2$$

(e) The utility is $v = 25$.

(f) The new expenditure is $e = 15$. Hence the agent needs \$7 extra.

6. Labour Supply (25 points)

An agent chooses leisure x_1 , and consumption x_2 to maximise her utility. Her utility function is given by

$$u(x_1, x_2) = x_1^2 x_2$$

The agent is endowed with T hours that she divides into leisure and work. The agent's wage is given by w , while we normalise the price of good 2 to 1. As a result, the agent's budget constraint is given by

$$w(T - x_1) \geq x_2$$

(a) Derive the agent's Marshallian demand for x_1 and x_2 .

(b) How does a change in the wage affect the demand for x_1 ? Interpret this result in terms of income and substitution effects.

(c) How does a change in the wage affect the demand for x_2 ? Interpret this result in terms of income and substitution effects.

Solution

(a) The Lagrangian is given by

$$\mathcal{L} = x_1^2 x_2 + \lambda[wT - wx_1 - x_2]$$

The FOCs are

$$\begin{aligned} 2x_1 x_2 &= \lambda w \\ x_1^2 &= \lambda \end{aligned}$$

These yield $wx_1 = 2x_2$. Using the budget equation,

$$x_1^* = \frac{2}{3} \frac{wT}{w} = \frac{2}{3} T \quad \text{and} \quad x_2^* = \frac{1}{3} wT$$

(b) The demand for leisure is independent of the wage. The increase in w has a negative substitution effect (as always). However, the income effect is positive (since the rise in w increases the value of the agent's endowment). These two effects exactly cancel out. Note this is not a Giffen good, since we are in a world with endowments.

(c) The demand for consumption is increasing in the wage. The increase in w has a positive substitution effect (since there are only two goods). The income effect is also positive (since the rise in w increases the value of the agent's endowment).

7. Cost Minimisation (25 points)

A firm has two inputs, z_1 and z_2 . It has production function

$$f(z_1, z_2) = (z_1 + z_2)^{1/2}$$

The firm faces input price r_1 and r_2 , where we assume $r_1 < r_2$.

- (a) Does the production technology satisfy increasing/constant/decreasing returns to scale, or none of the above.
- (b) The firm wishes to attain target output q . Derive the cost minimising inputs $z_i^*(r_1, r_2, q)$.
- (c) Derive the cost function $c(r_1, r_2, q)$.
- (d) Derive the average cost and the marginal cost functions.
- (e) Does the average cost increase or decrease in q ? Intuitively, how does this result relate to your finding in part (a)?

Solution

- (a) The firm has decreasing returns. Let $t > 1$

$$f(tz_1, tz_2) = (tz_1 + tz_2)^{1/2} = t^{1/2}(z_1 + z_2)^{1/2} < tf(z_1, z_2)$$

- (b) The inputs are perfect substitutes. Since $r_1 < r_2$, the firm will choose $z_1^* = q^2$ and $z_2^* = 0$.
- (c) The cost function is $c(q) = r_1q^2$.
- (d) The average cost is $AC(q) = r_1q$. The marginal cost is $MC(q) = 2r_1q$.
- (e) The average cost increases in q . This occurs because of decreasing returns to scale. Using part (a), doubling inputs means output goes up by $\sqrt{2}$. Hence if we double to target output the corresponding costs go up by q^2 .

Economics 11: Final

Instructions: The test is closed book/notes. Calculators are allowed. Please write your answers on this sheet. There are 200 points.

Name:

UCLA ID:

TA:

Question	Score
Short Questions 1–5	
Short Questions 6–10	
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Total	

Short Questions (50 points)

Question 1

Suppose demand is perfectly elastic. Do consumers or producers bear the burden of a tax?

Question 2

Suppose, in a perfectly competitive industry, each firm has an identical cost function $c(q) = q^2 + 2q + 4$, and the market demand is $X = 320 - 20p$, where p is the price of the good. Find the long-run equilibrium price p^* , the output of each firm q , the number of firms in the industry N and the total industry output Q .

Question 3

Demand and supply are given by $D(p) = 10 - p$ and $S(p) = p$. Calculate the deadweight loss if the government sets a quantity constraint of 3 units.

Question 4

What is an inferior good? What is a Giffen good? What is the relationship between them?

Question 5

Suppose there are two goods. For good 1, the own-price elasticity, the cross-price elasticity and the income elasticity sum to zero. That is,

$$e_{x_1,p_1} + e_{x_1,p_2} + e_{x_1,m} = 0$$

Explain why this is the case.

Question 6

There are two goods, x_1 and x_2 . A consumer has income 10 and faces prices $p_1 = 1$ and $p_2 = 2$. Good 1 is also rationed, so the consumer can buy at most 5 units. Draw the consumer's budget set.

Question 7

An agent has convex indifference curves over two goods, x_1 and x_2 .

- (a) Provide an economic interpretation for the convexity of the agent's indifference curves.
- (b) How does the agent's marginal rate of substitution vary as the agent moves along an indifference curve?

Question 8

True or False? Explain your answers.

- (a) Supply curves are always upward sloping. That is, $q^*(p') \geq q^*(p)$ for $p' > p$.
- (b) Demand curves are always downward sloping. That is, $x_1^*(p'_1, p_2, m) \leq x_1^*(p_1, p_2, m)$ for $p'_1 > p_1$.

Question 9

When AC is minimised, $AC=MC$. Why?

Question 10

An agent has quasi-linear utility, $u(x_1, x_2) = v(x_1) + x_2$. She faces prices $\{p_1, p_2\}$ and has income, m , that is sufficiently high so we can ignore boundary constraints. Show that demand for good 1, $x_1^*(p_1, p_2, m)$, is independent of income.

Long Questions (25 points each)

Question 1

There are three goods, $\{x_1, x_2, x_3\}$. An agent has utility function given by $u(x_1, x_2, x_3) = x_1 x_2 x_3$. Let p_1 , p_2 and p_3 denote the price of the three goods.

(a) Show that the expenditure function is given by $e(p_1, p_2, p_3, \bar{u}) = 3(\bar{u} p_1 p_2 p_3)^{1/3}$. [You can derive this however you wish.]

(b) Suppose the agent has income $m = 6$, and the prices are $\{p_1, p_2, p_3\} = \{4, 2, 1\}$. Derive the agent's utility.

(c) Suppose prices change, and become $\{p'_1, p'_2, p'_3\} = \{3, 3, 3\}$. How much money do we need to give the agent to compensate her for the price change?

Space for Question 1.

Question 2

An agent consumes two goods. She has utility $u(x_1, x_2) = \min\{x_1, x_2\}$. She has income m and faces prices $\{p_1, p_2\}$

(a) Calculate the agent's Marshallian demand.

(b) Calculate the agent's Hicksian demand.

(c) The own-price Slutsky equation states that

$$\frac{\partial}{\partial p_1} x_1^*(p_1, p_2, m) = \frac{\partial}{\partial p_1} h_1(p_1, p_2, \bar{u}) - x_1^*(p_1, p_2, m) \frac{\partial}{\partial m} x_1^*(p_1, p_2, m)$$

Verify that this holds. What does the Slutsky equation tell us about these preferences?

Space for Question 2.

Question 3

we are interested in the market for good 1. Suppose there are 450 agents with $u(x_1, x_2) = x_1 + x_2$. Each agent has income $m = 100$. The price of good 1, p , is to be determined; the price of good 2 is 40. There are also 100 firms who produce good 1, each with cost function $C(q) = q^2 + 100$.

- (a) Find the market demand function for good 1.
- (b) Find the market supply function for good 1.
- (c) In the short run, there is no entry or exit. What is the short-run equilibrium price of good 1?
- (d) In the long-run, there is free entry. What is the long-run equilibrium price of good 1? How many firms enter in the long-run?

Space for Question 3.

Question 4

An agent has utility function $u(c_1, c_2) = c_1^{1/2} + c_2^{1/2}$ where c_1 is consumption in period 1 and c_2 is consumption in period 2. The agent has income $m_1 = 75$ in period 1, and income $m_2 = 50$ in period 2.

- (a) For a given interest rate, r , write down the budget constraint.
- (b) Solve for first and second period consumption if $r = 1$.
- (c) What happens to consumption if the agent's income is $m_1 = 100$ and $m_2 = 0$? Explain how this result compares to part (b).

Space for Question 4.

Question 5

There is one good. Demand is given by $X = 30 - p$. Domestic supply is $Q = 2p$. [In the following questions, we are only concerned with the welfare of the domestic country.]

(a) Find the market equilibrium price and output. Find the producer surplus and consumer surplus.

(b) Suppose that the goods can be imported at a price of \$5 per unit. Find the market equilibrium price and the amount of goods imported. Find the new consumer surplus and new producer surplus. How much does total welfare change by?

(c) Suppose the government has decided to impose a tariff of \$3 per unit to protect the domestic producers. Find the new market equilibrium price and the amount of goods imported. Find the new consumer surplus, new producer surplus and government tariff revenue. When compared to (b), how much does total welfare change by?

Space for Question 5.

Question 6

A firm has two inputs, z_1 (labour) and z_2 (a new factory). The firm can hire any $z_1 \in \mathfrak{R}$, however it is restricted to choosing either $z_2 = 0$ or $z_2 = 1$. The production function is given by $f(z_1, z_2) = z_1^{1/2}(1 + z_2)$. The price of labour is $r_1 = 4$, while the price of the factory is $r_2 = 3$.

- (a) Suppose we fix $z_2 = 0$. Derive the cost function, $c_0(q)$.
- (b) Suppose we fix $z_2 = 1$. Derive the cost function, $c_1(q)$.
- (c) Suppose the firm can now choose $z_2 = 0$ or $z_2 = 1$ optimally. Putting (a) and (b) together, derive the overall cost function, $c(q)$. [Hint: you might wish to calculate the output, q' , where the firm is indifferent between choosing $z_2 = 0$ and $z_2 = 1$. Plotting the functions is also useful.]
- (d) Calculate the marginal cost corresponding to the cost function in (c).
- (e) Calculate the supply function, $q^*(p)$ corresponding to the cost curve in (c).

Space for Question 6.

Space for Rough Work (Do not write your answers here)

Space for Rough Work (Do not write your answers here)

Economics 11: Solutions to Final

Instructions: The test is closed book. Calculators are allowed.

Short Questions (50 points)

Question 1

Suppose demand is perfectly elastic. Do consumers or producers bear the burden of a tax?

Solution

If demand is perfectly elastic, then the supplier bears the full burden of the tax.

Question 2

Suppose, in a perfectly competitive industry, each firm has an identical cost function $c(q) = q^2 + 2q + 4$, and the market demand is $X = 320 - 20p$, where p is the price of the good. Find the long-run equilibrium price p^* , the output of each firm q , the number of firms in the industry N and the total industry output Q .

Solution

In the long-run free-entry equilibrium, $p^* = MC = AC$. Hence,

$$p^* = 2q + 2 = q + 2 + \frac{4}{q}$$

If we solve this for q ,

$$\begin{aligned} q &= \frac{4}{q} \\ q &= 2 \end{aligned}$$

By plugging this into AC, yields $p^* = 6$. At this price, market demand is

$$\begin{aligned} X &= 320 - 20 \times 6 \\ &= 200 \\ &= Q \end{aligned}$$

This must be equal to the market supply which is $2N$, each firm's output times the number of firms in the industry. Hence,

$$2N = 200$$

$$N = 100$$

Question 3

Demand and supply are given by $D(p) = 10 - p$ and $S(p) = p$. Calculate the deadweight loss if the government sets a quantity constraint of 3 units.

Solution

The equilibrium quantity is $q^* = 5$, with a price $p^* = 5$.

When the constraint is introduced the marginal agent is willing to pay 7, while the marginal firm has a cost 3. The deadweight loss is $(7 - 3)(5 - 3)/2 = 4$.

Question 4

What is an inferior good? What is a Giffen good? What is the relationship between them?

Solution

With an inferior good, an increase in income lowers demand.

With a Giffen good, an increase in price raises demand.

A Giffen good is a special case of an inferior good. It is a super-inferior good, where the income effect outweighs the substitution effect.

Question 5

Suppose there are two goods. For good 1, the own-price elasticity, the cross-price elasticity and the income elasticity sum to zero. That is,

$$e_{x_1, p_1} + e_{x_1, p_2} + e_{x_1, m} = 0$$

Explain why this is the case.

Solution

A one percent increase in income and both prices has no effect on the budget set. Hence it has no effect on the demand of good 1.

Question 6

There are two goods, x_1 and x_2 . A consumer has income 10 and faces prices $p_1 = 1$ and $p_2 = 2$. Good 1 is also rationed, so the consumer can buy at most 5 units. Draw the consumer's budget set.

Solution

The budget line intersects the x_2 -axis at 5. It has a slope of $-1/2$, until $x_1 = 5$ when it drops vertically to the axis.

Question 7

An agent has convex indifference curves over two goods, x_1 and x_2 .

- (a) Provide an economic interpretation for the convexity of the agent's indifference curves.
- (b) How does the agent's marginal rate of substitution vary as the agent moves along an indifference curve?

Solution

- (a) Convexity says that the consumer prefers averages to extremes.
- (b) When x_1 rises the MRS falls.

Question 8

True or False? Explain your answers.

- (a) Supply curves are always upward sloping. That is, $q^*(p') \geq q^*(p)$ for $p' > p$.
- (b) Demand curves are always downward sloping. That is, $x_1^*(p'_1, p_2, m) \geq x_1^*(p_1, p_2, m)$ for $p'_1 > p_1$.

Solution

(a) True. This follows from the convexity of the profit function and Hotelling's Lemma.

(b) False. A Giffen good exhibits upward sloping demand.

Question 9

When AC is minimised, AC=MC. Why?

Solution

If $MC > AC$ then the marginal unit is more costly than the average unit, implying AC is increasing. Conversely, if $MC < AC$ then AC is decreasing.

Question 10

An agent has quasi-linear utility, $u(x_1, x_2) = v(x_1) + x_2$. She faces prices $\{p_1, p_2\}$ and has income, m , that is sufficiently high so we can ignore boundary constraints. Show that demand for good 1, $x_1^*(p_1, p_2, m)$, is independent of income.

Solution

The consumer maximises

$$\mathcal{L} = v(x_1) + x_2 + \lambda[m - p_1x_1 - p_2x_2]$$

The FOC for x_1 is

$$v'(x_1) = \lambda p_1$$

The FOC for x_2 is

$$1 = \lambda p_2$$

Hence

$$v'(x_1) = \frac{p_1}{p_2}$$

and x_1 is independent of income, m .

1 Long Questions (25 points each)

Question 1

There are three goods, $\{x_1, x_2, x_3\}$. An agent has utility function given by $u(x_1, x_2, x_3) = x_1x_2x_3$. Let p_1, p_2 and p_3 denote the price of the three goods.

(a) Show that the expenditure function is given by $e(p_1, p_2, p_3, \bar{u}) = 3(\bar{u}p_1p_2p_3)^{1/3}$. [You can derive this however you wish.]

(b) Suppose the agent has income $m = 6$, and the prices are $\{p_1, p_2, p_3\} = \{4, 2, 1\}$. Derive the agent's utility.

(c) Suppose prices change, and become $\{p'_1, p'_2, p'_3\} = \{3, 3, 3\}$. How much money do we need to give the agent to compensate her for the price change?

Solution

(a) The agent minimises

$$\mathcal{L} = p_1x_1 + p_2x_2 + p_3x_3 + \lambda[\bar{u} - x_1x_2x_3]$$

The FOCs are

$$p_1 = \lambda x_2x_3$$

$$p_2 = \lambda x_1x_3$$

$$p_3 = \lambda x_1x_2$$

Rearranging, $p_1x_1 = p_2x_2 = p_3x_3$. Using the constraint we find that

$$\bar{u} = x_1^3 \frac{p_1^2}{p_2p_3}$$

Hence the Hicksian demand is

$$h_1 = \left(\frac{\bar{u}p_2p_3}{p_1^2} \right)^{1/3}$$

And similarly for the other goods.

Substituting, the expenditure function is

$$e = p_1 h_1 + p_2 h_2 + p_3 h_3 = 3(\bar{u} p_1 p_2 p_3)^{1/3}$$

(b) Inverting, the indirect utility function is

$$v = \frac{m^3}{27 p_1 p_2 p_3}$$

Given the prices, $v = 1$.

(c) When the prices change the expenditure becomes $e = 9$. Thus the agent needs \$3 extra.

Question 2

An agent consumes two goods. She has utility $u(x_1, x_2) = \min\{x_1, x_2\}$. She has income m and faces prices $\{p_1, p_2\}$

(a) Calculate the agent's Marshallian demand.

(b) Calculate the agent's Hicksian demand.

(c) The own-price Slutsky equation states that

$$\frac{\partial}{\partial p_1} x_1^*(p_1, p_2, m) = \frac{\partial}{\partial p_1} h_1(p_1, p_2, \bar{u}) - x_1^*(p_1, p_2, m) \frac{\partial}{\partial m} x_1^*(p_1, p_2, m)$$

Verify that this holds. What does the Slutsky equation tell us about these preferences?

Solution

(a) When maximising utility, the consumer chooses $x_1 = x_2$. Given the budget constraint, this becomes

$$x_1^* = x_2^* = \frac{m}{p_1 + p_2}$$

(b) When minimising expenditure, the consumer chooses $x_1 = x_2$. Given the utility constraint,

this becomes

$$h_1 = h_2 = \bar{u}$$

(c) The LHS of Slutsky is

$$\frac{\partial}{\partial p_1} x_1^*(p_1, p_2, m) = -\frac{m}{(p_1 + p_2)^2}$$

The RHS of Slutsky has two components. The first:

$$\frac{\partial}{\partial p_1} h_1(p_1, p_2, \bar{u}) = 0$$

The second:

$$-x_1^*(p_1, p_2, m) \frac{\partial}{\partial m} x_1^*(p_1, p_2, m) = -\frac{m}{p_1 + p_2} \frac{m}{p_1 + p_2}$$

And therefore Slutsky holds. This tells us that demand effects are due to income effects only: there are no substitution effects.

Question 3

we are interested in the market for good 1. Suppose there are 450 agents with $u(x_1, x_2) = x_1 + x_2$. Each agent has income $m = 100$. The price of good 1, p , is to be determined; the price of good 2 is 40. There are also 100 firms who produce good 1, each with cost function $C(q) = q^2 + 100$.

- (a) Find the market demand function for good 1.
- (b) Find the market supply function for good 1.
- (c) In the short run, there is no entry or exit. What is the short-run equilibrium price of good 1?
- (d) In the long-run, there is free entry. What is the long-run equilibrium price of good 1? How many firms enter in the long-run?

Solution

- (a) $x_1 = 100/p$ if $p < 40$ and $x_1 = 0$ otherwise. Market demand is $45000/p$ when $p < 40$.
- (b) Each firm supplies $q = p/2$, so market supply is $Q = 50p$.

(c) Setting demand equal to supply yields $p = 30$.

(d) The average cost is

$$AC = q + \frac{100}{q}$$

This is minimised at $q = 10$, yielding a price of $p = 20$. At this price demand is $X = 2250$. Hence there are 225 firms.

Question 4

An agent has utility function $u(c_1, c_2) = c_1^{1/2} + c_2^{1/2}$ where c_1 is consumption in period 1 and c_2 is consumption in period 2. The agent has income $m_1 = 75$ in period 1, and income $m_2 = 50$ in period 2.

(a) For a given interest rate, r , write down the budget constraint.

(b) Solve for first and second period consumption if $r = 1$.

(c) What happens to consumption if the agent's income is $m_1 = 100$ and $m_2 = 0$? Explain how this result compares to part (b).

Solution

(a) The budget constraint is

$$c_1 + \frac{1}{1+r}c_2 \leq m_1 + \frac{1}{1+r}m_2$$

(b) When $r = 1$, $1/(1+r) = 1/2$. The agent maximises

$$\mathcal{L} = c_1^{1/2} + c_2^{1/2} = \lambda[75 + 25 - c_1 - c_2/2]$$

The FOCs are

$$\frac{1}{2}c_1^{-1/2} = \lambda \frac{1}{2}c_2^{-1/2} = \frac{1}{2}\lambda \tag{1}$$

Rearranging, $c_2 = 2c_1$. Hence $c_1^* = 50$ and $c_2^* = 100$.

(c) Consumption is unchanged. Both income streams have the same present value, so consumption is the same.

Question 5

There is one good. Demand is given by $X = 30 - p$. Domestic supply is $Q = 2p$. [In the following questions, we are only concerned with the welfare of the domestic country.]

(a) Find the market equilibrium price and output. Find the producer surplus and consumer surplus.

(b) Suppose that the goods can be imported at a price of \$5 per unit. Find the market equilibrium price and the amount of goods imported. Find the new consumer surplus and new producer surplus. How much does total welfare change by?

(c) Suppose the government has decided to impose a tariff of \$3 per unit to protect the domestic producers. Find the new market equilibrium price and the amount of goods imported. Find the new consumer surplus, new producer surplus and government tariff revenue. When compared to (b), how much does total welfare change by?

Solution

(a) The equilibrium can be found by letting the demand equals the supply, $30 - p = 2p$, hence $p = 10$. By plugging this into the demand (or the supply), we get the equilibrium output, $Q = 20$. Consumer surplus is $(30 - 10)(20)/2 = 200$. Producer surplus is $(10)(20)/2 = 100$. Welfare is 300

b) The market equilibrium price will be the price at which the goods are imported, \$5. At $p = 5$, the demand will be $30 - 5 = 25$, and the supply will be $2 \times 5 = 10$. Hence the amounts of import will be $25 - 10 = 15$. Consumer surplus is $(30 - 5)(25)/2 = 312.5$. Producer surplus is $(5)(10)/2 = 25$. Total welfare is 337.5, an increase of 37.5.

c) The new market equilibrium price under the \$3 tariff is $\$5 + \$3 = \$8$. At this price, the demand is $30 - 8 = 22$, and supply is $2 \times 8 = 16$. Hence the amounts of import will be $22 - 16 = 6$.

The CS after the tariff is $(30 - 8)(22)/2 = 242$. The PS after the tariff is $(8)(16)/2 = 64$. The government revenues equal by $6 * 3 = 18$. Hence total surplus is 324, a decrease of 13.5.

Question 6

A firm has two inputs, z_1 (labour) and z_2 (a new factory). The firm can hire any $z_1 \in \mathfrak{R}$, however it is restricted to choosing either $z_2 = 0$ or $z_2 = 1$. The production function is given by $f(z_1, z_2) = z_1^{1/2}(1 + z_2)$. The price of labour is $r_1 = 4$, while the price of the factory is $r_2 = 3$.

- (a) Suppose we fix $z_2 = 0$. Derive the cost function, $c_0(q)$.
- (b) Suppose we fix $z_2 = 1$. Derive the cost function, $c_1(q)$.
- (c) Suppose the firm can now choose $z_2 = 0$ or $z_2 = 1$ optimally. Putting (a) and (b) together, derive the overall cost function, $c(q)$. [Hint: you might wish to calculate the output, q' , where the firm is indifferent between choosing $z_2 = 0$ and $z_2 = 1$. Plotting the functions is also useful.]
- (d) Calculate the marginal cost corresponding to the cost function in (c).
- (e) Calculate the supply function, $q^*(p)$ corresponding to the cost curve in (c).

Solution

(a) Here $q = z_1^{1/2}$. Hence the cost is $c_0(q) = 4q^2$.

(b) Here $q = 2z_1^{1/2}$. The cost is $c_1(q) = 3 + q^2$.

(c) The firm is indifferent when

$$4q^2 = 3 + q^2$$

Rearranging, $q' = 1$. Hence the cost function is

$$\begin{aligned} c(q) &= 4q^2 && \text{if } q < 1 \\ &= 3 + q^2 && \text{if } q > 1 \end{aligned}$$

(d) The marginal cost is

$$\begin{aligned}c(q) &= 8q && \text{if } q < 1 \\ &= 2q && \text{if } q > 1\end{aligned}$$

(e) When $p < 8$, the firm can choose to produce on either marginal cost curve. When $z_2 = 0$, the firm chooses $q = p/8$ and profits are

$$\pi = pq - 4q^2 = \frac{p^2}{8}$$

When $z_2 = 1$, the firm chooses $q = p/2$ and profits are

$$\pi = pq - q^2 - 3 = \frac{p^2}{4} - 3$$

The firm is indifferent when $p = \sqrt{24} \approx 4.9$. The supply curve is thus

$$\begin{aligned}q^*(p) &= 8q && \text{if } p \leq \sqrt{24} \\ &= 2q && \text{if } p \geq \sqrt{24}\end{aligned}$$