## **Exercise 3: Sequences**

September 10, 2007

- 1. Show the set  $\{..., -2, -1, 0, 1, 2, ...\}$  is countable.
- 2. Suppose  $x_k \to x$  and  $x_k \to y$ . Show x = y.
- 3. Show  $\frac{1}{n} \to 0$  as  $n \to \infty$ .
- 4. Prove the sandwich property.
- 5. Prove that a convergent sequence is bounded.
- 6. Suppose  $x_n \to x$  and  $y_n \to y$ . Show  $x_n + y_n \to x + y$ .

7. [Hard] Suppose  $x_n \to x$  and  $y_n \to y$ . Show  $x_n y_n \to xy$ . [Hint: Notice that  $x_n y_n - xy = x_n y_n - x y_n + x y_n - x y$ .]

8. Let  $x_n$  be an increasing sequence with  $x_n - x_{n-1} \leq \frac{1}{n}$ . Does  $x_n$  necessarily converge? [Hint: If you're stuck, look up "harmonic sequence".]

9. Show  $\sqrt{n^2 + 1/n!} \to 0$  as  $n \to \infty$ . [Hint: Approximate the numerator from above, and show this new fraction converges to zero].