Economics 326: Suggested Solutions 3

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1. Job market signaling with productive education.
(a) Agents θ chooses e to maximise

$$\theta + e - \frac{e^2}{2\theta}$$

Maximising, $e^* = \theta$.
(b) The low type obtains the efficient education level, $e_L = \theta_L$. The high type must take enough to separate himself, i.e. the low type's (IC) constrain binds.

$$\theta_L + e_L - \frac{e_L^2}{2\theta_L} = \theta_H + e_H - \frac{e_H^2}{2\theta_L}$$

Since $e_L = \theta_L$ this yields quadratic

$$e_H^2 - 2\theta_L e_H + 2\theta_L(\frac{3}{2}\theta_L - \theta_H)$$

this yields $e_H = \theta_L + \sqrt{2\theta_L(\theta_H - \theta_L)}$. One can verify that the assumption $\theta_L \geq \theta_H/3$ implies $e_H \geq \theta_H$.
(c) If $\theta_L \leq \theta_H/3$ then $e_H = \theta_H$, and the (IC) constraint is irrelevant.

2. If agent 1 does not exist, social welfare if the road is built is $7 + 5 - 10 = 2$, while social welfare if the road is not built is 0. If agent 1 declares a number greater that -2, the road is built and agent 1 pays nothing. If agent 1 declares a number less than -2, the road is not built and agent 1 pays 2, their effect on the welfare of others. (Of course, if valuations are positive, this latter case is not relevant).

If agent 2 does not exist social welfare is -1 ($= 1 + 7 - 10$) if the road is built and 0 if it is built. Thus agent 2 must pay 1 is the road is built and 0 is the road is not built. Similarly, agent 3 must pay 3 is the road is built and 0 is the road is not built.

In equilibrium, the road is built (since $2 + 5 + 7 > 10$) and their payments are (0,1,3). Observe that the government loses money.

3. For agent 1, (not built payment, built payment)=(2,0). For agent 2, payments are (3,0). For agent 3, payments are (1,0). In equilibrium not agent pays anything as none are pivitol.

4. (a) With one judge it costs 1.
(b) Now it costs 2.
(c) With the pivitol mechanism, it costs nothing!

5. Everyone flees, apart from the highest possible type. In any symmetric equilibrium types $[\theta^*, 1]$ approach. Yet $\theta^* < 1$ will surely lose their gold, so they should also flee. Thus $\theta^* = 1$. Notice that 'approach' strategy unravels in a similar way to the stag hunt cheaptalk example in class.

6. (a) $p_t = E[\theta|\theta \leq \theta_t] = (\theta_t + 1)/2$.
(b) $r(\theta_{t+1}) = p_t$. Hence $\theta_{t+1} = p_t - 1$.
(c) We have,

t	θ_t	p_t
0	11	6
1	7	4
2	5	3
3	4	5/2
4	7/2	9/4
5	13/4	17/8

Here, $p_t \to 2$ and $\theta_t \to 3$.
(d) The equilibrium price is 2, while sellers $[1, 3]$ participate. We've just calculated the dynamics I drew in class.