Exercise 9: Unconstrained Optimisation

September 19, 2007

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is strictly quasi–concave, in that

$$f(\alpha x + (1-\alpha)x') > \min\{f(x), f(x')\}$$

for $x \neq x'$. Suppose $\bigtriangledown f(\overline{x}) = 0$. Show, by example, that $\overline{x}$ may not be a global maximum.

2. Provide an example where $\overline{x}$ is a local maximum and $D^2 f(\overline{x}) = 0$.

3. Provide an example where $\overline{x}$ is not a local maximum and $D^2 f(\overline{x}) = 0$.

4. Consider the problem of choosing $x \in \mathbb{R}$ to maximise $x^3 - x$. Solve the FOCs. Does this satisfy the SOCs? What is the solution to the problem?

5. Consider the function $f(x_1, x_2) = 3x_1x_2 - x_1^2 - x_2^2$. Solve the FOCs. Solve for the values of $f_{11}(x_1^*, x_2^*)$ and $f_{22}(x_1^*, x_2^*)$. Is this a local max?

6. Let $f(x, y) = 8x_1^3 + 2x_1x_2 - 3x_1^2 + x_2^2 + 1$. Solve for the local minima.

Riley's "Essential Microeconomics" has good economic exercises. For example, I recommend 1.3.3 or 1.4.1. By this point you should be able to do any question from Chapter 1.