Economics 385: Midterm 2

21 March, 2007

This test is closed book. It is marked out of 100. You have 60 minutes. Good luck.

Question 1 (30 points)

Consider the competitive screening model covered in class. To be specific, two firms offer contracts of wages and tasks (w_i, t_i) among which workers select. A worker's utility if given by

$$U(w, t|\theta) = w - c(t, \theta)$$

where θ is the productivity of the agent. Suppose $\theta \in \{\theta_L, \theta_H\}$, and $\Pr(\theta = \theta_H) = \lambda$. As in class, suppose $c(t, \theta)$ is increasing in t and satisfies the single crossing property.

(a) Consider the least–cost pooling outcome, where both types choose $(w, t) = (E[\theta], 0)$. Explain why or why not this is an equilibrium.

(b) Consider the least–cost separating outcome, where θ_L chooses $(w, t) = (\theta_L, 0)$ and θ_H chooses $(w, t) = (\theta_H, \hat{t})$, and $\hat{t}$ has the property that

$$U(\theta_H, \hat{t}|\theta_L) = U(\theta_L, 0|\theta_L)$$

Explain why or why not this is an equilibrium.

Question 2 (30 points)

Consider Spence's signaling model. To be specific, suppose agents have productivity $\theta \in \{\theta_L, \theta_H\}$, where $\Pr(\theta = \theta_H) = \lambda$. Agents then choose education level $e \in [0, \infty)$ and are paid $w(e)$, as determined by a competitive market for labour. An agent's utility is given by

$$U(w, e|\theta) = w - c(e, \theta).$$

In addition, suppose the cost of education is $c(e, \theta) = e\theta$. Describe all the equilibria of this model. In particular, provide education choices and a supporting wage function.

Question 3 (35 points)

Consider Spence's signaling model with productive education. If agent θ gets e years of education then their productivity is $\theta + e$. The cost of education for type θ is $\frac{e^2}{2\theta}$. There are two types, $\theta_H \geq \theta_L$, where proportion λ are type θ_H.

As an aside, the solution to the quadratic

$$ax^2 + bx + c = 0$$

is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) Under which condition is $(w, e) = (E[\theta], 0)$ a pooling equilibrium?

(b) Assume the condition in your answer to part (a) holds. Characterise the set of pooling equilibria.