Economics 326: Suggested Solutions to Final

Question 1

(a) There are two pooling equilibria.

1. $e_L = e_H = e_1$. The wages must satisfy $w(e_1) = 20$, $w(e_2) \leq 24$ and $w(e_3) \leq 28$.
2. $e_L = e_H = e_2$. The wages must satisfy $w(e_1) \leq 11$, $w(e_2) = 20$ and $w(e_3) \leq 24$.

(b) There are no separating equilibria. The low type will always copy the high type since $c(e_3, 10) \leq 20$.

Question 2

(a) There is a unique equilibrium: only the low type puts their car on the market. The price is $p = 10$. To see this, note that if the medium type also puts their car on the market the price is $p = 15 < r(\theta_M)$. Similarly, if all agents put their car on the market then $p = 20 < r(\theta_H)$

(b) For the high type, revealing dominates staying at home. For the middle type, staying at home dominates revealing. Hence this is just like Akerlof with reservation values $r(\theta_L) = 8$, $r(\theta_L) = 16$, and $r(\theta_L) = 25$. In the unique equilibrium, the low type goes on the market, the middle type stays at home, while the high type gets the mechanics report.

Question 3

(a) The problem is

$$\max_{w_1, w_0, e} \quad p(e)V(x_1 - w_1) + (1 - p(e))V(x_0 - w_0)$$

$$s.t. \quad p(e)u(w_1) + (1 - p(e))u(w_0) - e \geq \underline{U} \qquad \text{(IR)}$$

(b) The Lagrangian is

$$\begin{aligned} \mathcal{L} &= p(e)V(x_1 - w_1) + (1-p(e))V(x_0 - w_0) && (1)\\ &\quad +\lambda\Big[p(e)u(w_1) + (1-p(e))u(w_0) - e - \underline{U}\Big] && (2) \end{aligned}$$

The FOC with respect to w_1 yields

$$V'(x_1 - w_1) = \lambda u'(w_1)$$

The FOC with respect to w_0 yields

$$V'(x_0 - w_0) = \lambda u'(w_0)$$

Thus the ratio of the marginal utilities is constant across states. Thus is known as the Borsch rule.

(c) The problem is

$$\begin{aligned} \max_{w_1,w_0,e} \quad & p(e)V(x_1 - w_1) + (1-p(e))V(x_0 - w_0) \\ s.t. \quad & p(e)u(w_1) + (1-p(e))u(w_0) - e \geq \underline{U} && \text{(IR)}\\ & p(e)u(w_1) + (1-p(e))u(w_0) - e \geq p(e')u(w_1) + (1-p(e'))u(w_0) - e' \quad (\forall e') && \text{(IC)} \end{aligned}$$

(d) The agent chooses e to maximise

$$p(e)u(w_1) + (1-p(e))u(w_0) - e$$

The problem is thus

$$\begin{aligned} \max_{w_1,w_0,e} \quad & p(e)V(x_1 - w_1) + (1-p(e))V(x_0 - w_0) \\ s.t. \quad & p(e)u(w_1) + (1-p(e))u(w_0) - e \geq \underline{U} && \text{(IR)}\\ & p'(e)(u(w_1) - u(w_0)) = 1 && \text{(ICFOC)} \end{aligned}$$

(e) The Lagrangian is

$$\begin{aligned} \mathcal{L} &= p(e)V(x_1 - w_1) + (1 - p(e))V(x_0 - w_0) \quad (3) \\ &+\lambda\Big[p(e)u(w_1) + (1 - p(e))u(w_0) - e - \underline{U}\Big] \quad (4) \\ &+\mu\Big[p'(e)(u(w_1) - u(w_0)) - 1\Big] \quad (5) \end{aligned}$$

The FOC with respect to w_1 yields

$$-p(e)V'(x_1 - w_1) + \lambda p(e)u'(w_1) + \mu p'(e)u'(w_1) = 0$$

Rearranging,

$$\frac{V'(x_1 - w_1)}{u'(w_1)} = \lambda + \mu \frac{p'(e)}{p(e)}$$

The FOC with respect to w_0 yields

$$-(1 - p(e))V'(x_0 - w_0) + \lambda(1 - p(e))u'(w_0) - \mu p'(e)u'(w_0) = 0$$

Rearranging,

$$\frac{V'(x_0 - w_0)}{u'(w_0)} = \lambda - \mu \frac{p'(e)}{1 - p(e)}$$

Question 4

There are three equilibria.

- θ_1 trades. $p = 5$
- θ_1 and θ_3 trade. $p = 15$.
- All three trade. $p = 16\frac{2}{3}$.

Question 5

First let's check the corner solutions. If no agents trade, then $p = 0$ and $r(0) = -3/10$, so this cannot be an equilibrium. If all agents trade then $p = 2/3$ and $r(1) = 7/10$, so this cannot be an equilibrium.

The buyers willingness to pay is

$$\begin{aligned} E\left[\theta \,\middle|\, \theta - \frac{3}{10} \leq p\right] =& E\left[\theta \,\middle|\, \theta \leq p + \frac{3}{10}\right] \\ =& \frac{2}{3}\left[p + \frac{3}{10}\right] \\ =& \frac{2}{3}p + \frac{2}{10} \end{aligned}$$

In equilibrium the price is $p = 3/5$, so types below $9/10$ enter the market. The cumulative density is $F(\theta) = \theta^2$, so proportion $81/100$ trade.