Economics 326: Homework 3

30 March, 2004

1. Consider Spence's signaling model with productive education. If agent θ gets e years of education then their productivity is $\theta + e$. The cost of education for type θ is $\frac{e^2}{2\theta}$.
(a) Suppose type θ (along with e) is observable. How many years of education would type θ get?
(b) Now suppose there are two types $\{\theta_L, \theta_H\}$, where the proportion of type θ_L agent is π, and $\theta_H \geq \theta_L \geq \theta_H/3$. Characterise the least–cost separating equilibrium.
(c) Suppose $\theta_L \leq \theta_H/3$. Characterise the least–cost separating equilibrium.

2. The government considers building a road for three agents. The cost of the road is 10. The agents valuations are 2, 5 and 7. If we use the clarke tax (i.e. pivitol mechanism), what will the payments be for the three agents if the road is built, and if it is not built? In equilibrium, what are their payments?

3. Repeat question 2, but assume the three agents' valuations are 6, 5 and 7. What are the payments under the pivitol mechanism if the road is built, and if it is not built? What are payments in equilibrium?

4. You have committed a heinous crime and have been brought to trial where you face N corrupt judges. If you are convicted judge i gets utility $v_i + t_i$, where t_i is any payment they receive, and $v_i \sim U[0,1]$ is private information. If you are not convicted judge i gets utility t_i. The court decision is by majority vote.
(a) Suppose $N = 1$. You offer the judge t_1 to set you free. How much will is cost to guarantee you are not convicted?
(b) Suppose $N = 3$. You offer judges 1 and 2 money to set you free. How much will is cost to guarantee you are not convicted?
(c) Suppose you use the pivitol mechanism. You offer each judge 2 if you are set free *and* they are pivitol. In equilibrium, how much will you pay?

5. Two man–of–war ships see each other in the distance. The ships simultaneously decide to 'flee' of 'approach'. If either ship flees they both get payoff zero. If they approach, one won with more guns steals the gold of the one with fewer guns. If the approach and have the same number of guns, they neither gain nor lose their gold. Ship i has $\theta_i \in [0,1]$ guns, which cannot be seen by the opponent at distance (i.e. this is private information). In the symmetric equilibrium, what types θ_i flee and which approach?

6. Consider Akerlof's model with $\theta \sim U[1, 11]$ with $r(\theta) = \theta - 1$.

(a) Suppose sellers $\theta \in [1, \theta_t]$ participate. Show the generates price $p_t = (\theta_t + 1)/2$.

(b) Suppose the price is p_t. Show that sellers $\theta \in [1, \theta_{t+1}]$ participate, where $\theta_{t+1} = p_t - 1$.

(c) Initially suppose all sellers are in the market, $\theta_0 = 11$. Calculate the price p_0 this generates. Calculate the sellers who participate under p_0, given by $[1, \theta_1]$. Similarly, calculate (p_t, θ_t) for $t = 0, 1, \ldots, 5$. What does this converge to?

(d) In equilibrium, what sellers participate in the market? What is the price level? Why does (c) give the same answer as (d)?