

Eco200: The Quiz

September 21st, 2007

This quiz has four questions and will be marked out of 100. Since the individual questions add up to 120, this gives you some room for manoeuvre. Good luck!

Question 1 (25 points)

A, B, and C are statements. The following theorem is true:

if A is true and B is not true then C is true.

Which of the following statements follow from this theorem?

- (a) If A is true then C is true.
- (b) If A is not true and B is true then C is not true.
- (c) If either A is not true or B is true (or both) then C is not true.
- (d) If C is not true then A is not true and B is true.
- (e) If C is not true then either A is not true or B is true (or both).

Question 2 (25 points)

Suppose $\{x_n\}$ is a sequence of real numbers such that $x_n > a$ ($\forall n$) and $x_n \rightarrow x$.

- (i) Is it necessarily true that $x \geq a$?
- (ii) Is it necessarily true that $x > a$?

Your answers should either contain a proof or a counterexample.

Question 3 (30 points)

Suppose a firm has a convex cost function, in that $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ satisfies

$$c(\alpha x + (1 - \alpha)x') \leq \alpha c(x) + (1 - \alpha)c(x')$$

for $\alpha \in (0, 1)$. Suppose also that $c(0) = 0$ and $c(x)$ is increasing in x . Show that average costs are increasing in output. That is, $c(x)/x$ is increasing in x .

Question 4 (40 points)

Consider the problem of choosing (x, y) to maximise

$$f(x, y) = 3x + y$$

subject to

$$(x + 1)^2 + (y + 1)^2 \leq 5 \quad \text{and} \quad x \geq 0, y \geq 0$$

- (a) Suppose that (x^*, y^*) solves this problem. Is there necessarily a value of λ such that (x^*, y^*, λ) satisfies the Kuhn-Tucker conditions?
- (b) Now suppose that (x^*, y^*, λ) satisfies the Kuhn-Tucker conditions. Does (x^*, y^*) necessarily solve the problem?
- (c) Given the information in your answers to (a) and (b), use the Kuhn-Tucker method to solve the problem.