Economics 326: Suggested Solutions 2

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Question 1

(a) In the pooling equilibrium the high type gets $w_H = \lambda\theta_H + (1-\lambda)\theta_L$ and $e_H = 0$.
(b) In the separating equilibrium the high type gets $w_H = \theta_H$ and $e_H = \tilde{e} > 0$ independent of λ.
(c) If $\lambda = 1$ the high type gets $w_H = \theta_H$ and $e_H = 0$.
(d) Under the pooling equilibrium, $w_H \to \theta_H$ and $e_H \to 0$ as $\lambda \to 1$.
(e) Under the separating equilibrium, $w_H \to \theta_H$ and $e_H \to \tilde{e} > 0$ as $\lambda \to 1$. Yet in the limit $e_H = 0$.

Question 2

(a) In the separating equilibria $\theta_H(\theta_H - \theta_L) \geq e \geq \theta_L(\theta_H - \theta_L)$.
(b) In the pooling equilibria $\lambda\theta_L(\theta_H - \theta_L) \geq e \geq 0$.

Question 3

(a) In the least cost separating equilibrium, $e_L = 0$ and $e_H = (\theta_H - \theta_L)\theta_L$. In equilibrium, $w(e_L) = \theta_L$ and $w(e_L) = \theta_H$. Outside equilibrium, $w(e) \in [\theta_L, \theta_H]$ and $w(e) \leq \theta_L + e/\theta_L$.
(b) In the least cost pooling equilibrium, $e_L = e_H = 0$. In equilibrium, $w(e_L) = E[\theta]$. Outside equilibrium, $w(e) \in [\theta_L, \theta_H]$ and $w(e) \leq E[\theta] + e/\theta_L$.

Question 4

Job market signaling with productive education.

(a) Agents θ chooses e to maximise

$$\theta + e - \frac{e^2}{2\theta}$$

Maximising, $e^* = \theta$.
(b) The low type obtains the efficient education level, $e_L = \theta_L$. The high type must take enough to separate himself, i.e. the low type's (IC) constrain binds.

$$\theta_L + e_L - \frac{e_L^2}{2\theta_L} = \theta_H + e_H - \frac{e_H^2}{2\theta_L}$$

Since $e_L = \theta_L$ this yields quadratic

$$e_H^2 - 2\theta_L e_H + 2\theta_L(\frac{3}{2}\theta_L - \theta_H)$$

this yields $e_H = \theta_L + \sqrt{2\theta_L(\theta_H - \theta_L)}$. One can verify that the assumption $\theta_L \geq \theta_H/3$ implies $e_H \geq \theta_H$.
(c) If $\theta_L \leq \theta_H/3$ then $e_H = \theta_H$, and the (IC) constraint is irrelevant.

Question 5

(a) The wage is $E[\theta] = \frac{1}{3}(\theta_1 + \theta_2 + \theta_3)$. The pooling equilibrium requires that the low type does not deviate, i.e. $(E[\theta] - \theta_1)\theta_1 \geq e \geq 0$. The beliefs must be accurate in equilibrium but can be set to θ_1 elsewhere.
(b) The lowest type take no education, $e_1 = 0$. The middle takes enough to separate herself from the lowest type, $e_2 = (\theta_2 - \theta_1)\theta_1$. The high type takes enough to separate themselves from the middle type, $e_3 = e_2 + (\theta_3 - \theta_2)\theta_2$. The beliefs must be accurate in equilibrium but can be set to θ_1 elsewhere.
(c) There is an equilibrium where types θ_1 and θ_2 take education $e_1 = e_2 = 0$ and receive wage $\theta_{12} := \frac{1}{2}(\theta_1 + \theta_2)$. Agent θ_3 takes education $e_3 = (\theta_3 - \theta_{12})\theta_2$ and receives wage θ_3. The beliefs must be accurate in equilibrium but can be set to θ_1 elsewhere.

Question 6

There is no such equilibrium. If θ_1 chooses e_1 and θ_2 chooses e_2, then θ_3 must choose e_2. The formal proof is as follows. Types θ_1 and θ_3 choose e_1 and get paid w_1. Type θ_2 chooses e_2 and gets paid w_2. The (IC) constraint for θ_1 says

$$w_1 - \frac{e_1}{\theta_1} \geq w_2 - \frac{e_2}{\theta_1}$$

The (IC) constraint for θ_2 says

$$w_2 - \frac{e_2}{\theta_2} \geq w_1 - \frac{e_1}{\theta_2}$$

Putting these together,

$$\theta_2(w_2 - w_1) \geq e_2 - e_1 \geq \theta_1(w_2 - w_1)$$

Hence $w_2 \geq w_1$. This means that

$$\theta_3(w_2 - w_1) \geq \theta_2(w_2 - w_1) \geq e_2 - e_1$$

and θ_3 prefers (e_2, w_2) over (e_1, w_1).

Question 7

Spence's signaling with re-normalised utility.

(a) A separating equilibrium exists: $e_L = 0$ and $e_H = \theta_L(\theta_H - \theta_L)$.
(b) A separating equilibrium does not exist. If the high type is willing to undertake any education level, the high type will copy them.