# Economics 326: Suggested Solutions to Final 

19th April, 2004

1.(a) In the least cost separating equilibrium, $e_{L}=0$ and $e_{H}=\left(\theta_{H}-\theta_{L}\right) \theta_{L}$. In equilibrium, $w\left(e_{L}\right)=\theta_{L}$ and $w\left(e_{L}\right)=\theta_{H}$. Outside equilibrium, $w(e) \in\left[\theta_{L}, \theta_{H}\right]$ and $w(e) \leq \theta_{L}+e / \theta_{L}$.
(b) In the least cost pooling equilibrium, $e_{L}=e_{H}=0$. In equilibrium, $w\left(e_{L}\right)=E[\theta]$. Outside equilibrium, $w(e) \in\left[\theta_{L}, \theta_{H}\right]$ and $w(e) \leq E[\theta]+e / \theta_{L}$.
2. (a) As $\theta_{2}$ increases, $q_{2}$ increases in $q_{1}$ decreases. An increase in $\theta_{2}$ means the high type gains more utility from quality and so should be given a higher quality. However, the high type's information rents $q_{1}\left(\theta_{2}-\theta_{1}\right)$ increase, so the seller responds by reducing $q_{1}$. As $\theta_{2} \rightarrow \theta_{1}$, so $q_{2}$ and $q_{1}$ converge to $q_{1}^{*}$, which satisfies $c^{\prime}\left(q_{1}^{*}\right)=\theta_{1}$.
(b) As $\theta_{1}$ increases, $q_{1}$ is increases and $q_{2}$ is unaffected. An increase in $\theta_{1}$ means that agent 1 prefers wine more, so should be given a higher quality. In addition, the information rent of agent 2 disappears. As $\theta_{2} \rightarrow \theta_{1}$, so $q_{1}$ converges to $q_{2}^{*}$, which satisfies $c^{\prime}\left(q_{2}^{*}\right)=\theta_{2}$.
3. (a) We should not ban smoking.
(b) A majority vote would ban smoking.
(c) A randomly assigned dictatorship would ban smoking with probability $2 / 3$.
(d) No! Agents 1 and 2 would claim their value is $+\infty$, while agent 3 would claim their value is $-\infty$.
(e) Clarke tax. Agent 1 must pay $\$ 1300$ if smoking is banned. Agent 2 must pay $\$ 1000$ if smoking is banned. Agent 3 must pay - $\$ 1500$ if smoking is banned. With truthtelling, smoking will not be banned and payments will be zero.
4. (a) The price equals $q$.
(b) The good firm sells computers if $p \geq c_{H}$. If $\left(q_{L}+q_{H}\right) / 2 \geq c_{H}$ the price is $p=\left(q_{L}+q_{H}\right) / 2$ and both types of firms sell computers. If $\left(q_{L}+q_{H}\right) / 2<c_{H}$ the price is $p=q_{L}$ and only bad firms sell computers.
(c) A consumer is willing to pay 1 . Profit from quality $q_{H}$ is $q_{H}-c_{H}$, which exceeds the profit in part (b) as $p<q_{H}$. Thus the high type offers the warranty.
(d) In the signalling game, the good quality firm offers a warranty. The bad quality firm is indifferent between offering one and not.

