Exercise 7: Convexity

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1. Show that the intersection of two convex sets is convex. Is the union of two convex sets necessarily convex?

2. Show that $f : \mathbb{R} \to \mathbb{R}$ is concave if and only if

$$f(\sum_i \alpha_i x_i) \geq \sum_i \alpha_i f(x_i)$$

for $\alpha_i \in [0, 1]$ such that $\sum_i \alpha_i = 1$. [Hint: Induction.]

3. Show that the sum of 2 concave functions is concave.

4. Show that the pointwise minimum (i.e. $\min\{f(x), g(x)\}$) of two concave functions is concave.

5. Suppose $f : \mathbb{R} \to \mathbb{R}$ is concave, and $g : \mathbb{R} \to \mathbb{R}$ is increasing and concave. Show that $g(f(x))$ is concave.

6. Suppose $f : \mathbb{R} \to \mathbb{R}$ is concave, and $g : \mathbb{R} \to \mathbb{R}$ is increasing. Show, by example, that $g(f(x))$ may not be concave.

7. Suppose $f : \mathbb{R} \to \mathbb{R}$ is quasi–concave, and $g : \mathbb{R} \to \mathbb{R}$ is increasing. Show that $g(f(x))$ is quasi–concave.