Exercise 12: Linear Algebra

September 20, 2007

1. Consider the following matrices

$$A = \begin{pmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$$

Calculate the following (if possible):
(a) $A + B$
(b) $A - 2B$
(c) AC
(d) CA

2. Calculate the inverse of

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$$

3. Compute the determinant of

$$A = \begin{pmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{pmatrix}$$

4. Suppose we have

$$A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix} \qquad u = \begin{pmatrix} 6 \\ -5 \end{pmatrix} \qquad v = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Are u and v eigenvectors of A? Also, show that 7 is an eigenvalue and find the corresponding eigenvector.

5. Find the eigenvalues of:

$$A = \begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$$

6. Diagonalise the following:

$$A = \begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}$$

7. Find the spectral decomposition of

$$A = \begin{pmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

Show that the resulting eigenvectors and orthogonal.