Economics 326: Homework 1

18 January, 2006

The following questions concern Akerlof's model of asymmetric information. In all the following questions we assume that there are a large number of buyers (in particular, more buyers than sellers). The later questions are harder than the earlier ones.

1. Suppose there are equal numbers of high and low quality cars held by sellers. If the quality is high the seller values the car at 8 and the buyer at 10. If the quality is low the seller values the car at 2 and the buyer at 5. Which types will trade in the competitive equilibrium?

2. Suppose there are equal numbers of high and low quality cars held by sellers. If the quality is high the seller values the car at 4 and the buyer at 10. If the quality is low the seller values the car at 2 and the buyer at 5. Which types will trade in the competitive equilibrium?

3. Suppose there are equal numbers of high and low quality cars held by sellers. If the quality is high the seller values the car at 7 and the buyer at 10. If the quality is low the seller values the car at 2 and the buyer at 5. Which types will trade in the competitive equilibrium?

4. Suppose the buyer's value is θ is distributed uniformly on $[1, 11]$ and the seller's reservation value is $r(\theta) = \theta - 1$.
(a) If there were perfect information what is the value of trade?
(b) With imperfect information, what types will trade? What is the value of trade?

5. Consider Akerlof's model with $\theta \sim U[1, 11]$ with $r(\theta) = \theta - 1$.
(a) Suppose sellers $\theta \in [1, \theta_t]$ participate. Show the generates price $p_t = (\theta_t + 1)/2$.
(b) Suppose the price is p_t. Show that sellers $\theta \in [1, \theta_{t+1}]$ participate, where $\theta_{t+1} = p_t - 1$.
(c) Initially suppose all sellers are in the market, $\theta_0 = 11$. Calculate the price p_0 this generates. Calculate the sellers who participate under p_0, given by $[1, \theta_1]$. Calculate the price p_1 this generates. Carry on with the algorithm, and calculate (p_t, θ_t) for $t = 0, 1, \ldots, 5$. What does this converge to?
(d) How does the answer to (c) compare with the competitive price?

6. Suppose the buyer's value, θ, is distributed uniformly on $[1, 11]$.
(a) Suppose the seller is leaving the country and has a reservation value of $r(\theta) = 0$. What is the competitive price?
(b) Suppose the seller is staying at home and has reservation value $r(\theta) = \theta - 1$.
(c) Consider the population of agents who have cars for sale. Half of these are leaving the country. Half of these are staying at home. Given a price p, what is the probability an agent staying at home will trade? Given a price p, what is the probability the agent a buyer who trades is staying at home? What will the competitive price be?

7. Suppose the buyer's value θ is distributed uniformly on $[0, 1]$ and the seller's reservation value is $r(\theta) = 1 - \theta$. (Note: $r(\theta)$ is decreasing in θ).
(a) If θ is known by both seller and buyer for what values of θ will trade occur?
(b) Suppose θ is only known by the seller. Given price p, which sellers will trade? What is the competitive equilibrium price? Which sellers trade in equilibrium?
(c) Compare the level of trade in (a) and (b). How does this outcome differ from the standard Akerlof model, where $r(\theta)$ is increasing in θ?

8. Suppose there is a seller who's reservation price r is uniform over $[0, 10]$. There is a continuum of identical buyers whose valuations v are identical and uniform over $[1, 11]$. Quality is purely subjective: r and v are independent of each other.
(a) First suppose r and v are observed by everyone. Who will trade?
(b) Suppose only the seller knows r; neither party knows the buyers' value v. What price are the buyers willing to pay? What types of seller will trade?
(c) How does trade in (a) and (b) compare?