Economics 326: Homework 2

23 February, 2004

Question 1

Consider Akerlof's lemons model with competitive buyers (as in the second half of lecture 1). Suppose the buyer's value, θ, is distributed uniformly on $[1/10, 11/10]$.

(a) Suppose the seller is leaving the country and has a reservation value of $r(\theta) = 0$. What is the competitive price?
(b) Suppose the seller is staying at home and has reservation value $r(\theta) = \theta - 1/10$.
(c) Consider the population of agents who have cars for sale. Half of these are leaving the country. Half of these are staying at home. Given a price p, what is the probability an agent staying at home will trade? Given a price p, what is the probability the agent a buyer who trades is staying at home? What will the competitive price be?

Question 2

Consider Spence's signaling model. There are two types $\theta_2 > \theta_1$, where the cost of eduction for type θ_i is $c(e, \theta_i) = e/\theta_i$. The proportion of type θ_2 agents is λ.

(a) What education levels are chosen in the set of separating equilibria?
(b) What education levels are chosen in the set of pooling equilibria?

Question 3

Consider Spence's signalling model. Suppose there are three types of agents $\theta_3 > \theta_2 > \theta_1$, where there are equal numbers of each type. The costs of education is $c(e, \theta_i) = e/\theta_i$.

(a) Describe the set of fully pooling equilibria (i.e. where all three types pool).
(b) Describe the least cost fully separating equilibrium.
(c) Describe an equilibria where θ_2 and θ_1 pool, and θ_3 separates.

Question 4

Suppose a seller of wine faces two types of customers, θ_1 and θ_2, where $\theta_2 > \theta_1$. The proportion of type θ_1 agents is $\pi \in [0,1]$. Let q be the quality of the wine and t the price. Agent θ_i has utility

$$u(\theta_i) = \theta_i q - \frac{1}{2}q^2 - t$$

Let type θ_1 buy contract (q_1, t_1) and type 2 buy (q_2, t_2). The cost of production is zero, $c(q) = 0$, and the seller maximises profit

$$\pi t_1 + (1-\pi)t_2 \tag{1}$$

(a) Suppose the seller observes the agent's types. Solve for the first best qualities. That is, maximise profit subject to the (IR) constraints of the two agents.
(b) Now suppose the seller cannot observe which agent is which. Write down the seller's optimisation problem subject to the two (IR) and two (IC) constraints.
(c) As in the lecture, argue that (IR_2) can be ignored and that (IR_1) and (IC_2) will bind. Next argue that we can replace (IC_1) with the monotonicity constraint, that $q_2 \geq q_1$.
(d) Write down the relaxed problem, ignoring the monotonicity constraint. Show the optimal qualities are

$$\begin{aligned} q_2 &= \theta_2 \\ q_1 &= \theta_1 - \left(\frac{1-\pi}{\pi}\right)(\theta_2 - \theta_1) \end{aligned}$$

Finally, verify this satisfies the monotonicity constraint.

Question 5

Suppose a seller of wine faces two types of customers, θ_1 and θ_2, where $\theta_2 > \theta_1$. The proportion of type θ_1 agents is $\pi \in [0,1]$. Let q be the quality of the wine and t the price. Agent θ_i has utility

$$u(\theta_i) = \theta_i(q - \frac{1}{2}q^2) - t$$

Let type θ_1 buy contract (q_1, t_1) and type 2 buy (q_2, t_2). The cost of production is zero, $c(q) = 0$, and the seller maximises profit

$$\pi t_1 + (1-\pi)t_2 \tag{2}$$

(a) Suppose the seller observes the agent's types. Solve for the first best qualities and prices. That is, maximise profit subject to the (IR) constraints of the two agents.
(b) Now suppose the seller cannot observe which agent is which. Write down the seller's optimisation problem subject to the two (IR) and two (IC) constraints.
and two (IC) constraints.
(c) As in the lecture, argue that (IR_2) can be ignored and that (IR_1) and (IC_2) will bind. Next argue that we can replace (IC_1) with the monotonicity constraint, that $q_2 \geq q_1$.
(d) Write down the relaxed problem, ignoring the monotonicity constraint. Show θ_2 gets quality 1, while θ_1 gets either quality 0 or 1 (i.e. a bang–bang solution). Finally, show the optimal quantity satisfies the monotonicity constraint.

Question 6

Consider the price discrimination model considered in the lecture, except that there are now three types, where $\theta_3 > \theta_2 > \theta_1$. The utility of agent θ_i is

$$u(\theta_i) = \theta_i q - t$$

Denote the bundles assigned to agent θ_i by (q_i, t_i). Now there are three (IR) constraints, one for each type. There are also six (IC) constraints, since we must ensure type θ_1 does not want to copy type θ_2 or θ_3, and similarly for the other agents. For example, (IC_1^2) says that θ_1 must not want to copy θ_2, i.e.

$$\theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2 \qquad (\text{IC}_1^2)$$

The firm's profit is

$$\sum_{i=1}^{3} \pi_i [t_i - c(q_i)]$$

where π_i is the proportion of type θ_i agents and $c(q)$ is increasing and convex.

(a) Show that (IR_2) and (IR_3) can be ignored.
(b) Show that $q_3 \geq q_2 \geq q_1$.
(c) Using (IC_2^1) and (IC_3^2) show that we can ignore (IC_3^1). Using (IC_2^3) and (IC_1^2) show that we can ignore (IC_1^3).
(d) Show that (IR_1) will bind.
(e) Show that (IC_2^1) will bind.
(f) Show that (IC_3^2) will bind.
(g) Assume that $q_3 \geq q_2 \geq q_1$. Show that (IC_1^2) and (IC_2^3) can be ignored.