Economics 385: Suggested Solutions to Midterm 2

21 March, 2007

Question 1

(a) This is not an equilibrium. One firm will offer a contract to skim away the high types, with $w \in (E[\theta], \theta_H)$ and t between the two indifference curves.

(b) This is an equilibrium if

$$U(\theta_H, \hat{t}|\theta_H) \geq U(E[\theta], 0|\theta_H)$$

If this condition is not satisfied, then a firm can offer a pooling contract which is accepted by both types and has $w < E[\theta]$ and $t = 0$.

Question 2

The key point about this question is that the single–crossing property is not satisfied.

(a) The pooling equilibria are given by

$$0 \geq e \geq \frac{E[\theta] - \theta_L}{\theta_H}$$

Note the most–cost pooling equilibria is determined by θ_H's indifference curve.

(b) There are no separating equilibria.

Question 3

(a) The least–cost pooling outcome is an equilibrium if the θ_H does not deviate. That is,

$$\begin{aligned} E[\theta] &\geq \max_e \left[\theta_L + e - \frac{e^2}{2\theta_H}\right] \\ &= \theta_L + \theta_H - \frac{\theta_H^2}{2\theta_H} \\ &= \theta_L + \frac{\theta_H}{2} \end{aligned}$$

(b) Assuming the condition in part (a) holds, the set of pooling equilibria is given by $[0, \hat{e}]$. The most–cost pooling outcome is characterised by

$$\max_e \left[\theta_L + e - \frac{e^2}{2\theta_L}\right] = E[\theta] + \hat{e} - \frac{\hat{e}^2}{2\theta_L}$$

That is,

$$\frac{3}{2}\theta_L = E[\theta] + \hat{e} - \frac{\hat{e}^2}{2\theta_L}$$

Rewriting,

$$\frac{\hat{e}^2}{2} - \theta_L \hat{e} + \theta_L \left[\frac{3}{2}\theta_L - E[\theta]\right] = 0$$

Solving,

$$\hat{e} = \theta_L + \sqrt{2\theta_L(E[\theta] - \theta_L)}$$

Notice that we've taken the positive root. This is because we must have $\hat{e} \geq \theta_L$.