Economics 385: Homework 2

7 March, 2007

Signalling

The following questions concern variants of Spence's education model. Unless other stated, the utility of type θ who take e years of education and is paid wage w is $u(w,e|\theta) := w - c(e,\theta)$.

Question 1 (Discrete Action Set)

Consider Spence's signalling model where there are two types $\theta \in \{10, 20\}$ and $\Pr(\theta = 20) = 1/2$. Workers can obtain one of three levels of education: $e \in \{e_1, e_2, e_3\}$. Agents get utility $u(w,e|\theta) = w - c(e,\theta)$, where wages must obey $w \in [10, 20]$. The cost if given by

$$\begin{aligned} c(e_1,10) &= 0 \qquad & c(e_1,20) &= 0 \\ c(e_2,10) &= 7 \qquad & c(e_2,20) &= 4 \\ c(e_3,10) &= 14 \qquad & c(e_3,20) &= 8 \end{aligned}$$

(a) What are the separating equilibria in this model? [Your description of any equilibrium should include a supporting wage function].

(b) What are the pooling equilibria in this model? [Again, your description of any equilibrium should include a supporting wage function].

Question 2 (Discrete Action Set)

Consider Spence's signalling model where there are two types of workers $\theta \in \{10, 30\}$ and $\Pr(\theta = 30) = 1/2$. Firms are competitive and obtain profit θ from a worker of quality θ. Workers can obtain one of three levels of education: $e \in \{e_1, e_2, e_3\}$. Agents get utility $u(w,e|\theta) = w - c(e,\theta)$, where wages must obey $w \in [10, 30]$. The cost if given by

$$\begin{aligned} c(e_1, 10) &= 0 \qquad & c(e_1, 30) &= 0 \\ c(e_2, 10) &= 9 \qquad & c(e_2, 30) &= 4 \\ c(e_3, 10) &= 18 \qquad & c(e_3, 30) &= 8 \end{aligned}$$

(a) What are the pooling equilibria in this model? [You should justify your answer. Your description of any equilibrium should also include a supporting wage function].

(b) What are the separating equilibria in this model? [You should justify your answer. Your description of any equilibrium should also include a supporting wage function].

Question 3 (Discontinuities)

Suppose there are two types of agents, $\theta_H \geq \theta_L$, where proportion λ are type θ_H. The cost function $c(e, \theta)$ satisfies the assumptions in the lecture (i.e. $c(0, \theta) = 0$, $c_e(e, \theta) > 0$ and $c_{e\theta}(e, \theta) < 0$).

(a) If $1 > \lambda > 0$ describe the lowest–cost pooling equilibrium.
(b) If $1 > \lambda > 0$ describe the lowest–cost separating equilibrium.
(c) If $\lambda = 1$ describe the equilibrium.
(d) In the lowest–cost pooling equilibrium, argue that θ_H's utility is continuous as $\lambda \to 1$.
(e) In the lowest–cost separating equilibrium, argue that θ_H's utility is discontinuous as $\lambda \to 1$.

Question 4 (Linear Cost)

Suppose there are two types of agents, $\theta_H \geq \theta_L$, where proportion λ are type θ_H. The cost of education is $c(e, \theta) = e/\theta$.

(a) What education levels can be chosen in the set of separating equilibria?
(b) What education levels can be chosen in the set of pooling equilibria?

Question 5 (Linear Cost)

Suppose there are two types of agents, $\theta_H \geq \theta_L$, where proportion λ are type θ_H. The cost of education is $c(e, \theta) = e/\theta$. In this question, your answers can be either pictorial or algebraic.

(a) What are the education choices in the least cost separating equilibrium? What is the wage, $w(e)$, at the chosen education levels? What restrictions must the wage function, $w(e)$, obey for other education levels? [That is, describe the set of wage functions, rather than just give one example].

(b) What are the education choices in the least cost pooling equilibrium? What is the wage, $w(e)$, at the chosen education level? What restrictions must the wage function, $w(e)$, obey for other education levels?

Question 6 (Square Root Cost)

Consider Spence's signalling model where there are two types $\theta \in \{\theta_L, \theta_H\}$ and $\Pr(\theta = \theta_H) = \lambda$. Workers can obtain education $e \geq 0$. The agents utility function is $u(w, e|\theta) = w - c(e, \theta)$, where $c(e, \theta) = \sqrt{e}/\theta$.

(a) Verify the $c(e, \theta)$ satisfies the three conditions in the lecture. That is (i) cost is zero when $e = 0$, (ii) cost is increasing in e and (iii) the single crossing condition.

(b) What education levels are chosen in the least–cost separating equilibrium?

(c) What education levels are chosen in the most–cost separating equilibrium?

(d) What education levels are chosen in the least–cost pooling equilibrium?

(e) What education levels are chosen in the most–cost pooling equilibrium?

Question 7 (Productive Education)

Consider Spence's signaling model with productive education. If agent θ gets e years of education then their productivity is $\theta + e$. The cost of education for type θ is $\frac{e^2}{2\theta}$.

(a) Suppose type θ (along with e) is observable. How many years of education would type θ obtain?
(b) Now suppose there are two types, $\theta_H \geq \theta_L$, where proportion λ are type θ_H. Also assume that $\theta_H \geq \theta_L \geq \theta_H/3$. Characterise the least–cost separating equilibrium.
(c) Suppose $\theta_L \leq \theta_H/3$. Characterise the least–cost separating equilibrium.

Question 8 (Three Types)

Suppose there are three types of agents $\theta_3 > \theta_2 > \theta_1$, where there are equal numbers of each type. The costs of education is $c(e, \theta) = e/\theta$.

(a) Describe the set of fully pooling equilibria (i.e. where all three types pool).
(b) Describe the least cost fully separating equilibrium.
(c) Describe an equilibria where θ_2 and θ_1 pool, and θ_3 separates.

Question 9 (Three Types)

Suppose there are three types of agents $\theta_3 > \theta_2 > \theta_1$, where there are equal numbers of each type. The costs of education is $c(e, \theta) = e/\theta$. Is there an equilibrium where types θ_1 and θ_3 choose education level e_1, while θ_2 chooses education level $e_2 \neq e_1$? If there is, please describe it. If not, please explain why.

Question 10 (Changing Payoffs)

Consider a version of Spence's signaling model where θ denotes an agent's productivity. There are two types $\{\theta_L, \theta_H\}$, with equal numbers of each type. Assume the cost of education is the *same* for both agents, $c(e) = e$.

(a) Suppose the utility of agent θ who is paid wage w and undertakes education e is $\theta w - e$, so the high type is more desperate for money. Describe the separating equilibrium, if it exists.
(b) Suppose the utility of agent θ who is paid wage w and undertakes education e is $w/\theta - e$, so the low type is more desperate for money. Describe the separating equilibrium, if it exists.

Competitive Screening

Question 1

Consider the competitive screening model from class.

(a) Show that a pooling equilibrium cannot exist.

(b) Under what conditions does a separating equilibrium also not exist?

(c) Do you view this result as troubling? How might we be able to alter the model to get a more reasonable prediction?

Question 2

Consider the competitive screening model from class, except that the single crossing condition does not hold, i.e. $c_{t\theta}(t, \theta) \geq 0$. For example, we are looking to employ an accountant, and t is the number of pushups required for the job.

(a) Show that in any equilibrium, both firms make zero profits.

(b) Show that no pooling equilibrium can exist where $t > 0$.

(c) Can a pooling equilibrium exist where $t = 0$?

Question 3

Consider the competitive screening model from class. Suppose that utility is given by

$$u(w, t|\theta) = w - \frac{t}{\theta}$$

and that $\theta_L = 10$, $\theta_H = 20$. The proportion of high types is $\lambda = 1/2$.

Consider the following contracts. Are there equilibria? If not, why not?

(a) Firm A offers $(w, t) = (8, 10)$, while firm B offers $(w, t) = (18, 120)$.

(b) Firm A offers $(w,t) = (15,10)$, while firm B offers $(w,t) = (10,0)$

(c) Firm A offers $(w,t) = (15,0)$, while firm B offers $(w,t) = (20,60)$

(d) Firm A offers $(w,t) = (10,10)$ and $(w,t) = (20,110)$, while firm B offers no contract.

(e) Firm A offers $(w,t) = (10,0)$, while firm B offers $(w,t) = (18,100)$.

(f) Firm A offers $(w,t) = (10,0)$, while firm B offers $(w,t) = (20,110)$.

(g) Firm A offers $(w,t) = (10,0)$ and $(w,t) = (20,100)$, while firm B offers no contract. [Note: this is slightly subtle].

(h) Firms A offers $(w,t) = (10,0)$ and $(w,t) = (20,100)$. Firm B offers $(w,t) = (10,0)$, $(w,t) = (20,100)$ and $(w,t) = (15,90)$.