Economics 385: Suggested Solutions 1

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Akerlof with Discrete Types

Question 1

Suppose there is an equilibrium where both types trade. Then the buyers are willing to pay 7.5. But at this price, the high types will drop out of the market. Next suppose there is an equilibrium where only the low types trade, yielding a price $p = 5$. At this price the low types are happy to trade while the high types are happy not to trade. Hence the unique equilibrium is for low types to trade.

Question 2

First suppose both types trade, generating price $p = 7.5$. Since both types of seller are indeed happy to trade at this price, this is an equilibrium. Next, suppose only the low types trade, generating price $p = 5$. Since the high types only value the car at 4, they would put their cars on the market, contradicting the supposition that only low types trade. Hence the unique equilibrium is for both types to trade.

Question 3

There are two equilibria in this model. One is where both types trade and the price is $p = 7.5$. The second equilibrium is where only the low types trade and the price is $p = 5$. Notice that the high–price equilibrium pareto dominates the low–price equilibrium.

Question 4

Suppose proportion π are low quality.
(a) The equilibrium where only low types trade always exists.
(b) The equilibrium where both types trade exists if $20\pi + 30(1 - \pi) \geq 25$. That is, $\pi \leq 1/2$.

Question 5

There are two equilibria. First, only low types trade and $p = 8$. Second, low and medium types trade and $p = 10.5$.

Question 6

There are three equilibria.

- θ_1 trades. $p = 5$
- θ_1 and θ_3 trade. $p = 15$.
- All three trade. $p = 16\frac{2}{3}$.

Akerlof with Continuous Types

Question 1

(a) With perfect information the value of trade is the difference in valuations, i.e. 1.
(b) Suppose the market price is p. Then the expected reservation price of those participating is $E[r(\theta)|r(\theta) \leq p] = p/2$. Hence the willingness to pay of the buyer is $p/2 + 1$. Equilibrium means $p = 1 + p/2$ implying $p = 2$. Thus trade occurs 1/5 of the time and the value of this trade is is $1/5 \times 1 = 1/5$.

Question 2

(a) $p_t = E[\theta|\theta \leq \theta_t] = (\theta_t + 1)/2$.
(b) $r(\theta_{t+1}) = p_t$. Hence $\theta_{t+1} = p_t + 1$.
(c) We have,

t	θ_t	p_t
0	11	6
1	7	4
2	5	3
3	4	5/2
4	7/2	9/4
5	13/4	17/8

Here, $p_t \to 2$ and $\theta_t \to 3$.
(d) The equilibrium price is 2, while sellers $[1, 3]$ participate. We've just calculated the dynamics I drew in class.

Question 3

(a) $p = 6$.
(b) As above, $p = 2$.
(c) At price p an agent staying at home trades with probability $p/10$, while an agent leaving the country trades with probability 1. It follows that fraction $[1+p/10]/2$ of all sellers sell their

cars. Using Bayes' rule, the expected quality is thus

$$\frac{[1]/2}{[1+p/10]/2}\,6+\frac{[p/10]/2}{[1+p/10]/2}\left[\frac{p}{2}+1\right]=\frac{10}{10+p}\,6+\frac{p}{10+p}\left[\frac{p}{2}+1\right]$$

Setting this equals to the price yields the quadratic

$$\frac{p^2}{2}+9p-60=0$$

Solving, the positive solution is $p=-9+\sqrt{201}\approx 5.18$. Note how this is closer to $p=6$ than $p=2$. This happens for two reasons. First, the number of nonstrategic sellers choosing to trade outweighs the number of strategic sellers. [Question: what are the relative proportions?]. Second, the introduction of the nonstrategic sellers has a multiplier effect. Their introduction first leads to a direct price increase. This induces higher quality strategic sellers to participate which further increases the price. This induces even more strategic sellers to participate which further increases the price, and so on. Eventually the process settles down at $p=5.18$.

Question 4

Multiplicative reserve values:
(a) The expected quality, given price p, is $E[\theta|r(\theta)\le p]=E[\theta|\theta\le 4p/3]$. This equals $2p/3$ if $p\le 3/4$, and $1/2$ if $p\ge 3/4$. Hence the unique equilibrium is $p=0$. There is complete unravelling.
(b) The expected quality, given price p, is $2p$ if $p\le 1/4$, and $1/2$ if $p\ge 1/4$. Hence there are two equilibria: $p=0$ and $p=1/2$. The latter seems more reasonable and is the only stable one.

Question 5

First let's check the corner solutions. If no agents trade, then $p=0$ and $r(0)=-3/10$, so this cannot be an equilibrium. If all agents trade then $p=2/3$ and $r(1)=7/10$, so this cannot be an equilibrium.

The buyers willingness to pay is

$$\begin{aligned} E\left[\theta \,\middle|\, \theta - \frac{3}{10} \le p\right] =& E\left[\theta \,\middle|\, \theta \le p + \frac{3}{10}\right] \\ =& \frac{2}{3}\left[p + \frac{3}{10}\right] \\ =& \frac{2}{3}p + \frac{2}{10} \end{aligned}$$

In equilibrium the price is $p = 3/5$, so types below $9/10$ enter the market. The cumulative density is $F(\theta) = \theta^2$, so proportion $81/100$ trade.

Question 6

(a) Under perfect information trade occurs if $\theta \ge 1/2$.
(b) A seller trades if $\theta \ge 1 - p$. Hence $E[\theta|\ r(\theta) \le p] = 1 - p/2$. The unique competitive price is $p = 2/3$. (How do I know this is unique?) Thus trade occurs if $\theta \ge 1/3$.
(c) Too much trade! This is the opposite to the problem when $r(\theta)$ is increasing.

Question 7

(a) Trade will occur if $v \ge r$.
(b) Since valuations are independent, the buyer is willing to pay 6. A seller will sell if $r \le 6$, so that 60% of sellers will sell.
(c) In (a) there is trade if $v \ge r$. In (b) there is trade if $E[v] \ge r$. For example, if $(v, r) = (1, 2)$ trade will occur under (b) but not (a). Trade under (a) is more efficient (in the pareto sense).

Information Disclosure

Question 1

(a) Suppose all three hide. Then the price equals $p = 23.3$ and the highest type will reveal. Suppose the low and middle types hide, then the price equals 15, and the middle type will reveal.

(b) There are two equilibria. In the first, the low and middle types hide. The middle type will not reveal since the price will only rise by 5, while the cost of revelation is 8. In the second equilibrium, only the low types hide. The middle types will not hide, since the price will fall by 10 and they will save the cost of 8.

Question 2

(a) There is a unique equilibrium: only the low type puts their car on the market. The price is $p = 10$. To see this, note that if the medium type also puts their car on the market the price is $p = 15 < r(\theta_M)$. Similarly, if all agents put their car on the market then $p = 20 < r(\theta_H)$

(b) For the high type, revealing dominates staying at home. For the middle type, staying at home dominates revealing. Hence this is just like Akerlof with reservation values $r(\theta_L) = 8$, $r(\theta_L) = 16$, and $r(\theta_L) = 25$. In the unique equilibrium, the low type goes on the market, the middle type stays at home, while the high type gets the mechanics report.