Exercise 6: Derivatives

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1. Let $f : \mathbb{R}_+ \to \mathbb{R}$ be defined by $f(x) = x \sin(1/x)$ for $x > 0$ and $f(0) = 0$. Is $f$ differentiable at $x = 0$?

2. Let $f : \mathbb{R}_+ \to \mathbb{R}$ be defined by $f(x) = x^2 \sin(1/x)$ for $x > 0$ and $f(0) = 0$. Is $f$ differentiable at $x = 0$?

3. Show a differentiable function is continuous.

4. Let $f(x) = x^2$ for $x \in \mathbb{R}$. Calculate $f'(x)$ from first principles.

5. Let $f(x) = |x|$ for $x \in \mathbb{R}$. Is $f$ continuous? Is it differentiable?

6. A function $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing on $[a, b]$ if $f(y) > f(x)$ for $y > x$. Suppose $f'(x) > 0$ on $x \in (a, b)$. Show that $f$ is strictly increasing on $[a, b]$. [Hint: mean value theorem].