## **Exercise 6: Derivatives**

September 12, 2007

1. Let  $f : \mathbb{R}_+ \to \mathbb{R}$  be defined by  $f(x) = x \sin(1/x)$  for x > 0 and f(0) = 0. Is f differentiable at x = 0?

2. Let  $f : \mathbb{R}_+ \to \mathbb{R}$  be defined by  $f(x) = x^2 \sin(1/x)$  for x > 0 and f(0) = 0. Is f differentiable at x = 0?

3. Show a differentiable function is continuous.

4. Let  $f(x) = x^2$  for  $x \in \mathbb{R}$ . Calculate f'(x) from first principles.

5. Let f(x) = |x| for  $x \in \mathbb{R}$ . Is f continuous? Is it differentiable?

6. A function  $f : \mathbb{R} \to \mathbb{R}$  is strictly increasing on [a, b] if f(y) > f(x) for y > x. Suppose f'(x) > 0 on  $x \in (a, b)$ . Show that f is strictly increasing on [a, b]. [Hint: mean value theorem].