Economics 385: Suggested Solutions 2

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Signalling

Question 1 (Discrete Action Set)

(a) In the separating equilibrium, $e^*(10) = e_1$. The high type needs to obtain enough education to separate themselves, so $e^*(20) = e_3$. Wages are as follows: $w(e_1) = 10$, $w(e_3) = 20$ and $w(e_2) \leq 16$.

(b) There is one pooling equilibrium, where $e^* = e_1$. Wages are as follows: $w(e_1) = 15$, $w(e_2) \leq 19$ and $w(e_3) \leq 20$. Notice there cannot be a pooling equilibrium with more education, since the low type would deviate.

Question 2 (Discrete Action Set)

(a) There are two pooling equilibria.

1. $e_L = e_H = e_1$. The wages must satisfy $w(e_1) = 20$, $w(e_2) \leq 24$ and $w(e_3) \leq 28$.
2. $e_L = e_H = e_2$. The wages must satisfy $w(e_1) \leq 11$, $w(e_2) = 20$ and $w(e_3) \leq 24$.

(b) There are no separating equilibria. The low type will always copy the high type since $c(e_3, 10) \leq 20$.

Question 3 (Discontinuities)

(a) In the pooling equilibrium the high type gets $w_H = \lambda\theta_H + (1 - \lambda)\theta_L$ and $e_H = 0$.
(b) In the separating equilibrium the high type gets $w_H = \theta_H$ and $e_H = \tilde{e} > 0$ independent of λ.
(c) If $\lambda = 1$ the high type gets $w_H = \theta_H$ and $e_H = 0$.

(d) Under the pooling equilibrium, $w_H \to \theta_H$ and $e_H \to 0$ as $\lambda \to 1$.
(e) Under the separating equilibrium, $w_H \to \theta_H$ and $e_H \to \tilde{e} > 0$ as $\lambda \to 1$. Yet in the limit $e_H = 0$.

Question 4 (Linear Cost)

(a) In the separating equilibria $\theta_H(\theta_H - \theta_L) \geq e \geq \theta_L(\theta_H - \theta_L)$.
(b) In the pooling equilibria $\lambda\theta_L(\theta_H - \theta_L) \geq e \geq 0$.

Question 5 (Linear Cost)

(a) In the least cost separating equilibrium, $e_L = 0$ and $e_H = (\theta_H - \theta_L)\theta_L$. In equilibrium, $w(e_L) = \theta_L$ and $w(e_L) = \theta_H$. Outside equilibrium, $w(e) \in [\theta_L, \theta_H]$ and $w(e) \leq \theta_L + e/\theta_L$.
(b) In the least cost pooling equilibrium, $e_L = e_H = 0$. In equilibrium, $w(e_L) = E[\theta]$. Outside equilibrium, $w(e) \in [\theta_L, \theta_H]$ and $w(e) \leq E[\theta] + e/\theta_L$.

Question 6 (Square Root Cost)

(a) Clearly $c(0, \theta) = 0$ and $c(e, \theta)$ is increasing in e. The cross–partial is $c_{e\theta}(e, \theta) = -(1/2)e^{-1/2}\theta^{-2}$, which is negative.

(b) $e_L = 0$, while e_H is given by the indifference condition

$$\theta_L - \frac{0}{\theta_L} = \theta_H - \frac{\sqrt{e_H}}{\theta_L}$$

Rearranging, $e_H = (\theta_H - \theta_L)^2\theta_L^2$.

(c) $e_L = 0$ and $e_H = (\theta_H - \theta_L)^2\theta_H^2$.

(d) $e_L = e_H = 0$.

(e) $e_L = e_H = (E[\theta] - \theta_L)^2\theta_L^2$

Question 7 (Productive Education)

(a) Agents θ chooses e to maximise

$$\theta + e - \frac{e^2}{2\theta}$$

Maximising, $e^* = \theta$.
(b) The low type obtains the efficient education level, $e_L = \theta_L$. The high type must take enough to separate himself, i.e. the low type's (IC) constrain binds.

$$\theta_L + e_L - \frac{e_L^2}{2\theta_L} = \theta_H + e_H - \frac{e_H^2}{2\theta_L}$$

Since $e_L = \theta_L$ this yields quadratic

$$e_H^2 - 2\theta_L e_H + 2\theta_L(\frac{3}{2}\theta_L - \theta_H) = 0$$

this yields $e_H = \theta_L + \sqrt{2\theta_L(\theta_H - \theta_L)}$. One can verify that the assumption $\theta_L \geq \theta_H/3$ implies $e_H \geq \theta_H$.
(c) If $\theta_L \leq \theta_H/3$ then $e_H = \theta_H$, and the (IC) constraint is irrelevant.

Question 8 (Three Types)

(a) The wage is $E[\theta] = \frac{1}{3}(\theta_1 + \theta_2 + \theta_3)$. The pooling equilibrium requires that the low type does not deviate, i.e. $(E[\theta] - \theta_1)\theta_1 \geq e \geq 0$. The beliefs must be accurate in equilibrium but can be set to θ_1 elsewhere.
(b) The lowest type take no education, $e_1 = 0$. The middle takes enough to separate herself from the lowest type, $e_2 = (\theta_2 - \theta_1)\theta_1$. The high type takes enough to separate themselves from the middle type, $e_3 = e_2 + (\theta_3 - \theta_2)\theta_2$. The beliefs must be accurate in equilibrium but can be set to θ_1 elsewhere.
(c) There is an equilibrium where types θ_1 and θ_2 take education $e_1 = e_2 = 0$ and receive wage $\theta_{12} := \frac{1}{2}(\theta_1 + \theta_2)$. Agent θ_3 takes education $e_3 = (\theta_3 - \theta_{12})\theta_2$ and receives wage θ_3. The beliefs must be accurate in equilibrium but can be set to θ_1 elsewhere.

Question 9 (Three Types)

There is no such equilibrium. If θ_1 chooses e_1 and θ_2 chooses e_2, then θ_3 must choose e_2. The formal proof is as follows. Types θ_1 and θ_3 choose e_1 and get paid w_1. Type θ_2 chooses e_2 and gets paid w_2. The (IC) constraint for θ_1 says

$$w_1 - \frac{e_1}{\theta_1} \geq w_2 - \frac{e_2}{\theta_1}$$

The (IC) constraint for θ_2 says

$$w_2 - \frac{e_2}{\theta_2} \geq w_1 - \frac{e_1}{\theta_2}$$

Putting these together,

$$\theta_2(w_2 - w_1) \geq e_2 - e_1 \geq \theta_1(w_2 - w_1)$$

Hence $w_2 \geq w_1$. This means that

$$\theta_3(w_2 - w_1) \geq \theta_2(w_2 - w_1) \geq e_2 - e_1$$

and θ_3 prefers (e_2, w_2) over (e_1, w_1).

Question 10 (Changing Payoffs)

Spence's signaling with re-normalised utility.

(a) A separating equilibrium exists: $e_L = 0$ and $e_H = \theta_L(\theta_H - \theta_L)$.
(b) A separating equilibrium does not exist. If the high type is willing to undertake any education level, the high type will copy them.

Competitive Screening

Question 1

(a) Suppose A is offering a pooling contract. B can make money by skimming off the high types.

(b) A separating equilibrium is beaten by a pooling contract if the high type prefers $(w, t) = (E[\theta], 0)$ to the least cost separating equilibrium.

(c) There is no right answer. Personally, I don't think this result is particularly troubling: separating equilibria don't exist when they are Pareto dominated, which is no bad thing. In practice, it seems the ability for one firm to skim off another is somewhat limited. One approach, due to Wilson (1979), is to allow the pooling firm to withdraw his pooling contract after the separating firm has skimmed off the top. This means that the separating firm is then lumbered with both types. One also may be able to obtain existence by allowing mixed strategies or more types.

Question 2

(a) Same as Lemma 1 in class. Let (w_L, t_L) and (w_H, t_H) be the proposed equilibrium contracts, and suppose industry profits are positive. The firm making less profits should undercut and offer $(w_L - \epsilon, t_L)$ and $(w_H - \epsilon, t_H)$.

(b) Suppose the equilibrium contract is $(E[\theta], t)$, where $t > 0$. Then one firm can offer $(E[\theta] - \epsilon, 0)$ and make a profit.

(c) Yes. It is now impossible to skim off the high type.

Question 3

Below I given one deviation in the cases that are not equilibria. There may be other perfectly good deviations.

(a) Firm B can offer $(w, t) = (9, 10)$ and steal the low type.

(b) Firm B can offer $(w, t) = (16, 25)$ and steal the high type.

(c) Firm A should withdraw his contract, since he's making a loss.

(d) Firm B can offer $(w, t) = (9.5, 0)$ and steal the low type.

(e) Firm A can offer $(w, t) = (19, 100)$ and steal the high type.

(f) Firm A can offer $(w, t) = (19.6, 100)$ and steal the high type.

(g) Since B offers no contracts, A can unilaterally lower his wages and offer $(w, t) = (9, 0)$ and $(w, t) = (19, 100)$ and make a positive profit.

(h) This is an equilibrium. Observe that B's third contract, $(w, t) = (15, 90)$, is not accepted by either agent.