Exercise 10: Constrained Optimisation

September 18, 2007

1. Consider maximising $f(x_1, x_2) = x_1 x_2$ subject to $x_1 + x_2 = 6$. Solve the FOCs. Are your solution(s) a local max? A global max?

2. Consider maximising $f(x_1, x_2) = x_1^2 x_2$ subject to $2x_1^2 + x_2^2 = 3$. Solve the FOCs. Are your solution(s) a local max? A global max?

3. A firm that uses two inputs to produce output has the production function $3x_1^{1/3}x_2^{1/3}$, where x_1 is the amount of input 1 and x_2 is the amount of input 2. The price of output is 1 and the prices of the inputs are w_1 and w_2 . The firm is constrained by the government to use exactly 1000 units of input 1.

(i) How much of input 2 does it use?

(ii) What is the most that it is willing to bribe an inspector to allow it to use another unit of input 1?

4. Consider minimising $f(x_1, x_2) = 2x_1^2 + 3x_2^2$ subject to $x_1 + x_2 - 1 \ge 0$.

5. Consider maximising f(x) = x subject to $x^2 \leq 0$.

(i) For which values of x is the constraint qualification satisfied?

(ii) Solve the problem.

6. Consider maximising $f(x_1, x_2) = 10x_1 + x_2$ subject to $-(x_1 - 2)^3 - x_2 \ge 0$.

(i) For which values of x is the constraint qualification satisfied?

(ii) Solve the problem.