Economics 385: Homework 1

19 January, 2007

Akerlof with Discrete Types

The following questions concern Akerlof's model of asymmetric information. In all the following questions we assume that there are a large number of buyers (in particular, more buyers than sellers). The later questions are harder than the earlier ones.

Question 1

Suppose there are equal numbers of high and low quality cars held by sellers. If the quality is high the seller values the car at 8 and the buyer at 10. If the quality is low the seller values the car at 2 and the buyer at 5. Which types will trade in the competitive equilibrium?

Question 2

Suppose there are equal numbers of high and low quality cars held by sellers. If the quality is high the seller values the car at 4 and the buyer at 10. If the quality is low the seller values the car at 2 and the buyer at 5. Which types will trade in the competitive equilibrium?

Question 3

Suppose there are equal numbers of high and low quality cars held by sellers. If the quality is high the seller values the car at 7 and the buyer at 10. If the quality is low the seller values the car at 2 and the buyer at 5. Which types will trade in the competitive equilibrium?

Question 4

Consider Akerlof's model with two types of sellers. Proportion $\pi \in (0, 1)$ are of low quality.

- If quality is low, the buyers' and sellers' values are $v_B^L = 20$, $v_S^L = 10$.
- If quality is high, the buyers' and sellers' values are $v_B^H = 30$, $v_S^H = 25$.

(a) For what values of π does a competitive equilibrium exist where only low types trade?

(b) For what values of π does a competitive equilibrium exist where both low and high types trade?

Question 5

Consider Akerlof's model with equal numbers of three types of sellers.

- If quality is low, the buyers' and sellers' values are $v_B^L = 8$, $v_S^L = 5$.
- If quality is medium, the buyers' and sellers' values are $v_B^M = 13$, $v_S^M = 10$.
- If quality is high, the buyers' and sellers' values are $v_B^H = 18$, $v_S^H = 15$.

Describe which types trade and the resulting prices in the competitive equilibria of this model.

Question 6

Consider Akerlof's model, where there are three types of car. There are equal numbers of each type, where θ is the buyers' valuation and $r(\theta)$ is the sellers' valuation. Unlike Akerlof, buyers and sellers have different preference orderings over the cars: buyers prefer θ_3 over θ_2 over θ_1, while sellers prefer θ_2 over θ_3 over θ_1. The agents' valuations for the three types of car are:

$$\begin{aligned} \theta_1 &= 5 \qquad & r(\theta_1) &= 4 \\ \theta_2 &= 20 \qquad & r(\theta_2) &= 16 \\ \theta_3 &= 25 \qquad & r(\theta_3) &= 12 \end{aligned}$$

Describe which types trade and the resulting prices in the competitive equilibria of this model.

Akerlof with Continuous Types

Question 1

Suppose the buyer's value is θ is distributed uniformly on $[1, 11]$ and the seller's reservation value is $r(\theta) = \theta - 1$.

(a) If there were perfect information what is the value of trade?

(b) With imperfect information, what types will trade? What is the value of trade?

Question 2

Consider Akerlof's model with $\theta \sim U[1, 11]$ with $r(\theta) = \theta - 1$.

(a) Suppose sellers $\theta \in [1, \theta_t]$ participate. Show the generates price $p_t = (\theta_t + 1)/2$.

(b) Suppose the price is p_t. Show that sellers $\theta \in [1, \theta_{t+1}]$ participate, where $\theta_{t+1} = p_t + 1$.

(c) Initially suppose all sellers are in the market, $\theta_0 = 11$. Calculate the price p_0 this generates. Calculate the sellers who participate under p_0, given by $[1, \theta_1]$. Calculate the price p_1 this generates. Carry on with the algorithm, and calculate (p_t, θ_t) for $t = 0, 1, \ldots, 5$. What does this converge to?

(d) How does the answer to (c) compare with the competitive price?

Question 3 (hard)

Suppose the buyer's value, θ, is distributed uniformly on $[1, 11]$.

(a) Suppose the seller is leaving the country and has a reservation value of $r(\theta) = 0$. What is the competitive price?

(b) Suppose the seller is staying at home and has reservation value $r(\theta) = \theta - 1$.

(c) Consider the population of agents who have cars for sale. Half of these are leaving the country. Half of these are staying at home. Given a price p, what is the probability an agent staying at home will trade? Given a price p, what is the probability the agent a buyer who trades is staying at home? What will the competitive price be?

Question 4

Consider Akerlof's model with a continuum of types. Each seller has a good of quality $\theta \sim U[0,1]$. As in the lecture, a buyer values a good of quality θ at θ.

In the following two cases, identify the equilibrium price levels. If there are multiple equilibria, which is most reasonable?

(a) A seller's reserve value for a good of quality θ is given by $r(\theta) = 3\theta/4$.

(b) A seller's reserve value for a good of quality θ is given by $r(\theta) = \theta/4$

Question 5

Consider Akerlof's model with a continuum of types. The buyers' valuations, θ, are distributed on $[0,1]$ with density $f(\theta) = 2\theta$ (i.e. the triangular distribution). The conditional expectation is

$$E[\theta|\theta \leq x] = \frac{2}{3}x$$

The reservation value of type θ is $r(\theta) = \theta - \frac{3}{10}$.

Describe the competitive equilibria of this model. In your description you should state (a) the equilibrium price and (b) what proportion of the agents trade.

Question 6

Suppose the buyer's value θ is distributed uniformly on $[0,1]$ and the seller's reservation value is $r(\theta) = 1 - \theta$. (Note: $r(\theta)$ is decreasing in θ).

(a) If θ is known by both seller and buyer for what values of θ will trade occur?

(b) Suppose θ is only known by the seller. Given price p, which sellers will trade? What is the competitive equilibrium price? Which sellers trade in equilibrium?

(c) Compare the level of trade in (a) and (b). How does this outcome differ from the standard Akerlof model, where $r(\theta)$ is increasing in θ?

Question 7

Suppose there is a seller who's reservation price r is uniform over $[0, 10]$. There is a continuum of identical buyers whose valuations v are identical and uniform over $[1, 11]$. Quality is purely subjective: r and v are independent of each other.

(a) First suppose r and v are observed by everyone. Who will trade?

(b) Suppose only the seller knows r; neither party knows the buyers' value v. What price are the buyers willing to pay? What types of seller will trade?

(c) How does trade in (a) and (b) compare?

Information Disclosure

Question 1

Consider the information disclosure model. There are equal numbers of three types of sellers, $\theta \in \{10, 20, 40\}$. As in class, agents can either reveal or hide their information, but cannot lie. Buyers are competitive, so the price is bid up to the seller's type (if the seller reveals) or the expected type of the sellers who hide (if the seller chooses to hide).

(a) Argue that, in equilibrium, all agents will fully disclose their information (except possibly the lowest type).

(b) Suppose there is cost 8 to revealing information (e.g. the cost of the mechanic's report). Describe the equilibria of this game. [Hint: there are two of them].

Question 2

There are a large number of agents selling their cars. There are three type of cars (low, medium, high), with equal numbers of each type. Sellers initially know the type of car they own, however buyers cannot observe this quality. Buyers are competitive and bid the price up to their expected willingness to pay. The willingness to pay, θ, and reservation utilities, $r(\theta)$, for each of the three types of cars are:

$$\begin{aligned} \theta_L = 10 \qquad & r(\theta_L) = 8 \\ \theta_M = 20 \qquad & r(\theta_M) = 16 \\ \theta_H = 30 \qquad & r(\theta_H) = 24 \end{aligned}$$

(a) As in Akerlof, suppose sellers can choose whether or not to participate in the market. If a seller trades, she get the market price, p; if a seller does not trade, she gets her reservation utility, $r(\theta)$. Describe the equilibria of the model.

(b) Now suppose sellers have three choices: (i) stay at home and get $r(\theta)$; (ii) have a mechanic certify the car for \$5, and get $\theta - 5$; or (iii) put the car on the market without the mechanic's report and get the market price p. Describe the equilibria of this model.