Economics 326: Final

Date: 24th April, 2006

Course: Economics 326
Instructor: Simon Board
Duration: 2 hours
Aids: Calculators, Brain.
Total marks: 200

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Question 1 (35 points)

Consider Spence's signalling model where there are two types of workers $\theta \in \{10, 30\}$ and $\Pr(\theta = 30) = 1/2$. Firms are competitive and obtain profit θ from a worker of quality θ. Workers can obtain one of three levels of education: $e \in \{e_1, e_2, e_3\}$. Agents get utility $u(w, e|\theta) = w - c(e, \theta)$, where wages must obey $w \in [10, 30]$. The cost if given by

$$\begin{aligned} c(e_1, 10) &= 0 & c(e_1, 30) &= 0 \\ c(e_2, 10) &= 9 & c(e_2, 30) &= 4 \\ c(e_3, 10) &= 18 & c(e_3, 30) &= 8 \end{aligned}$$

(a) What are the pooling equilibria in this model? [You should justify your answer. Your description of any equilibrium should also include a supporting wage function].

(b) What are the separating equilibria in this model? [You should justify your answer. Your description of any equilibrium should also include a supporting wage function].

Question 2 (35 points)

There are a large number of agents selling their cars. There are three type of cars (low, medium, high), with equal numbers of each type. Sellers initially know the type of car they own, however buyers cannot observe this quality. Buyers are competitive and bid the price up to their expected willingness to pay. The willingness to pay, θ, and reservation utilities, $r(\theta)$, for each of the three types of cars are:

$$\theta_L = 10 \qquad r(\theta_L) = 8$$
$$\theta_M = 20 \qquad r(\theta_M) = 16$$
$$\theta_H = 30 \qquad r(\theta_H) = 24$$

(a) As in Akerlof, suppose sellers can choose whether or not to participate in the market. If a seller trades, she get the market price, p; if a seller does not trade, she gets her reservation utility, $r(\theta)$. Describe the equilibria of the model.

(b) Now suppose sellers have three choices: (i) stay at home and get $r(\theta)$; (ii) have a mechanic certify the car for \$5, and get $\theta - 5$; or (iii) put the car on the market without the mechanic's report and get the market price p. Describe the equilibria of this model.

Question 3 (60 points)

A principal employs a risk–averse agent to work on a project. The worker chooses effort $e \in [0, \infty)$ at cost $c(e) = e$. The project succeeds with probability $p(e)$, which is increasing and concave and satisfies $\lim_{e \to 0} p'(e) = \infty$. The principal pays wage w_0 if the project fails and w_1 if it succeeds. The agent's utility if given by $u(w_s) - c(e)$, where $s \in \{0, 1\}$. The agent's utility function is strictly increasing and strictly concave. The agent has reservation utility $\underline{U}$.

Suppose that the principal is also risk–averse with strictly increasing, concave utility function $V(\cdot)$. The principal's utility in state $s \in \{0, 1\}$ is $V(x_s - w_s)$ where $x_1 > x_0$.

First, suppose that the agent's effort is verifiable.

(a) The principal chooses (w_0, w_1, e) to maximise profit subject to individual rationality. Write down this problem.

(b) Using the Lagrangian associated with the problem in (a), write down the FOCs that characterise the optimal wages (w_0, w_1).

Next, suppose that the agent's effort is not observable.

(c) The principal chooses (w_0, w_1, e) to maximise profit subject to individual rationality and incentive compatibility. Write down this problem.

(d) Use the first–order approach to replace the incentive compatibility constraint with the agent's first–order condition. Write down the new problem.

(e) Using the Lagrangian associated with the problem in (d), write down the FOCs that characterise the optimal wages (w_0, w_1).

Question 4 (35 points)

Consider Akerlof's model, where there are three types of car. The sellers know the type of car they own, but the buyers do not. There are equal numbers of each type, where θ is the buyers' valuation and $r(\theta)$ is the sellers' valuation. Unlike Akerlof, buyers and sellers have different preference orderings over the cars: buyers prefer θ_3 over θ_2 over θ_1, while sellers prefer θ_2 over θ_3 over θ_1. The agents' valuations for the three types of car are:

$$\begin{aligned} \theta_1 &= 5 \qquad & r(\theta_1) &= 4 \\ \theta_2 &= 20 \qquad & r(\theta_2) &= 16 \\ \theta_3 &= 25 \qquad & r(\theta_3) &= 12 \end{aligned}$$

Describe which types trade and the resulting prices in the competitive equilibria of this model.

Question 5 (35 points)

Consider Akerlof's model with a continuum of types. The buyers' valuations, θ, are distributed on $[0, 1]$ with density $f(\theta) = 2\theta$ (i.e. the triangular distribution). The conditional expectation is

$$E[\theta|\theta \le x] = \frac{2}{3}x$$

The reservation value of type θ is $r(\theta) = \theta - \frac{3}{10}$.

Describe the competitive equilibria of this model. In your description you should state (a) the equilibrium price and (b) what proportion of the agents trade.