Economics 385: Homework 3

1 April, 2007

Moral Hazard

Question 1

A principal employs an agent to work on a project. The worker chooses effort $e \in \{L, H\}$ at costs $c_L = 0$ and $c_H = 1$. The project succeeds with probability

$$\begin{aligned} p_H &= 1/2 \qquad \text{if} \quad e = H \\ p_L &= 0 \qquad \text{if} \quad e = L. \end{aligned}$$

The principal pays wage w_0 if the project fails and w_1 if it succeeds. The agent's utility if given by $u(w_s) - c_e$, where $s \in \{0, 1\}$. Utility $u(\cdot)$ is strictly increasing and concave. The agent has reservation utility $\underline{U} = 0$. The principal's profit is $x_s - w_s$ where $x_1 > x_0$.

(a) Suppose the principal can contract on effort, e. Assume the principal wishes to implement e_H. The principal maximises her profit subject to individual rationality. Write down this maximisation problem.

(b) What wages will the principal offer in the profit–maximising contract?

(c) Now suppose that the principal cannot observe effort, e. Again, assume the principal wishes to implement e_H. The principal maximises her profit subject to individual rationality and incentive compatibility. Write down this maximisation problem.

(d) What wages will the principal offer in the profit–maximising contract?

(e) How does the answer from (b) differ from that in (d)? Why?

Question 2

Consider the setup in Question 1, except that the probability of success is

$$\begin{aligned} p_H &= 1 \qquad \text{if} \quad e = H \\ p_L &= 1/2 \qquad \text{if} \quad e = L. \end{aligned}$$

(a) Suppose the principal can contract on effort, e. Assume the principal wishes to implement e_H. The principal maximises her profit subject to individual rationality. Write down this maximisation problem.

(b) What wages will the principal offer in the profit–maximising contract?

(c) Now suppose that the principal cannot observe effort, e. Again, assume the principal wishes to implement e_H. The principal maximises her profit subject to individual rationality and incentive compatibility. Write down this maximisation problem.

(d) What wages will the principal offer in the profit–maximising contract?

(e) How does the answer from (b) differ from that in (d)? Why?

(f) How does the second–best solution in this question compare to the second–best solution in Question 1? Why?

Question 3

This questions show another way of tackling the moral hazard problem. One can use this approach to solve games where the agent has many actions or many outputs.

A principal employs an agent to work on a project. The worker chooses unobserved effort $e \in \{L, H\}$ at costs c_L and c_H. The project succeeds with probability p_H if $e = H$ and p_L if $e = L$, where $p_H > p_L$. The principal pays wage w_0 if the project fails and w_1 if it succeeds. The agent's utility if given by $u(w_s) - c_e$, where $s \in \{0, 1\}$. Utility $u(\cdot)$ is strictly increasing and strictly concave. The agent has reservation utility $\underline{U}$. The principal's profit is $x_s - w_s$ where $x_1 > x_0$.

First, suppose the principal chooses to implement $e = L$.

(a) Write down the principal's maximisation problem.

(b) Ignore the (IC) constraint. Write down the Lagrangian associated with the principal's problem. Denote the multiplier associated with the (IR) constraint by λ.

(c) Derive the first–order conditions of the Lagrangian with respect to (w_0, w_1). Use the strict concavity of utility to show the wage is constant. Use (IR) to derive this wage.

(d) Finally, show that the (IC) constraint is satisfied by the optimal wage.

Next, suppose the principal chooses to implement $e = H$.

(e) Write down the principal's maximisation problem.

(f) Write down the Lagrangian associated with the principal's problem. Denote the multiplier associated with the (IR) constraint by λ, and that associated with the (IC) constraint by μ. Derive the first–order conditions of the Lagrangian with respect to (w_0, w_1).

(g) Since (IC) is an inequality constraint $\mu \geq 0$. Using the strict concavity of utility, show that $\mu > 0$. [Hint: Prove by contradiction]. Finally, show that that $w_1 > w_0$ at the optimal solution.

Question 4 (optional)

The following question considers the two–action continuous–output model, which is a bit harder than the stuff in the lecture. It uses the same solution technique as Question 3, expect that sums are replaced by integrals.

A principal employs an agent to work. The worker chooses unobserved effort $e \in \{L, H\}$ at costs c_L and c_H. Output q is distributed according to $f(q|e)$ if the agent takes effort e. Suppose a higher effort induces higher output in the sense of first order stochastic dominance, i.e. $F(q|H) \leq F(q|L)$, where $F(q|e)$ is the cumulative distribution of output. The principal pays the worker $w(q)$ if they produce output q.

The agent's utility if given by $u(w(q)) - c_e$, where utility $u(\cdot)$ is strictly increasing and strictly concave. Thus the expected utility of an agent who chooses e is

$$\int (u(w(q)) - c_e) f(q|e)\, dq - c_e$$

The agent has reservation utility $\underline{U}$.

The principal's profit is $q - w(q)$ when output q is realised. Expected profit if the agent chooses e is thus

$$\int (q - w(q)) f(q|e)\, dq$$

First, suppose the principal chooses to implement $e = L$.

(a) Write down the principal's maximisation problem (i.e. maximising profit subject to (IC) and (IR)).

(b) Ignore the (IC) constraint. Write down the Lagrangian associated with the principal's problem. Denote the multiplier associated with the (IR) constraint by λ.

(c) Derive the first–order condition of the Lagrangian with respect to $w(q)$. Use the strict concavity of utility to show the wage is constant. Use (IR) to derive this wage. [Technical note: the derivative of $u(w(q_1)) + u(w(q_2))$ with respect to $w(q_1)$ is $u'(w(q_1))$. Similarly, the derivative of $\int u(w(q))\, dq$ with respect to $w(q)$ is $u'(w(q))$ $(\forall q)$. This is known as differentiating "pointwise"].

(d) Finally, show that the (IC) constraint is satisfied by the optimal wage.

Next, suppose the principal chooses to implement $e = H$.

(e) Write down the principal's maximisation problem.

(f) Write down the Lagrangian associated with the principal's problem. Denote the multiplier associated with the (IR) constraint by λ, and that associated with the (IC) constraint by μ. Derive the first–order condition of the Lagrangian with respect to $w(q)$.

(g) Since (IC) is an inequality constraint $\mu \geq 0$. Argue that $\mu > 0$. [Hint: Prove by contradiction].