## Exercise 6: Derivatives

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1. Let $f: \mathbb{R}_{+} \rightarrow \mathbb{R}$ be defined by $f(x)=x \sin (1 / x)$ for $x>0$ and $f(0)=0$. Is $f$ differentiable at $x=0$ ?
2. Let $f: \mathbb{R}_{+} \rightarrow \mathbb{R}$ be defined by $f(x)=x^{2} \sin (1 / x)$ for $x>0$ and $f(0)=0$. Is $f$ differentiable at $x=0$ ?
3. Show a differentiable function is continuous.
4. Let $f(x)=x^{2}$ for $x \in \mathbb{R}$. Calculate $f^{\prime}(x)$ from first principles.
5. Let $f(x)=|x|$ for $x \in \mathbb{R}$. Is $f$ continuous? Is it differentiable?
6. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing on $[a, b]$ if $f(y)>f(x)$ for $y>x$. Suppose $f^{\prime}(x)>0$ on $x \in(a, b)$. Show that $f$ is strictly increasing on $[a, b]$. [Hint: mean value theorem].
