

Economics 326: Homework 2

22 February, 2006

The following questions concern variants of Spence's education model. Unless other stated, the utility of type θ who take e years of education and is paid wage w is $u(w, e|\theta) := w - c(e, \theta)$.

Question 1

Suppose there are two types of agents, $\theta_H \geq \theta_L$, where proportion λ are type θ_H . The cost function $c(e, \theta)$ satisfies the assumptions in the lecture (i.e. $c(0, \theta) = 0$, $c_e(e, \theta) > 0$ and $c_{e\theta}(e, \theta) < 0$).

- (a) If $1 > \lambda > 0$ describe the lowest-cost pooling equilibrium.
- (b) If $1 > \lambda > 0$ describe the lowest-cost separating equilibrium.
- (c) If $\lambda = 1$ describe the equilibrium.
- (d) In the lowest-cost pooling equilibrium, argue that θ_H 's utility is continuous as $\lambda \rightarrow 1$.
- (e) In the lowest-cost separating equilibrium, argue that θ_H 's utility is discontinuous as $\lambda \rightarrow 1$.

Question 2

Suppose there are two types of agents, $\theta_H \geq \theta_L$, where proportion λ are type θ_H . The cost of education is $c(e, \theta) = e/\theta$.

- (a) What education levels can be chosen in the set of separating equilibria?
- (b) What education levels can be chosen in the set of pooling equilibria?

Question 3

Suppose there are two types of agents, $\theta_H \geq \theta_L$, where proportion λ are type θ_H . The cost of education is $c(e, \theta) = e/\theta$. In this question, your answers can be either pictorial or algebraic.

- (a) What are the education choices in the least cost separating equilibrium? What is the wage, $w(e)$, at the chosen education levels? What restrictions must the wage function, $w(e)$, obey for

other education levels? [That is, describe the set of wage functions, rather than just give one example].

(b) What are the education choices in the least cost pooling equilibrium? What is the wage, $w(e)$, at the chosen education level? What restrictions must the wage function, $w(e)$, obey for other education levels?

Question 4

Consider Spence's signaling model with productive education. If agent θ gets e years of education then their productivity is $\theta + e$. The cost of education for type θ is $\frac{e^2}{2\theta}$.

(a) Suppose type θ (along with e) is observable. How many years of education would type θ obtain?

(b) Now suppose there are two types, $\theta_H \geq \theta_L$, where proportion λ are type θ_H . Also assume that $\theta_H \geq \theta_L \geq \theta_H/3$. Characterise the least-cost separating equilibrium.

(c) Suppose $\theta_L \leq \theta_H/3$. Characterise the least-cost separating equilibrium.

Question 5

Suppose there are three types of agents $\theta_3 > \theta_2 > \theta_1$, where there are equal numbers of each type. The costs of education is $c(e, \theta) = e/\theta$.

(a) Describe the set of fully pooling equilibria (i.e. where all three types pool).

(b) Describe the least cost fully separating equilibrium.

(c) Describe an equilibria where θ_2 and θ_1 pool, and θ_3 separates.

Question 6

Suppose there are three types of agents $\theta_3 > \theta_2 > \theta_1$, where there are equal numbers of each type. The costs of education is $c(e, \theta) = e/\theta$. Is there an equilibrium where types θ_1 and θ_3 choose education level e_1 , while θ_2 chooses education level $e_2 \neq e_1$? If there is, please describe it. If not, please explain why.

Question 7

Consider a version of Spence's signaling model where θ denotes an agent's productivity. There are two types $\{\theta_L, \theta_H\}$, with equal numbers of each type. Assume the cost of education is the *same* for both agents, $c(e) = e$.

- (a) Suppose the utility of agent θ who is paid wage w and undertakes education e is $\theta w - e$, so the high type is more desperate for money. Describe the separating equilibrium, if it exists.
- (b) Suppose the utility of agent θ who is paid wage w and undertakes education e is $w/\theta - e$, so the low type is more desperate for money. Describe the separating equilibrium, if it exists.