Economics 385: Suggested Solutions 3

1 April, 2007

Moral Hazard

Question 1

(a) The problem is

$$\begin{aligned} \max_{w_1,w_0} \quad & \frac{1}{2}[x_1 - w_1] + \frac{1}{2}[x_0 - w_0] \\ s.t \quad & \frac{1}{2}u(w_1) + \frac{1}{2}u(w_0) - 1 \geq 0 \qquad \text{(IR)} \end{aligned}$$

(b) Clearly, (IR) will bind (else the principal should reduce wages). Since the agent is risk averse, $w_0 = w_1$. Thus (IR) yields $u(w_0) = u(w_2) = 1$

(c) The problem is

$$\begin{aligned} \max_{w_1,w_0} \quad & \frac{1}{2}[x_1 - w_1] + \frac{1}{2}[x_0 - w_0] \\ s.t. \quad & \frac{1}{2}u(w_1) + \frac{1}{2}u(w_0) - 1 \geq 0 \qquad \text{(IR)} \\ & \frac{1}{2}u(w_1) + \frac{1}{2}u(w_0) - 1 \geq u(w_0) \qquad \text{(IC)} \end{aligned}$$

(d) Following the arguments in class, both (IR) and (IC) will bind. Solving yields

$$u(w_0) = 0$$
$$u(w_1) = 2$$

(e) Wages are more spread in (d) because the principal needs to provide incentives for the agent to work.

Question 2

(a) The problem is

$$\max_{w_1,w_0} \quad [x_1 - w_1]$$
$$s.t \quad u(w_1) - 1 \geq 0 \qquad \text{(IR)}$$

(b) Clearly (IR) will bind (else the principal should reduce wages). Thus $u(w_1) = 1$. Wage w_0 is not defined.

(c) The problem is

$$\max_{w_1,w_0} \quad [x_1 - w_1]$$
$$s.t. \quad u(w_1) - 1 \geq 0 \qquad \text{(IR)}$$
$$u(w_1) - 1 \geq \frac{1}{2}u(w_1) + \frac{1}{2}u(w_0) \qquad \text{(IC)}$$

(d) Optimal wages are given by

$$u(w_1) = 1$$
$$u(w_0) = -\infty$$

(e) The answer is essentially the same. The inability of the principal to observe effort has no affect wages or utilities.

(f) This example is peculiar: if the project fails, the principal knows that the agent shirked. Thus by punishing failure sufficiently strongly the principal can stop the agent from shirking and provide full insurance to an agent who works.

Question 3

(a) The problem is

$$\max_{w_1,w_0} \quad p_L[x_1 - w_1] + (1-p_L)[x_0 - w_0]$$

$$s.t. \quad p_L u(w_1) + (1-p_L)u(w_0) - c_L \geq \underline{U} \qquad \text{(IR)}$$

$$p_L u(w_1) + (1-p_L)u(w_0) - c_L \geq p_H u(w_1) + (1-p_H)u(w_0) - c_H \qquad \text{(IC)}$$

(b) Ignoring (IC), the Lagrangian is

$$\mathcal{L} = p_L[x_1 - w_1] + (1-p_L)[x_0 - w_0] \qquad (1)$$

$$+\lambda\Big[p_L u(w_1) + (1-p_L)u(w_0) - c_L - \underline{U}\Big] \qquad (2)$$

(c) The FOC with respect to w_0 can be rearranged to obtain

$$\frac{1}{u'(w_0)} = \lambda$$

The FOC with respect to w_1 can be rearranged to obtain

$$\frac{1}{u'(w_1)} = \lambda$$

By the strict concavity of utility, $u'(w)$ is strictly decreasing and the optimal wage is constant, $w_1 = w_0 =: w$. Using (IR), $u(w) = c_L + \underline{U}$.

(d) Since the wage is constant the agent is happy to do no work. One can easily check (IC) holds.

(e) The problem is

$$\max_{w_1,w_0} \quad p_H[x_1 - w_1] + (1-p_H)[x_0 - w_0]$$

$$s.t. \quad p_H u(w_1) + (1-p_H)u(w_0) - c_H \geq \underline{U} \qquad \text{(IR)}$$

$$p_H u(w_1) + (1-p_H)u(w_0) - c_H \geq p_L u(w_1) + (1-p_L)u(w_0) - c_L \qquad \text{(IC)}$$

(f) The Lagrangian is

$$\begin{aligned}\mathcal{L} &= p_H[x_1 - w_1] + (1-p_H)[x_0 - w_0] &(3)\\ &+\lambda\Big[p_H u(w_1) + (1-p_H)u(w_0) - c_H - \underline{U}\Big] &(4)\\ &+\mu\Big[p_H u(w_1) + (1-p_H)u(w_0) - c_H - p_L u(w_1) - (1-p_L)u(w_0) + c_L\Big] &(5)\end{aligned}$$

The FOC with respect to w_0 can be rearranged to obtain

$$\frac{1}{u'(w_0)} = \lambda + \mu\left[1 - \frac{1-p_L}{1-p_H}\right]$$

The FOC with respect to w_1 can be rearranged to obtain

$$\frac{1}{u'(w_1)} = \lambda + \mu\left[1 - \frac{p_L}{p_H}\right]$$

(g) Suppose $\mu = 0$. By the strict concavity of utility, the wage is flat. Yet this would induce the worker to choose e_L. Hence $\mu > 0$ and $w_1 > w_0$.

Question 4

(a) The problem is

$$\begin{aligned}\max_{w_1,w_0} \quad & \int [q - w(q)]f(q|L)\,dq \\ s.t. \quad & \int u(w(q))f(q|L)\,dq - c_L \geq \underline{U} &\text{(IR)}\\ & \int u(w(q))f(q|L)\,dq - c_L \geq \int u(w(q))f(q|H)\,dq - c_H &\text{(IC)}\end{aligned}$$

(b) Ignoring (IC), the Lagrangian is

$$\begin{aligned}\mathcal{L} &= \int [q - w(q)]f(q|L)\,dq &(6)\\ &+\lambda\left[\int u(w(q))f(q|L)\,dq - c_L - \underline{U}\right] &(7)\end{aligned}$$

(c) The FOC with respect to $w(q)$ is

$$-f(q|L) + u'(w(q))f(q|L) = 0$$

This can be rearranged to obtain

$$\frac{1}{u'(w(q))} = \lambda$$

By strict concavity of utility, the optimal wage is constant, $w_1 = w_0 =: w$. Using (IR), $u(w) = c_L + \underline{U}$.

(d) Since the wage is constant the agent is happy to do no work. One can easily check (IC) holds.

(e) The problem is

$$\begin{aligned}
\max_{w_1, w_0} \quad & \int [q - w(q)] f(q|H)\, dq \\
s.t. \quad & \int u(w(q)) f(q|H)\, dq - c_H \geq \underline{U} && \text{(IR)} \\
& \int u(w(q)) f(q|H)\, dq - c_H \geq \int u(w(q)) f(q|L)\, dq - c_L && \text{(IC)}
\end{aligned}$$

(f) The Lagrangian is

$$\begin{aligned}
\mathcal{L} \quad = \quad & \int [q - w(q)] f(q|L)\, dq && (8) \\
& + \lambda \left[\int u(w(q)) f(q|H)\, dq - c_H - \underline{U} \right] && (9) \\
& + \mu \left[\int u(w(q)) f(q|H)\, dq - c_H - \int u(w(q)) f(q|L)\, dq + c_L \right] && (10)
\end{aligned}$$

The FOC with respect to $w(q)$ is

$$-f(q|L) + \lambda u'(w(q))f(q|L) + \mu[u'(w(q))f(q|H) - u'(w(q))f(q|L)] = 0$$

this can be rearranged to obtain

$$\frac{1}{u'(w(q))} = \lambda + \mu \left[1 - \frac{f(q|L)}{f(q|H)} \right]$$

(g) Suppose $\mu = 0$. By strict concavity of utility, the wage is flat. Yet this would induce the worker to choose e_L.

To better understand the shape of the optimal wage, see MWG, Section 14.B.