Exercise 11: Comparative Statics

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1. A firm, who faces demand curve $p(q) = 1 - q$ and marginal cost c, chooses quantity q to maximise her profits. How does a change in the marginal cost c affect profits?

2. A firm chooses inputs (x_1, x_2) to minimise costs $c(w_1, w_2, q) = w_1x_1 + w_2x_2$ subject to producing output q, i.e. $f(x) \geq q$. How does a change in the input price w_1 affect costs? [Aside: This result is called Shepard's Lemma.]

3. Profits equal $\pi = k^\alpha l^{1-\alpha} - wk - rl$, where $\alpha \in (0, 1)$. Suppose there is an increase in r. How does this affect the optimal choices of capital, k, and labour, l?

4. A consumer faces prices p_t over time $t \in \{1, 2, \ldots\}$. The agent chooses her optimal purchase time τ to maximise utility $u(\theta, \tau) = \theta\delta^\tau - p_\tau$, where $\delta < 1$ and $\theta > 0$. How does the optimal purchase time vary with θ?

5. Consider a sealed bid auction, where all agents simultaneously submit bids, and the highest bidder wins and pays her bid. Consider the problem of agent 1. Suppose the highest bid of her opponents has cumulative distribution function $F(\cdot)$ on $[0, \infty)$, and the agent has value v. If she bids b, her expected utility is thus

$$u(v, b) = (v - b)F(b)$$

How does the agent's optimal bid change with v? [Hint: A comparative static holds for $f(x, t)$ if it also holds for $\log f(x, t)$. You can use, but do not have to prove, this result.]

6. Suppose $f(x, t)$ and $g(x, t)$ are supermodular. Show $f(x, t) + g(x, t)$ is supermodular.

7. Suppose $f(x, t)$ is supermodular and $g : \mathbb{R} \to \mathbb{R}$ is increasing. Show, by example, that $g(f(x, t))$ may not be supermodular.

As an aside, you should now be able to do any exercise in Chapter 3 of Riley's "Essential Microeconomics".