Economics 326: Suggested Solutions 1

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1. Suppose there is an equilibrium where both types trade. Then the buyers are willing to pay 7.5. But at this price, the high types will drop out of the market. Next suppose there is an equilibrium where only the low types trade, yielding a price $p = 5$. At this price the low types are happy to trade while the high types are happy not to trade. Hence the unique equilibrium is for low types to trade.

2. First suppose both types trade, generating price $p = 7.5$. Since both types of seller are indeed happy to trade at this price, this is an equilibrium. Next, suppose only the low types trade, generating price $p = 5$. Since the high types only value the car at 4, they would put their cars on the market, contradicting the supposition that only low types trade. Hence the unique equilibrium is for both types to trade.

3. There are two equilibria in this model. One is where both types trade and the price is $p = 7.5$. The second equilibrium is where only the low types trade and the price is $p = 5$. Notice that the high–price equilibrium pareto dominates the low–price equilibrium.

4. (a) With perfect information the value of trade is the difference in valuations, i.e. 1.
(b) Suppose the market price is p. Then the expected reservation price of those participating is $E[r(\theta)|r(\theta) \leq p] = p/2$. Hence the willingness to pay of the buyer is $p/2 + 1$. Equilibrium means $p = 1 + p/2$ implying $p = 2$. Thus trade occurs 1/5 of the time and the value of this trade is is $1/5 \times 1 = 1/5$.

5. (a) $p_t = E[\theta|\theta \leq \theta_t] = (\theta_t + 1)/2$.
(b) $r(\theta_{t+1}) = p_t$. Hence $\theta_{t+1} = p_t + 1$.
(c) We have,

t	θ_t	p_t
0	11	6
1	7	4
2	5	3
3	4	5/2
4	7/2	9/4
5	13/4	17/8

Here, $p_t \to 2$ and $\theta_t \to 3$.

(d) The equilibrium price is 2, while sellers $[1,3]$ participate. We've just calculated the dynamics I drew in class.

6. (a) $p = 6$.
(b) As above, $p = 2$.
(c) At price p an agent staying at home trades with probability p, while an agent leaving the country trades with probability 1. Hence the probability you a buyer trades with a seller staying at home is $p/10 + p$. The expected quality is thus

$$\frac{10}{10+p}6 + \frac{p}{10+p}\left[\frac{p}{2}+1\right]$$

Setting this equals to the price yields the quadratic

$$\frac{p^2}{2} + 9p - 60 = 0$$

Solving, the positive solution is $p = -9 + \sqrt{201} \approx 5.18$. Note how this is closer to $p = 6$ than $p = 2$. This happens for two reasons. First, the number of nonstrategic sellers choosing to trade outweighs the number of strategic sellers. [Question: what are the relative proportions?]. Second, the introduction of the nonstrategic sellers has a multiplier effect. Their introduction first leads to a direct price increase. This induces higher quality strategic sellers to participate which further increases the price. This induces even more strategic sellers to participate which further increases the price, and so on. Eventually the process settles down at $p = 5.18$.

7. (a) Under perfect information trade occurs if $\theta \geq 1/2$.
(b) A seller trades if $\theta \geq 1 - p$. Hence $E[\theta|\ r(\theta) \leq p] = 1 - p/2$. The unique competitive price is $p = 2/3$. (How do I know this is unique?) Thus trade occurs if $\theta \geq 1/3$.
(c) Too much trade! This is the opposite to the problem when $r(\theta)$ is increasing.

8. (a) Trade will occur if $v \geq r$.
(b) Since valuations are independent, the buyer is willing to pay 600. A seller will sell if $r \leq 600$, so that 60% of sellers will sell.
(c) In (a) there is trade if $v \geq r$. In (b) there is trade if $E[v] \geq r$. For example, if $(v, r) = (100, 200)$ trade will occur under (b) but not (a). Trade under (a) is more efficient (in the pareto sense).