Fast Bargaining in Bankruptcy

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Abstract

I combine two previously separate strands of the bargaining literature to present a bargaining model with both one-sided private information and a majority vote for proposals to go into effect. I use this model to show that the US bankruptcy code produces shorter delays and higher welfare than the UK law.

I consider the bargaining that occurs in bankruptcy between an informed firm and a set of uninformed creditors over a set of claims against the firm. The agents have an infinite horizon to bargain and cannot commit to a schedule of future offers. If individual creditors can be treated differently and a majority vote is required for the acceptance of new claims, adding creditors increases the probability of reaching agreement by the end of any given period. The US regime has these features. I give numerical examples which show the efficiency gains from increasing the number of creditors are significant.

The UK voting rule allows one creditor a veto of all plans. Replacing the majority voting rule with the UK voting rule and allowing only the creditor with the veto to suggest plans, I show that the UK regime has longer delays and is less efficient than the US regime as long as the US regime has multiple creditors.

1 Introduction

In this paper, I consider the process of debt renegotiation between a firm and its creditors in bankruptcy. The management and the creditors bargain over new claims that the firm pays. The current management has private information on its ability to run the firm and this information affects the outcome of the bargaining. I show that the nature of the bankruptcy law can have large effects on the size of the inefficiencies in bargaining.

I compare bargaining outcomes with two different bankruptcy laws. Under the first law, a firm can leave bankruptcy if a set of new claims, or a plan, passes a majority vote among the creditors and is approved by the firm. I label this law the "majority vote" law. Under the second law, one specific creditor is deemed to have a special priority which gives him the right to propose new contracts to the firm. The firm must in turn approve the plan. No other creditor needs to approve the plan. I refer to this law as the "controlling creditor" law. Neither law places any other restriction on acceptable plans.

There are two main results. The first is a comparison of delays in bankruptcy for the two systems. With just one creditor these two laws are identical in every detail and produce the same delays. With multiple creditors, however, the delays in bankruptcy are shorter under the majority vote law. With such a law, as more creditors are added the bargaining becomes faster. The intuition behind this result becomes apparent by examining the conflicts between the creditors. Every plan that passes requires support from a majority of creditors or a "winning coalition." Creditors in the winning coalition receive positive payments; outsiders receive zero. In addition, within the winning coalition, every plan must be suggested by one creditor. The plan's proposer asks for the best possible treatment for himself. Consequently, should the firm approve the plan, the proposer receives a higher payment than the other creditors in the winning coalition. Proposers always suggest plans that pass the creditors' vote. When a proposer suggests such a plan, he faces a trade-off between maximizing his own payment through a high offer to the firm and increasing the probability the firm accepts through a low offer. As the number of creditors increases, the fear of being an outsider and the reduced probability that a given creditor becomes a proposer in future winning coalitions leads the creditors to sacrifice the size of their payments for a higher probability of settlement. The majority vote is critical to this result. With a unanimity vote there is no fear of exclusion from future coalitions and the two laws are equivalent.

The second main result is a welfare comparison. For such a comparison, I consider the problem at the time the debt contracts are negotiated. I add an additional, initial period at which the firm must make a fixed investment which requires the negotiation of debt contracts with an exogenous set of creditors. The firm does not know its ability to run the firm until after it makes the investment. After it makes the investment, the firm's management learns its managerial ability and receives its profits. If the firm can afford the payments it has negotiated, it makes them to the creditors. Otherwise the firm enters bankruptcy. There are the same two bankruptcy laws as before and bankruptcy occurs as previously described. The majority vote law maximizes ex-ante welfare in this environment. The intuition here is as follows. A slow bankruptcy law, such as the controlling creditor law, creates a hold up problem. The long delays associated with a slow law create substantial deadweight losses because the firm receives reduced profits while it lingers in bankruptcy. The "controlling creditor" exacerbates the deadweight losses by adopting tough bargaining positions. On the positive side, the tough bargaining positions generate extra revenues to the controlling creditor in bankruptcy. Although these revenues keep the probability the firm enters bankruptcy lower than under the majority vote law, any decrease in this probability is overwhelmed by the increase in deadweight losses. The key feature in creating the hold up problem with the "controlling creditor" law is the lack of commitment ex-ante. Ex-ante welfare would increase if the controlling creditor could commit to softer bargaining positions.

I study the laws governing bankruptcy because they form an institution with important consequences. In the words of Douglass North:

> Institutions provide the incentive structure of an economy; as that structure evolves it shapes the direction of economic change towards growth, stagnation or decline. (1991)

A fast bankruptcy law makes debt finance more attractive. Potential managers and lenders can be unwilling to sign debt contracts if they believe that poor outcomes lead the firm to languish in the limbo of the bankruptcy court. A long stay in bankruptcy can bind key assets of the firm in unproductive uses and make long term relationships with clients and suppliers hard to establish and maintain. Similarly, potential lenders may be scared off by long bankruptcies. A prolonged bankruptcy can bind the lenders' collateral, create liquidity shortages, and generally introduce uncertainty. Slow bankruptcy laws can lead the participants to liquidate otherwise viable firms. Quick reorganizations can avoid these problems. By promoting debt finance, lowering bankruptcy costs can have significant effects on welfare.

The body of this paper is as follows. In the next section, I discuss the legal counterparts of these two laws, the US and UK bankruptcy laws. In Section 3, I discuss the related literature. In Section 4, I define the ex-post game focusing on the model with the majority vote law. I describe the equilibrium and present an existence and uniqueness result. In Section 5, I give the delay comparison. In Section 6, I perform the welfare comparison. Section 7 concludes.

2 Description of US and UK Bankruptcy Laws

I study the majority vote and controlling creditor laws because they capture distinguishing features of the US and the UK bankruptcy laws.[1] A majority (or, to be precise, a weighted supermajority less than unanimity) vote among the creditors plays a key role in the passage of plans under the US law, or Chapter 11. In addition, the inferior treatment of a particular group of creditors is not sufficient to prevent a plan from going into effect.

To see how the majority vote fits into the passage of plans and the roots of "discrimination" among creditors under US law, it is useful to describe the typical negotiations under Chapter 11. Consider a firm whose debt is held entirely by unsecured creditors. First the firm negotiates the plan with a subset of the creditors called the Creditors' Committee. The negotiations between the firm and the Creditors' Committee end when the latter is asked to approve the plan through a majority vote. Once the Creditors' Committee and the court approve the plan, the firm puts the plan to a vote among the entire set of creditors. The confirmation rule is fairly complicated. Generalizing, an affirmative vote from the creditors that is less than unanimous is required.

Next, suppose an arbitrary unsecured creditor is treated less favorably than his counterparts in a plan that has passed the above votes. To keep the plan from going into effect, the inferiorly treated creditor must contest the plan at a "cramdown" hearing. In a cramdown hearing, the creditor may be able block the plan by convincing the judge that he has been treated in an inferior manner to creditors he shares priority with. However, the court can rule that there is a principled basis for the unequal treatment.[2] Further, the hearing itself requires the court to estimate the values of both the creditors' old and new claims, which can be prohibitively expensive. Due to the costs and the difficulties of such a task, many judges and creditors are unwilling to force a hearing and accept the discrimination. If no creditor forces a cramdown

[1] In Appendix A, I give a more detailed description of the laws. I also document the details that are presented here. Particularly, the formal confirmation procedure is presented here.

[2] A frequent rationale for such a determination is that the extra-favorable treatment for another creditor occurs because such treatment is a "business necessity," or that a good relationship with the favored creditor is important for the continuation of the firm. A second rationale is that the discrimination is not significant enough to stop an otherwise acceptable plan.

Table 1a, Data Summary

Observation	US	UK
Duration in Months	21	52
Return to Creditors	0.51	0.36
Classes of creditors	8.57	few

From a variety of sources

hearing, the judge makes a determination whether the plan is in the firm's and the creditors' best interests and if so accepts the plan.

The controlling creditor law captures a unique feature of the UK law, or Receivership. In the UK, a necessary requirement for a firm to exit bankruptcy is that one specific, predetermined creditor supports the plan. This creditor has the powers the controlling creditor has in the model.

Since the model maps into the US and UK laws, comparing the potential inefficiencies in the data for these countries is informative. There are significant differences in the data. The delays in the reorganization of financial distressed firms in the US are typically much shorter. UK firms that successfully reorganize spend on average 52 months in receivership, whereas US firms spend 21 months in Chapter 11.[3] There is other evidence that greater inefficiencies are present in the UK system. I highlight one that is relevant to this paper; namely, the returns to the creditors in the US are greater than the returns to the creditors in the UK. In the US, creditors receive 51 cents on each dollar of debt they are owed on average. In the UK creditors receive 0.36 cents on a dollar of debt.[4]

3 Related Literature

I split the related literature into three categories. The first are papers which relate issues of financial distress to the number of creditors. There are two of these papers worth discussion, Gertner and Scharfstein (1991) and Bolton and Scharfstein (1996). These papers describe the effects of additional creditors on ex-ante and ex-post welfare, given that in financial distress a firm must enter a costly negotiation with their creditors. They find that additional creditors make initial contracts more difficult to renegotiate. My paper is similar to theirs in the assumption that creditors can be treated differently. However we have opposite results. They find that more creditors make agreements more difficult to reach. This contradicts the bankruptcy data, where the regime which allows more creditors to negotiate with the firm achieves agreements

[3]All numbers presented here are summarized in Table 1a. The sources for the duration numbers are in the Appendix.

[4]Franks, Nyborg, and Torous (1996) This number also includes liquidated firms which suggests falls in the value of the firm due to delays in entering bankruptcy may be an important explainer of this piece of data. In Benjamin 2003, I show how the basic analysis presented here can account for this observation.

faster.

The difference in results can be explained by the different environments these authors and myself consider. The central feature of these models is that the creditors can be negotiated with one at a time. However, my environment describes bankruptcy better. In bankruptcy, binding offers can be made to the entire body of creditors at the same time. Creditors later in line cannot use their position to demand additional rents or avoid writing down debt.

The second are papers that use a bargaining framework to model bankruptcy. These papers include Hart and Moore (1994), Eraslan (2002a), and Bebchuk and Chang (1992). These papers are primarily concerned with distributional issues between the unsecured creditors as a whole and equity. Further they usually assume complete information. I am interested in a different distributional issue and do not assume complete information.

More relevant to the theory in this paper is the third category, the repeated offer bargaining literature. I combine two classic bargaining models in this paper. As such, I rely heavily on previous work in understanding these models. The first is a bargaining model with one-sided incomplete information. In this model, the uniformed agent makes a series of offers to the agent with private information. With this model, I can formally discuss delays in renegotiation. The two papers which give the earliest theoretical results for this model are Gul, Sonnenschein, and Wilson (1986) and Fudenburg, Levine, and Tirole (1985), hereafter referred to as GSW and FLT respectively. These are the papers which originated the study of this model. A good summary article is Ausubel, Cramton, and Denerke (2001). Many of the proofs are based on results in this article.

The second is a complete information bargaining model with random proposers and a majority voting rule for plans to pass. The important works here are Baron and Ferejohn (1989), Eraslan and Merlo (2002) and Eraslan (2002b). The first is particularly useful whenever the private information is negligible. The second and third are particularly useful in the general description of the equilibrium.

4 The Ex-Post Game

I consider a firm entering bankruptcy with outstanding debts to a set of n identical creditors. If $n > 1$, I describe only bargaining with the US regime. For firms bargaining with the UK regime, I exogenously designate one creditor as the receiver and consider the other creditors to be null agents in the model. (This is reproduces the same outcomes as the majority vote regime with one creditor.)

There exists one firm with type $\theta \in [\theta_l, \theta_h]$. The firm's type, θ, is the present discounted value for the firm with its current management. It is only achievable for a firm *outside bankruptcy* and is private information to the firm, which is perfectly informed. The return to the firm if it is run by an outsider is θ_l. Thus $\theta - \theta_l$ is the return to the firm's management ability. Firms inside bankruptcy have no returns. This feature of the model captures the negative effects financial distress has with the management's ability to run the firm as it sees best. For example, judicial oversight can greatly reduce the profits the firm earns in bankruptcy. The creditors share an initial belief that θ has density f and distribution F. The upper bound on the distribution of θ is θ_h. I assume that f is smooth, continuous and bounded from above and below by a positive number.

The model also includes n creditors who have previously given financing to the firm and have identical claims. To have a consistent definition of a majority, n is an odd integer. The creditors are uninformed about θ. All agents are risk neutral and have a common discount factor δ, with which they discount any payments they give or receive.

4.1 Timing

The timing of the game is given in Figure 1 in the Appendix. At the beginning of period 1, one creditor is chosen at random to propose a plan consisting of a set of payments to the n creditors. This creditor is called the proposer. The proposer's index always takes the value, i.

All other creditors are called voters. After the proposer is chosen, the voters are randomly ordered. The firm is then placed at the end of the line. One at a time the members of the line vote to accept or reject the plan. If the firm and a majority of the creditors ($\frac{n-1}{2}$ of the voters) accept the plan, the plan passes, and the firm receives its valuation and makes the agreed upon payments. Otherwise the game proceeds to the next period. The next period has the same timing as the current period. The game concludes when a plan is passed.

4.2 Strategies

Proposers suggest new plans. The proposer in period t's strategy, σ_{it}, is a map from the history of previous offers and votes before he proposes to n payments; p_{it} (to himself) and $\{p_{jt}\}_{j\neq i}$ (to other creditors)[5]

The remaining creditors, or voters, vote on the plan. A voter decides to vote for or against the plan after learning how voters earlier in line have voted. A voter's strategy is either "Y" or "N". Formally, a voter's strategy in period t, σ_{jt}, is a map from the history of plans and votes prior to creditor j's vote to an element of $\{Y, N\}$.

A firm can approve or veto the proposed plan. If the firm and a majority of the creditors accept the plan, the firm leaves bankruptcy and operates under current management. If the firm or the creditors do not approve the plan, the bargaining continues into the next period. Formally, a firm's strategy in period t, $\sigma_{at}(\theta)$, is a map from a firm's type and the history of plans and votes prior to the firm's vote to an element of $\{Y, N\}$. I restrict its strategy to satisfy measurability requirements.

[5]Currently there is no restriction on plans that the proposer can suggest. I have extended the qualitative results in this paper to a model in which all creditors' individual payments are restricted to be no smaller than a constant fraction of the aggregate payment. This generalization is in the Technical Appendix, which is available from the author on request.

The strategy for the entire set of agents for the entire game is called σ. The strategy for agent k in period t is σ_{kt}. Finally one bit of standard notation I find useful is to let σ_{-kt} equal σ excluding σ_{kt} for $k = i, j$, or a.

I consider a relevant history for player k in period t to be all of the plans, votes, and acceptance decisions prior to k's move in period t. I refer to a relevant history as h_k^t. For example, for proposer i, h_i^t consists of $t-1$ plans, $(t-1) \times (n-1)$ votes from the creditors, and $t-1$ acceptance decisions from the firm. And if creditor j votes in position M, $h_j^t = h_i^t$ plus an additional plan and $M-1$ additional votes.

4.3 Payoffs

In this subsection, I define the payoffs of the agents in the model. The payoff to the firm is the discounted value of the firm's profits minus any payments the firm make. To make this concrete, fix the current period as T. Suppose the strategies σ are such that a type θ firm reaches agreement in t additional periods. When agreement is reached, the plan is $\{p_{it}\}_{i=1}^n$. Relevant to the firm's payoff, the aggregate payment P_t can be derived letting $P_t = \sum_j p_{jt}$. A type $\theta's$ payoff is:

$$U_T(\theta) = \delta^{t-T}(\theta - P_t)$$

The creditors' expected payoffs are the expected values of any payments it receives. Two examples I return to frequently are the payoffs to a proposer and the expected payoffs to a voter.

The payoff for a proposing creditor in period T is:

$$V_T = \delta^{t-T} p_{it}$$

Second are payoffs to the voters, which are referred to as W_{jT}. When dealing with voters it is easiest to not consider the specific proposal made to them. Instead the most useful payoff to consider is their expectation conditioned only on histories where they are *not* the proposer. For voter j, the expected payoff in period T is:

$$W_{jT} = \delta^{t-T} E[p_{jt}]$$

4.4 Equilibrium Concept

The equilibrium concept depends on the beliefs: $\{\mu_t\}_{t=1}^{\infty}$. In every period, creditors share a belief, μ_t, about the types of firms which have not yet reached agreement at the beginning of the period.

The equilibrium concept for this game is sequential equilibrium.

Definition 1. A sequential equilibrium is a strategy σ and a sequence of beliefs $\{\mu_t\}_{t=1}^{\infty}$, such that for all periods t,

I. For all relevant histories to the firm h_a^t, and for any type θ,

$$\sigma_{at}(\theta) \in \text{arg} \max U_t(\theta, \sigma_{at}(\theta), \sigma_{-at}(\theta))$$

II. For every proposing creditor and every relevant history to the proposer, h_i^t,

$$\sigma_{it} \in \text{arg} \max V_t(\sigma_{it}, \sigma_{-it}, \mu_t)$$

III. For every voting creditor j and every relevant history to the voter, h_j^t,

$$\sigma_{jt} \in \text{arg} \max W_{jt}(\sigma_{jt}, \sigma_{-jt}, \mu_t)$$

IV. μ_t is found through Bayes' Rule whenever possible.[6]

4.5 Functional Equation

Now, I describe the equilibrium outcome I am interested in. It satisfies a Functional Equation. Every equilibrium outcome has a common form, even those that do not satisfy the functional equation. Each period, the interval of the types of the firm still bargaining splits in two. Those with types above a given threshold accept the creditors' offer. Those with types below the threshold reject the offer. Hence, each successive belief is a truncation of previous beliefs to an interval containing θ_l. Also, in every equilibrium, a necessary condition for a settlement is that a "winning" coalition with sufficient votes emerges. Creditors in this coalition vote for the plan and receive positive payments. Creditors outside the coalition receive zero payments.

There are typically many equilibrium outcomes for this game. I use a **Functional Equation** to select an equilibrium that I contend is a good description of how these games are played. With this Functional Equation, I ignore other equilibria which rely on the asymmetric treatment of certain creditors or on the use of punishments besides the rejection of the current offer.

The functional equation depends upon three types of numbers and two value functions. The three types of numbers represent payments. They are: p_i, the payment of the game to the proposer; p_j, the payment of the game to non-proposing creditors who are included in the "winning" coalition; and zero, the payment excluded creditors receive. The value functions represent the creditors' *payoffs*. The payoffs depend on the upper bound on the belief entering the period, θ_b. The two value functions are: $V(\theta_b)$, the value to the proposer; and $W(\theta_b)$, the value to any non-proposing creditor. In the equilibrium I consider, V and W do not depend on the identity of either the proposer or the voters. To aid with the definition, I let Θ equal the set of all types. The functional equation is presented through the following proposition.

[6]In bargaining models, the extensive form is such that consistency does not apply any greater restrictions on μ_t other than it is derived via Bayes' Rule whenever possible. Thus any Bayesian Perfect Equilibria is also a Sequential Equilibrium

Proposition 1. Consider the following set of functions: $\Theta \to R$

a payment to the proposer p_1;

payment to the included voters p_{-1};

a belief updating function, g ;

value functions, V, W;

and *an aggregate payment function,* $C = p_1 + \frac{n-1}{2} p_{-1}$;

The above functions are a sequential equilibrium outcome if they satisfy **Program 1**:

Program 1

I. **Proposer's Problem**

For each θ_b, a proposer with an upper bound on beliefs of θ_b chooses θ_a, p_i, and p_j to satisfy **DP**

$$V(\theta_b) = \max_{p_i, p_j, \theta_a} p_i \left(\frac{F(\theta_b) - F(\theta_a)}{F(\theta_b)} \right) + \delta \left(\frac{1}{n} V(\theta_a) + \frac{n-1}{n} W(\theta_a) \right) \frac{F(\theta_a)}{F(\theta_b)} \tag{1}$$

s.t.

$$\theta_a - p_i - \frac{(n-1)}{2} p_j \geq \delta \left(\theta_a - C(\theta_a) \right) \tag{2}$$

with equality if $\theta_a > 0$

$$p_j \left(\frac{F(\theta_b) - F(\theta_a)}{F(\theta_b)} \right) + \delta \left(\frac{1}{n} V(\theta_a) + \frac{n-1}{n} W(\theta_a) \right) \frac{F(\theta_a)}{F(\theta_b)} = \delta \left(\frac{1}{n} V(\theta_b) + \frac{n-1}{n} W(\theta_b) \right) \tag{3}$$

$$\theta_l \leq \theta_a \leq \theta_b \tag{4}$$

II. And given p_i, p_j, θ_a and V from **DP** the following relationships complete a fixed point argument:

$$p_1(\theta_b) = p_i \tag{5}$$

$$p_{-1}(\theta_b) = p_j \tag{6}$$

$$C(\theta_b) = p_1(\theta_b) + \frac{n-1}{2} p_{-1}(\theta_b) \tag{7}$$

$$W(\theta_b) = \frac{1}{2} p_1(\theta_b) \left(\frac{F(\theta_b) - F(g(\theta_b))}{F(\theta_b)} \right) + \delta \left(\frac{1}{n} V(g(\theta_b)) + \frac{n-1}{n} W(g(\theta_b)) \right) \frac{F(g(\theta_b))}{F(\theta_b)} \quad (8)$$

The proof of this proposition is in the Appendix. I turn to a description of the program.

(1) is the objective function of the proposer. The proposer chooses a payment to himself, p_i, a non-zero payment to his coalition members, p_j, and a lowest type that accepts the current offer, θ_a, subject to incentive constraints. The first parenthetical term is the probability he receives p_i. The last term is the continuation value of the game. Specifically, the proposer in the next period receives $V(\theta_a)$. Non-proposing creditors receive $W(\theta_a)$. With probability $\frac{1}{n}\frac{F(\theta_a)}{F(\theta_b)}$ the next period is reached and the initial proposer proposes again in the next period. With probability $\frac{n-1}{n}\frac{F(\theta_a)}{F(\theta_b)}$ the next period is reached but the initial proposer does not propose again.

(2) constrains the proposer such that his suggestion of lowest type to approve the plan is the lowest type that incentives dictate approve the plan. The incentive constraints of all other types of the firm do not bind.

(3) constrains the proposer to only suggest plans such that creditors for whom a 'Y' is needed for plans to pass vote as required.

(4) constrains the proposer to choose an upper bound for the next period's belief that is in the support of the current belief.

(5) to (8) ensure that a proposer's choice after the current history is consistent with the choices of proposers after different histories. Of note is (8), which gives the value of W. Before the proposer is selected, creditors expect to be offered a positive payment with probability $\frac{1}{2}$ should they not be selected to propose.

4.6 Simpler Functional Equation

To derive the results I simplify the functional equation. Among other results, for $\theta_l > 0$, such a simplification is necessary to derive an existence result, which is the last of the intermediate results I present in the next subsection. A casual reader may which to skip directly to the main results. In this subsection I assume an equilibrium exists and derive a simpler Function Equation which the solution to the original functional equation must also satisfy.

To derive the simpler equation, I eliminate a value function and two payments. In particular I show that the model has the same equilibrium outcomes as a model with one creditor who proposes one payment in every period, but does so as if his discount factor is lower than the actual discount factor.

The new problem is presented through a proposition. To state the proposition, I need notation for the payment to the entire set of creditors. I choose the label X for this term. For most results, it is useful to work with unconditional probabilities. Hence, I let X include the probability the game has not ended, given both the initial beliefs and the current belief, θ_b.

$$X(\theta_b) = (V(\theta_b) + (n-1)W(\theta_b))\, F(\theta_b)$$

Proposition 2. The aggregate payment function, $C(\theta_b)$, belief updating function, $g(\theta_b)$, and value function, $X(\theta_b)$ of an equilibrium outcome that solves **Program 1** also solves:

Program 2

I. **New Proposer's Maximization Problem**

Given X and C, a proposer with an upper bound on beliefs of θ_b chooses P and θ_a to solve

DP2

$$\theta_a, P = \left(\arg\max_{\theta_a, P} P\left(F(\theta_b) - F(\theta_a)\right) + \bar{\delta}(n,\delta)X(\theta_a)\right) \tag{9}$$

s.t.

$$\theta_a - P \geq \delta\left(\theta_a - C(\theta_a)\right) \tag{10}$$

with strict equality if $\theta_a > 0$

$$\theta_l \leq \theta_a < \theta_b \tag{11}$$

$$\text{where } \bar{\delta}(n,\delta) = \delta\left(\frac{1}{2} + \frac{1}{2n}\right).$$

And given P and θ_a from **DP2**; C, g, and X satisfy:

$$g(\theta_b) = \theta_a \tag{12}$$

$$C(\theta_b) = P \tag{13}$$

$$X(\theta_b) = \left((1-\delta)g(\theta_b) + \delta C\left(g(\theta_b)\right)\right)\left(F(\theta_b) - F\left(g(\theta_b)\right)\right) + \delta X\left(g(\theta_b)\right) \tag{14}$$

The proof of this proposition is in the Appendix. The quasi-discount factor, $\bar{\delta}$, actually is composed of two different terms. The first is the share of the continuation value that the proposer expects to receive in the next period. The second is the compensation to the remaining members of the majority for any change in their beliefs if the firm rejects the current offer. A rejected offer teaches the creditors that the firm's type is below a given cut-off level. The lower the offer, the less optimistic the creditors are in their belief about the firm's type in the next period. To vote for the current offer, other creditors must be compensated for any decline in optimism about their future prospects. The amount of this compensation is equal to the difference between the continuation value under a non-serious offer and the continuation value under the current offer.

The main results rely on the quasi-discount factor decreasing in n. Hence, the results would be essentially the same if a supermajority short of unanimity is needed.[7] With unanimity, the results are different. For unanimity, regardless of n, the quasi-discount factor equals the actual discount factor and changing the number of creditors does not affect outcomes.

4.7 Intermediate Results

I have two results for this section. Both pertain to equilibria that satisfy the Functional Equation. I show that if $\theta_l > 0$, should an equilibria exist, the bargaining ends in a finite number of periods almost surely. I call this the Finite Ending Result. Using this result I present an existence and uniqueness theorem for equilibrium that satisfy the Functional Equation.

Proposition 3. Finite Ending Result If $\theta_l > 0$, in any equilibrium that satisfies the Functional Equation, the bargaining lasts a finite number of periods.

The proof of the Finite Ending Result requires multiple steps and is found in the Appendix. Although some generalizations are required, the basic strategy for the proof follows FLT. This result implies that if an equilibrium exists the bargaining must last a finite number of periods. Hence to show existence, I can consider only equilibrium candidates that must eventually produce agreement for sure. The equilibrium strategies must contain a plan that the lowest type θ_l accepts for sure. I can show such a plan exists and is part of an optimal strategy. From this plan I can construct the continuation payoffs for games in the second to last period of bargaining. Working inductively, I can prove both existence and uniqueness by calculating the unique equilibrium outcome in a given period when the expected continuation payoffs are derived from know equilibrium behavior.

Theorem 1. Let $\theta_l > 0$. There exists a unique equilibrium outcome which satisfies the (Simpler) Functional Equation generically up to a potential multiplicity of equilibrium outcomes in

[7] This is very important to the application where due to the complicated acceptance rule and judicial discretion, a better approximation of the US rule may be a supermajority rule greater than majority but short of unanimity.

the initial period over a set of measure zero.[8]

The proof of this theorem is in the Appendix and again is a generalization of FLT. That the equilibrium lacks mixed strategies (except in an at most countable set in the initial period) comes from a result in Ausubel and Deneckere (1989). The proof is of interest on its own merit because it gives a recursive description of the equilibrium which can be used in computations.

There remains two questions. First there is the matter of existence if $\theta_l = 0$. In this case I restrict the distribution to be uniform for all the results I derive. I prove existence by constructing the solution to the functional equation.

Second, there is the question of whether equilibria exist with outcomes that are not solutions to the Functional Equation. It is well known that if $n = 1$, and $\theta_l > 0$ then no such equilibria exist. With multiple creditors, even if $\theta_l = \theta_h$ (the standard Baron-Ferejohn model), other equilibria are known to exist. However, in this second case, it is well known what restrictions are needed to rule out these equilibria.[9] The traditional restriction is *stationarity* or that creditors' votes depend only on the current offer. For the model in this paper, the combination of $\theta_l > 0$ and the stationarity of the creditors' voting strategies are not sufficient to guarantee a unique equilibrium.

Unlike the Baron-Ferejohn model, equilibria may exist that are stationary but not symmetric in the treatment of the creditors. In this model non-symmetric equilibria exist when particular creditors favor particular offers, if they are more likely to be included in future coalitions after these offers are rejected. Also, unlike GSW and FLT, even if $\theta_l > 0$, equilibria exist where the firm's strategy depends on more than just the aggregate offer. In particular, equilibria frequently exist where the firm conditions its vote on the number of offers the creditors have rejected.

One difficulty that does not arise are equilibria where multiple creditors vote yes for plans they would prefer to reject, because of a coordination failure. Such strategies are not part of any equilibria because the creditors vote in a randomly determined order as opposed to simultaneously. The argument here follows from backwards induction through the creditors' votes. Suppose there is a significant mass of creditors who want to reject a plan. The sequential vote allows any voting creditor who wants the plan rejected to vote "no" and lead the equilibrium through a branch of the tree where subsequent creditors who similarly desire to reject the plan also vote honestly. Likewise creditors who desire that a particular plan passes do not need to worry that coordination problems may defeat the plan.

[8] In period 1, the initial proposer may have more than one optimal choice of P and θ_a. But after this period, the equilibrium is entirely without mixed strategies. The generic restriction is that such a multiplicity of outcomes can occur at most on a countable subset of initial beliefs.

[9] See Baron and Ferejohn (1989) and Eraslan (2002b)

5 Delay Comparison

In this section, I present the delay comparison. The delay comparison is useful in understanding the differences in outcomes that the bankruptcy laws produce. The bargaining between firms and their creditors that occurs in the US inside bankruptcy maps into the multiple creditor model, and the bargaining in the UK under bankruptcy maps into the one creditor model. The differences in delays associated with the two systems can be partially explained if agreements are quicker to be reached as the number of creditors increases.[10]

The main result is that, as n increases, the lowest type accepting the creditors' offer falls in every period. Hence, a lower percentage of the firm's types do not reach agreement by the end of any particular period. This result requires some qualification since even for the standard GSW and FLT models, the belief updating function, g, may not necessarily be monotone increasing in δ. For this particular model, I am concerned about situations in which g is monotone increasing in $\bar{\delta}$ for a fixed δ. In this event, I say that the "Impatience Effect" dominates, since in these situations a more impatient proposer engages in less screening of types.[11] I prove the result for two cases. One is the uniform distribution on $[0, \theta_h]$ and no restriction on θ_h. The second is an arbitrary distribution with $\theta_l > 0$ and restrictions on θ_h. I also provide an example to show the delay comparison holds for the uniform distribution and any value of θ_h when $\theta_l > 0$.

Some notation is necessary to state the theorem. The term I demonstrate the delay comparison with is called the **Continuation Rate**. The continuation rate at time t is the percentage of the firm's types who continue to bargain after t periods. The continuation rate in a period depends on two things. First, it depends on the what the belief is entering the period. Second, it depends on how the belief is updated in the period. For a given initial belief θ_b, I use the following notation for the values of subsequent belief updating functions. Let $\theta^n_{a1}(\theta_b) = g(\theta_b)$. And, given $\theta^n_{at-1}(\theta_b)$, let $\theta^n_{at}(\theta_b) = g\left(\theta^n_{at-1}(\theta_b)\right)$. In words, for an initial upper bound on beliefs of θ_b, θ^n_{at} is the highest type not to reach agreement by the end of period t.

The continuation rate is the probability the bargaining has not stopped given an initial upper bound on beliefs of θ_b, n creditors, and t rounds of bargaining have occurred. That is:

$$F^n_t(\theta_b) = \frac{F\left(\theta^n_{at}(\theta_b)\right)}{F(\theta_b)}$$

Next I state the theorem on delay comparisons. If θ is uniformly distributed on $[0, \theta_h]$, I consider only the equilibrium for which $C(\theta_b)$ is analytic. If $\theta_l > 0$, I consider only beliefs such that the equilibrium is unique for both n.

Theorem 2. [Delay Comparison] If either

(1) $\theta_l = 0$ and θ is uniformly distributed, or

(2) $\theta_l > 0$ and θ_h is sufficiently small,

[10] This assumes there are multiple creditors present at the beginning of period 1 in the US.

[11] Paradoxically, for certain densities, a more impatient proposer may actually engage in more screening.

then $F_t^n(\theta_h) \leq F_t^1(\theta_h)\ \forall t$, with the inequality strict if $F_t^1(\theta_h) > 0$.

The intuition is as follows. As the number of creditors increases, the exclusion of half of the creditors from receiving a positive offer in any period combined with the reduced probability a particular creditor expects to propose in the future decreases the proposer's quasi-discount factor and makes the proposer more impatient. More impatient proposers put less weight on the continuation play of the game. The less weight the proposer places on the continuation play of the game, the less willing he is to screen out different types of firms and, consequently, the lower his offers become. Lower offers lead to a higher probability of acceptance from the firm. Hence, $g(\theta_b)$ decreases and the "impatience" effect dominates.

5.1 Delay Comparison: "No Gap" Case

If $\theta_l = 0$ and θ is uniformly distributed, I can solve for the belief updating and value functions explicitly, because (as I show) the value function is quadratic; and both the aggregate payment function and the belief updating function are linear.

Proposition 4. Let θ be uniformly distributed on $[0, \theta_h]$. Consider an equilibrium such that $C(\theta_b)$ is analytic.[12] Theorem 2 holds.

Proof:

I begin by setting the notation for the proof. Let θ' be a candidate for a proposer who must select $g(\theta_b)$ when the upper bound on beliefs is θ_b. Also substitute $P\left(g(\theta_b)\right) = (1-\delta)g(\theta_b) + \delta C\left(g(\theta_b)\right)$ directly into the objective function of the simplified problem for P .

I conjecture that:

$$X(\theta_b) = \alpha\theta_b^2 \text{ and } P(\theta_b) = \gamma\theta_b$$

With this conjecture, I can find for the equilibrium outcome that is the solution to the functional equation. The first step is to show that the associated belief updating function is of the form $g(\theta_b) = \beta\theta_b$.

Normalize θ_h to one.[13] Three equations entirely determine this equilibrium. The first is the F.O.C. for the proposer's maximization problem in the simplified functional equation. After substituting $P(\theta)$ directly into the maximization problem, the F.O.C is:

$$\frac{d}{d\theta'}\left(P(\theta')(\theta_b - \theta') + \bar{\delta}X(\theta')\right)|_{\theta'=g(\theta_b)} = 0$$

Substituting functional forms and taking the derivative yields:

$$\gamma\theta_b - 2\gamma g(\theta_b) + 2\bar{\delta}\alpha g(\theta_b) = 0 \tag{15}$$

[12] An argument in GSW shows that there is at most one equilibrium with an analytic C.

[13] Otherwise θ_h appears in the denominator of α.

which is the first equation.

Second the value function X is found explicitly by:

$$X(\theta_b) = P\left(g(\theta_b)\right)\left(\theta_b - g(\theta_b)\right) + \delta X\left(g(\theta_b)\right)$$

After substituting in the specific functional forms, this yields:

$$\alpha\theta_b^2 = \gamma g(\theta_b)(\theta_b - g(\theta_b)) + \delta\alpha\left(g(\theta_b)\right)^2 \tag{16}$$

which is the second equation.

Finally after substituting the new notation for C, the constraint on the highest type accepting the current offer, becomes:

$$P(\theta_b) = (1-\delta)\theta_b + \delta P\left(g(\theta_b)\right)$$

Again utilizing the conjecture on functional forms yields:

$$\gamma\theta_b = (1-\delta)\theta_b + \delta g(\theta_b)\gamma \tag{17}$$

which is the third equation that completes the solution to the functional equation.

From (15-17), it is apparent that g is linear and also that the conjectures on previous functional forms hold.

From here, let $g(\theta_b) = \beta\theta_b$.

A sufficient condition for the Delay Comparison to hold is that, as an implicit function, β decreases as $\bar{\delta}$ decreases.

(15-17) yields a system of three equations and three unknowns. The resulting system is:

$$\gamma - 2\gamma\beta + 2\bar{\delta}\alpha\beta = 0 \tag{18}$$

$$\alpha = \gamma\beta(1-\beta) + \delta\alpha\beta^2 \tag{19}$$

$$\gamma = (1-\delta) + \delta\beta\gamma \tag{20}$$

Algebra yields the following equation relates β and $\bar{\delta}$:

$$1 - 2\beta + 2\bar{\delta}\frac{(1-\beta)\beta^2}{1-\delta\beta^2} = 0 \tag{21}$$

To complete the proof, fix δ, let N be an odd integer greater than 1 and adopt the notation that β_n and $\bar{\delta}_n$ are the values of β and $\bar{\delta}(n,\delta)$ associated with a given n and δ. For example, β_1 is the solution to Equation (21) for $n = 1$.

Split the LHS of (21) into two functions, A and B:

$$[A(\beta)] = 1 - 2\beta \text{ and } [B(\beta, \bar{\delta}, \delta)] = 2\bar{\delta}\frac{(1-\beta)\beta}{1-\delta\beta^2}$$

$B(\beta, \bar{\delta}, \delta)$ is always positive for any $\beta, \bar{\delta}, \delta \in [0, 1]$.

Hence:

$$B(\beta_1, \bar{\delta}_N, \delta) < B(\beta_1, \bar{\delta}_1, \delta)$$

and

$$A(\beta_1) + B(\beta_1, \bar{\delta}_N, \delta) < 0.$$

Next consider the function $D(\beta) = A(\beta) + B(\beta, \bar{\delta}_N, \delta)$.

I complete the proof by computing $D(0)$ and showing that this term is positive. Since $A(0) = 1$ and $B(0, \bar{\delta}_N, \delta) = 0$, $D(0) = 1$.

Since D is continuous in β and $D(\beta) = 0$, for some $\beta \in (0, \beta_1)$, $\beta_N < \beta_1$ as desired.

□

The quantitative effects of increasing the number of creditors can be seen through a numerical example. In Figure 5-5c, I normalize θ_h to 1, and let $n = 1$ and 9. The number of creditors is chosen to match US and UK observations. The first observation, $n = 1$, is chosen to capture the equivalence between the UK law and the US law with one creditor. The second observation, $n = 9$, is chosen to match the average number of classes of impaired debt for a firm in bankruptcy in the US as found by LoPucki and Whitford (1990). In Figure 5, I plot g as a function of δ for both values of n. From this figure, it is apparent that the belief updating function is strictly higher for the one creditor model, and for high discount factors, significantly so. In Figure 5a, I show the equilibrium aggregate payment, $C(1)$, for $n = 1$ and 9. From this figure, the more impatient proposers in the multiple creditor environment offer significantly lower payments. Facing lower payments, the firm settles sooner.

I also report the expected delays associated with introducing more creditors in Figure 5b. As this figure shows, the expected delays in the one creditor model can be many multiples of the expected delays in the multiple creditor model. This feature is driven by the limiting behavior of $\bar{\delta}$ and $g(1)$ as δ converges to 1. For high discount factors, the proposer in the one creditor environment has a quasi-discount factor that converges to one, which leads $g(1)$ to also converge to one. But for the multiple creditor environment the proposer's quasi-discount factor is strictly bounded by $\frac{1}{2}$. Hence $g(1)$ is bounded away from one. Thus, in the limit, the expected delays are bounded in this case.

Finally, Figure 5c shows the effect additional creditors have on the expected payoff to the entire set of creditors, or $X(1)$. Adding more creditors strictly reduces their payoff.

5.2 Delay Comparison: Gap Case

For the gap case, that is when $\theta_l > 0$, if θ_h is sufficiently low I can derive the functions C and X explicitly. In this case I can prove the following proposition.

Proposition 5. Let $\theta_l > 0$ and θ_h be sufficiently small. Theorem (2) holds.

The proposition is a simple consequence of the following lemma.

Lemma 1. As $\bar{\delta}$ increases, there exists a value θ^* such that $g(\theta_b)$ increases everywhere on $(\theta_l, \theta^*]$.

The proof of the lemma is contained in the Appendix. In order to have analytic forms for the function, I require that the bargaining lasts no more than 2 periods. For more general distributions and loosened restrictions on θ_h, I use examples to demonstrate what is a slightly weaker result. The weaker result is that the expected length of bargaining decreases in n for many standard distributions. I can no longer guarantee that the belief updating function, g, is monotone.

For the uniform distribution and a given discount factor, it is possible to check the entire parameter space from which θ_l and θ_h are drawn through one example. To do this simply extend the range of θ_h and the result does indeed hold. I pick $\theta_l = 0.1$, $N = 9$ and $\delta = 0.97$ for the example I show. The output shown is the expected length of bargaining.[14] As the graph shows, decreasing patience can have a strong effect.

However sufficient conditions for more general analytic theorems are difficult to show. In particular, global comparative statics on the belief updating function are rare. In the "No Gap" case the forward looking nature of beliefs implies that the payment function C is not differentiable and that the belief updating function, g can have jumps. Hence even for smooth distributions, the behavior of g can be somewhat chaotic.

In the next section, I consider the game before the initial contracts are signed which, should the firm default, lead to the ex-post model of bankruptcy. For the ex-ante game, there are two important variables from the ex-post game: X and U. To set notation, $X(\theta_h, n)$ and $U(\theta_h, n, \theta)$ are the expected payoffs to the entire set of creditors and type θ firm respectively, if the highest type in bankruptcy is θ_h and there are n creditors who actively bargain with the firm in bankruptcy. In the next section, I firmly identify the US system with the majority vote regime and UK system with the controlling creditor regime. Previously, it has not been necessary to explicitly discus the UK law with more than one creditor. In the next section, in order to describe the contracting environment, I must explicitly refer to the UK law with multiple creditors. I adopt the notational convention that n refers to the number of creditors who actively participate in the ex post game, not the number of creditors who sign the initial contracts.

[14]The length of bargaining is multiplied by $F(\theta_b)$ to produce a monotonic picture.

6 Ex-Ante Contracts and Welfare

I conclude the paper with an ex-ante welfare comparison for the two bankruptcy systems. I consider the problem for a firm which must sign a contract at $t = 0$ in order to receive the necessary finance to earn θ.[15] I show that welfare is higher with the US system.

The model for this section is simple and is a natural extension of the previous model. Default in this model leads to the ex-post game described in Section 4. New to this model is a trade-off between avoiding delay costs should bankruptcy occur and obtaining sufficient funding for projects. I show that the US regime with n creditors is ex-ante superior to the UK regime if the following assumption holds:

Assumption 1. Solvent firms cannot enter bankruptcy.[16]

Assumption (1) is satisfied by the US bankruptcy code if judges can observe whether a firm's type is above a given threshold. Under US law, judges in bankruptcy courts can dismiss bankruptcies entered into in bad faith through "abstention." Such a dismissal is costless to judges and not subject to appeal. I make the assumption because I do not want the payments negotiated by firms that settle in the first period of bankruptcy to determine the payments firms make outside bankruptcy.

The intuition behind the result is as follows. Consider the UK law. Whenever the controlling creditor in the UK makes a proposal, he asks for higher payments than his US counterpart. The tougher he bargains, the more revenues he expects to generate, but also the more deadweight losses that he expects to occur. For very reasonable discount factors, any expected decrease in revenues from increasing the number of creditors is overwhelmed by an expected decrease in deadweight losses. In this case, the "hold up" problem is severe enough for ex-ante welfare to be higher with one creditor.

This section is organized as follows. The ex-ante model is described first and welfare defined. Then, the welfare comparison is presented. (The Delay Comparison continues to hold trivially.)

6.1 Ex-Ante Environment and Timing

Consider a firm which can earn a stock return of θ, where θ is restricted to be uniformly distributed on $[0, 1]$. To achieve θ, the firm must borrow I to begin the project. At the time the firm seeks financing in period 0, θ is unknown to the firm. To obtain financing, a firm signs a debt contract with a value of p, which in the US, is split evenly by an exogenous set

[15]This section is part of a larger work, Benjamin(2003), which analyzes a richer environment than the one present here.

[16]The qualitative result applies to a model in which managers must pay a sufficiently large fixed cost (in terms of their personal reputation for example) to enter bankruptcy. It does not apply to environments where entering bankruptcy is costless. In this case the qualitative results shift.

of creditors. In the UK, this restriction must be modified such that the creditors other than the receiver receive a second, higher interest rate for their payments outside bankruptcy. In the US, the burden of supplying the investment I is split evenly among the creditors who sign the contract. This gives me identical creditors in bankruptcy. In the UK, the investment can be divided arbitrarily between the receiver and the other creditors. The investment market is perfectly competitive and all creditors have enough resources to meet whatever is demanded of them. The players are restricted to sign contracts such that all payments outside bankruptcy must be made in the first period.

The firm's ex-ante problem has the timing outlined in Figure 2 in the Appendix. In period 0, the firm proposes the initial contract. This is a take-it-or-leave-it offer. The initial contract consists of the fraction of the required investment from each of the creditors in period zero and the payment the creditors are entitled to in period 1. If all the creditors approve, the initial contract is signed and I is delivered. In period 1, the firm learns its type. If the firm has sufficient resources, it makes the aggregate payment, p, and consumes $\theta - p$. If θ is beneath the promised aggregate payment, the firm enters bankruptcy. From here the timing is the same as in the ex-post model. The first offer occurs in the period the firm enters bankruptcy.

The distribution of the firm's type before any contracts are signed is called G. Once the contracts are signed and the types of the firm above the bankruptcy threshold make their payments, the distribution of firms inside bankruptcy, F, can be found from $F(\theta) = \frac{G(\theta)}{G(\theta_h)}$. Associated with G is a density g_a.

6.2 Ex-Ante Welfare Defined

In this subsection, I present the formal definitions of ex-ante welfare under the two different systems. I consider only welfare with n creditors in both systems. I call welfare in the UK, Z_{UK}. Welfare in the US system is called Z_{US}.

Welfare to the firm in the US in period 0 satisfies:

$$Z_{US} = \max_{\{\theta_h, p\}} \int_0^{\theta_h} U(\theta_h, n, \theta) g_a(\theta) d\theta + \int_{\theta_h}^1 (\theta - p)\, g_a(\theta) d\theta \tag{22}$$

such that

$$\theta_h - p \geq 0 \tag{23}$$

$$X(\theta_h, n) + p\,(1 - G(\theta_h)) = I \tag{24}$$

Constraint (23) is a wealth constraint which puts firms that cannot afford the prescribed payment into bankruptcy. Constraint (24) is a zero expected profit constraint on the entire set of creditors.

Welfare in the UK system requires a different definition as the constraints change. For ease of exposition, let creditor one be the receiver. This is exogenous to the model. Creditor one receives p_1 outside bankruptcy. All other creditors receive p_{-1} outside bankruptcy. Inside bankruptcy creditor one is the only creditor negotiating with the firm and the only one to receive positive payments. The other creditors receive zero payments. Likewise, I_1 and I_{-1} refer to the expected investment from the receiver and from any other creditor, respectively. With this new notation, I can define welfare. Welfare in the UK to the firm in period 0 satisfies:

$$Z_{UK} = \max_{\{\theta_h, p, p_1, p_{-1}, I_1, I_{-1}\}} \int_0^{\theta_h} U(\theta_h, 1, \theta) g_a(\theta) d\theta + \int_{\theta_h}^1 (\theta - p)\, g_a(\theta) d\theta \tag{25}$$

such that

$$\theta_h - p \geq 0 \tag{26}$$

$$X(\theta_h, 1) + p_1 (1 - G(\theta_h)) = I_1 \tag{27}$$

$$p_{-1} (1 - G(\theta_h)) = I_{-1} \tag{28}$$

$$p_1 + (n-1) p_{-1} = p \tag{29}$$

$$I_1 + (n-1) I_{-1} = I \tag{30}$$

There are two zero profit constraints: (27) for creditor one and (28) for the other creditors. Constraints (29) and (30) are identities that insure that the creditors as a whole receive the an aggregate payment equal to what the aggregate payment the firm makes and supply the aggregate investment the firm receives.

The actual values of p_1, p_{-1}, I_1 and I_{-1} are indeterminant. Whenever the non-negativity constraint is satisfied, the definition of ex-ante welfare in the UK system is identical to the definition of ex-ante welfare in the US system (with a different bankruptcy policy).

6.3 Welfare Comparison

I present the welfare comparison through a proposition.

Proposition 6. Suppose

I. $n > 1$

II. δ is sufficiently high [17]

III. $I \leq \max_{\theta_h} X(\theta_h, n) + \theta_h (1 - G(\theta_h))$

then $Z_{US} > Z_{UK}$.

[17] Sufficiently high=0.54 if $n = 9$.

The proof is in the Appendix.[18] The result occurs because there is a hold up problem that reduces ex-ante welfare. For high discount factors, the patience of the firm implies that more patient proposers generate more deadweight losses than revenues. Multiple creditors sacrifice these small revenues gains but avoid the larger deadweight losses. This eases the hold up problem.

For the above model, the Delay Comparison continues to hold. This is true simply because the continuation rate is constant in θ_h whenever θ has the uniform distribution.

7 Conclusion

From this study, advice can be found for governments choosing bankruptcy laws. On the simplest level, if the choice is a zero-one choice between the US and the UK bankruptcy laws, then they should choose the US law.

If the choice is for general principles to govern bankruptcy, the bankruptcy system should be designed, in part, to prevent inefficiencies ex-post. Picking a voting rule that facilitates agreements, such as a majority voting rule, can significantly reduce inefficiencies that are the result of lengthy delays. Conversely, requiring creditors to consent to any write down of their debt increases ex-post inefficiencies and as such is undesirable to a country choosing a bankruptcy law.

However, there is a caveat to the importance of a majority vote. A majority vote is only effective in limiting delays if individual creditors can be rewarded, relative to other creditors, for supporting plans that produce agreements. An equally key feature of bankruptcy design is that the division of claims between creditors must be open to negotiation. This is a much maligned feature of Chapter 11 but a key part of its success.

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[18]A consequence of this exercise is that when the number of creditors is endogenous, firms in the US prefer to contract with multiple creditors and, in such a case, the two bankruptcy systems produce different outcomes. The choice of the number of creditors in the UK for such an exercise has some indeterminacy.

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8 Appendix A: Detail of Law

8.1 US Law

In Chapter 11, bargaining is done over bankruptcy plans. A plan specifies a division of creditors into classes. Though judges have some discretion here, creditors with similar claims can be and frequently are placed in different classes.[19]

> [Bankruptcy Law] explicitly permits the separate classification of all similar claims or interests.

A plan also specifies a new claim for each of the classes. The assignment of different claims to similar classes is frequently a heated problem in bankruptcy. From Ayer, Bernstein, and Friedland (2003):

> [The assignment of different claims to similar classes] happens enough that some of the worst fights in chapter 11 involve plan classification, with the dissenters arguing that the plan proponent is "gerrymandering" the classes, while the proponent argues that there is a principled basis for its classification scheme.

The plan is negotiated between the firm and a subset of the creditors called the Creditors' Committee. The court in Jones-Mansville states:[20]

> Reorganization committees are the primary negotiating bodies for the plan of reorganization.

Generally speaking it is easy for unsecured creditors to join the Creditors' Committee. The committee always includes the seven largest creditors. By statute, it also includes any creditor who has previously entered negotiations with the firm. Finally, the court can add any creditor whose interests the court deems are not adequately represented by the committee. Secured creditors who serve on the committee are not allowed to vote, as they are generally considered adverse to reorganization.

At the first meeting of the Creditors' Committee, the creditors select a lawyer and other professionals by majority vote. From Andrews (1985):

> The Creditors' Committee acts through the professionals whom they employ. [Bankruptcy law] permits a committee, by a majority vote of those present at a scheduled meeting to select and authorize the appointment of attorneys, accountants and other agents.

[19] All quotations are from a standard textbook in the field: Baird, Jackson, and Adler (2001) unless otherwise noted.

[20] See Andrews (1985)

Traditionally one attorney is the chief attorney for the committee. Once chosen, he suggests a voting rule for all actions the committee takes excluding the hiring and firing of the committee's employees. This voting rule must be adopted by a majority vote, but as a show of solidarity is typically adopted unanimously. Almost always, the voting rule is a majority rule. According to Klee and Schaffer (1993)

> Seasoned committee counsel provides voting rules in the committee's by-laws that are almost always adopted unanimously. Experience teaches that [majority vote] "one person, one vote" regardless of the size of claim is the standard adopted in most cases.

The Creditors' Committee approves the plan if it passes a majority vote of its members. Once the Creditors' Committee approves the plan, the firm files a Disclosure Statement with the court. The court approves and distributes the Disclosure Statement and a vote is scheduled among the creditors as a whole. Approving and distributing the Disclosure Statement is typically a matter of months.[21] Though the Creditors' Committee approval is not needed for the court to approve the Disclosure Statement (or to eventually confirm the plan), it is a practical requirement for both.

There are two procedures for the plan to pass the vote of the entire set of creditors. First, a plan can go in effect if it passes a majority vote (in number and two thirds in amount) in every class.

> Approval requires positive votes by those who hold two thirds in amount and a majority by number [of the firm's claims within a class].

If a class receives full payment, then that class is deemed to have voted for the plan (They do not actually vote). Other classes are deemed impaired. I am more interested in the second procedure for plans to go into effect, for which the impairment of certain classes plays a larger role in the eventual adoption of plans. The second procedure is called "cramdown." A plan can go into effect through "cramdown" if at least one impaired class votes to accept the plan. A "crammed down" plan must satisfy certain technical requirements, the most important one I describe shortly. But, first I document my description of cramdown. I quote the description in Baird, Jackson, and Adler (2001) of the formal rule governing cramdown:

> If at least one impaired class of claims accepts the plan, the proponent of the plan can seek to have it confirmed over the objections of the other classes [through cramdown].

Formal cramdown procedures are frequently costly and many creditors avoid them and vote for lower payments then they would otherwise be entitled to.

[21] See LoPucki and Whitford (1990)

> Cramdown is a time consuming and expensive affair...Creditors may be better off accepting a plan even if [the law] gives them the right to insist on better treatment.

A more detailed commentary on the costs of cramdown can be found in LoPucki and Whitford (1990). They find:

> While the absolute priority rule governs any adjudication of the rights of unsecured creditors and shareholders to share in the distribution under the plan, a variety of reasons may cause the representatives of creditors and shareholders to wish to settle their rights on a different basis. A principal reason is to avoid litigation over whether the standards for cram down are satisfied. A cram down determination requires a potentially difficult valuation of properties, such as debt instruments and shares, distributed to particular classes. Such valuation can be expensive and time consuming.

In their sample, no unsecured creditor forces a cramdown hearing, though many of the plans that pass contain grounds for such hearings.

It should be noted that should a hearing occur, the rules that govern cramdown reject the extreme contracts I assume in the model. Though not universally applicable, the Fair and Equitable test gives creditors the right to insist on equal treatment to creditors with identical priority. From Baird, Jackson, and Adler (2001):

> Each class can insist on being treated at least as well as classes that enjoy the same priority under nonbankruptcy law.

There are exceptions to the Fair and Equitable Test which do allow for some discrimination among similar classes.[22] The court may allow a discriminatory plan if it finds a principled reason for the discrimination. a common reason is called the ”business necessity” exception.[23] Firms are allowed to discriminate against certain classes in order to preserve ongoing business relationships with other classes of similar priority. Also, if the discrimination is not severe, the court may it allow it to avoid potentially costly future negotiations.

I conclude this subsection of the appendix with a discussion of the assumptions in the model that potentially conflict with the US law. In the paper, I make the extreme assumption that creditors (and judges) approve plans whenever the payments in a plan are non-negative. This assumption is WLOG as the main qualitative results also apply to a model where for any $c < 1$, all creditors are restricted to receive at least $\frac{c}{n}$ of the payment to the entire set of creditors. Large values of c can produce significant differences in outcomes if the discount

[22] A popular method of limiting court enforcement of the Fair and Equitable Test is to offer debt with different maturities. These assets frequently have different bankruptcy risks. Courts generally allow great deference in comparing the valuations of such debts. See Markell (1998) for more detail on this and discrimination in general.

[23] See Norberg (1995) for a complete discussion.

factor is high enough. (The analysis of the more general model is in the Technical Appendix which is available on request.)

I also make the assumption that the creditors and the judge ratify the result of a majority vote. Allowing plans to pass with support from a different supermajority preserves the qualitative results, provided the vote is less than unanimous. In addition to the vote in the Creditors' Committee, the majority vote captures the judicial discretion that is present in the law. Although cramdown only requires the approval of one class of creditors, the judge has many subjective criteria by which he can reject plans, especially if there is substantial opposition from the creditors. The basic test for confirmation, Ayer, Bernstein, and Friedland (2003) include:

> ...the determination that the plan complies with all applicable law and has been proposed in good faith...[and that the firm] has a credible business plan and can reasonably be expected to perform its obligations and accomplish the objectives set forth in the plan.

It is reasonable to assume that without the support from a majority of the creditors, the judge will not rule affirmatively on the above criteria. Thus, the judicial discretion I assume may best be thought of as minimal in this regard.

8.2 UK Law

The UK law is biased towards one creditor, typically a bank. The law encourages firms to designate one and only one creditor by an initial floating charge as deserving special privileges in bankruptcy. This creditor has the right to appoint the receiver who has the right to determine the fate of a business in bankruptcy.

The most succinct description of the UK law appears in Franks, Nyborg, and Torous (1996):

> The receiver is appointed by the creditor with the floating charge and represents the interests of that creditor with virtually no duty of care to other creditors.
>
> The powers of the receiver are significant. He has complete control of the firm, and does not require permission from the court or from the other creditors for his actions.

If no creditor has the right to appoint a receiver, the firm cannot enter receivership.

The substantial constraints on the receiver are as follows. He may not pay his appointer more than he is owed. Also he requires the consent of secured creditors to use their collateral.

9 Appendix B

Proof of Proposition 1

I complete this proof by constructing strategies and beliefs such that the corresponding outcome given by functional equation is an equilibrium outcome.

For any period t and any history with an upper bound on belief of θ_b, consider the following profile:

$$\sigma_{it}(\theta_b) = (p_1(\theta_b), \{z_{ij}p_{-1}(\theta_b)\}_{j\neq i})$$

$$\text{where } \sum_{j\neq i} z_{ij} = \frac{n-1}{2}, \tag{31}$$

$$z_j \in \{0,1\} \tag{32}$$

$$\text{and } E[z_{ij}||j\neq i] = \frac{1}{2} \tag{33}$$

$$\sigma_{jt}(\theta_b) = \begin{cases} Y & \text{if } p_j \frac{F(\theta_b)-F(g(\theta_b))}{F(\theta_b)} \geq \delta\left(\frac{1}{n}V(\theta_b) + \frac{n-1}{n}W(\theta_b)\right) \\ & -\delta\left(\frac{1}{n}V\left(g(\theta_b)\right) + \frac{n-1}{n}W\left(g(\theta_b)\right)\right)\frac{F(g(\theta_b))}{F(\theta_b)}, \\ N & \text{otherwise} \end{cases}$$

$$\sigma_{at}(\theta) = \begin{cases} Y & \text{if } C(\theta_b) \leq (1-\delta)g(\theta_b) + \delta C\left(g(\theta_b)\right), \\ N & \text{otherwise} \end{cases}$$

$$\mu_t = \text{ the truncation of } f \text{ to } [\theta_l, \theta_b]$$

I claim that the above is an equilibrium. It is easy to show that the above strategies reproduce the continuation values in **Program 1**. It is also apparent that the beliefs satisfy Bayes' Rule. In this proof I simply check one period deviations to show the strategies are best responses. First consider the proposer. First I show that the proposer chooses the optimal division of resources among the creditors. There are two cases. In the first case the proposer makes an offer that is rejected by all types of the firm, say $P_t = \theta_h + 1$. This earns him the continuation value evaluated at the current upper bound on belief θ_b regardless of the division in his proposed plan among the creditors. Hence any equilibrium profile that includes such an aggregate payment can be a best response whenever making a non-serious offer is a best response. This includes the profile suggested here.

Suppose it is a best response to make an offer such that some types of the firm accept. I must show that the division described above is optimal. For a particular aggregate payment P_t, let's derive the highest payoff the proposer can achieve. Note the continuation payoff for all voters are identical at P_t given the equilibrium profile (and the restriction on the z's in (33)). Hence their continuation values are identical. To receive the necessary votes, the plan must satisfy (3) with inequality (or else the proposer can achieve the same welfare by making a trivial offer that satisfies (3)).

$$p_j \left(\frac{F(\theta_b) - F(\theta_a)}{F(\theta_b)} \right) + \delta \left(\frac{1}{n} V(\theta_a) + \frac{n-1}{n} W(\theta_a) \right) \frac{F(\theta_a)}{F(\theta_b)} \geq \delta \left(\frac{1}{n} V(\theta_b) + \frac{n-1}{n} W(\theta_b) \right)$$

Hence the only possible deviation is one that satisfies (3) with strict inequality. But such an offer is not a best response to the creditors' strategies, as the proposer could get the requisite votes and a higher share of the revenues by decreasing the offer to the creditor for whom the incentive constraint does not bind.

Next consider the proposer's choice of the aggregate plan $P_t^* = C(\theta_b)$. Based on the equilibrium strategies for every possible aggregate plan P_t^*, there exists a lowest type θ_a that accepts the plan. This type is given by (2), but with strict equality.

$$\theta_a - p_i - \frac{(n-1)}{2} p_j = \delta \left(\theta_a - C(\theta_a) \right)$$

It is obvious, in the solution to **DP** constraint (2) always holds with equality. Hence the proposer always considers the firm's equilibrium strategy when choosing a plan.

The equilibrium strategies give us that the welfare function in (1) corresponds to V, Hence the proposer in period t maximizes his choice of an aggregate plan by solving the maximization problem outlined in **DP**.

For voters and firms, the strategy profile is a best response if they vote 'Y' whenever their current payoff is higher than their continuation value. These are precisely the strategies outlined above.

Thus the proposed strategies are an equilibrium.

$\square$

Proof of Proposition (3)

Proposition. If $\theta_l > 0$, the bargaining lasts a finite number of periods.

The proposition can be shown with two lemmas. The first shows that if the upper bound on the belief is sufficiently low the game ends in the current period. The second shows such beliefs are always reached.

Lemma 2. In any equilibrium there exists a $\theta^* > \theta_l$, such that if the belief falls below θ^*, the game ends in one period, regardless of the proposer. Further, the aggregate payment equals θ_l in such a period.

Proof of Lemma

First I establish the complete set of strategies for games in their final period. This allows me to explicitly derive the proposer's payoff for settling with all of the remaining types in a period.

If after any relevant history for proposer i, h_i^t, and upper bound on belief, θ^*, it is common knowledge the game ends in the current period; then, for the proposer's value, v_o, and a voter's expected value, w_0, the proposer solves:

Last Period's Problem

v_0, w_0 are the fixed point of the following program:

(DP_0)

$$v_0 = \max_{p_i, p_j} p_i \tag{34}$$

$$\text{s.t. } \theta_l - p_i - \frac{(n-1)}{2} p_j \geq 0 \tag{35}$$

$$p_j \geq \delta \left(\frac{1}{n} v_0 + \frac{n-1}{n} w_0 \right) \tag{36}$$

And given p_j from DP_0,

$$w_0 = \frac{1}{2} p_j \tag{37}$$

This is the problem studied in Baron and Ferejohn (1989) which they show has a unique solution:

Definition 2. The Baron-Ferejohn payments or BF payments for surplus s are such that:

Let $BF = 1 - \delta \frac{n-1}{2n}$.

I. The proposer request BF times s for himself.

II. For half of the remaining creditors he requests $\frac{2(1-BF)s}{n}$ of the surplus

III. For the other half he requests a payment of zero.

The *equilibrium outcome* for any game in the final period of bargaining includes dividing θ_l according to the BF payments.

Next I show that there exists a θ^* such that for all beliefs beneath θ^*, all plans with $P > \theta_l$ are dominated by splitting θ_l according to the BF payments.

The first step in this process is to show that there exists a θ^* such that the expected payoff to the entire set of creditors is maximized by offering $P = \theta_l$.

Claim 1. There exists a θ^* such that if $\theta_b < \theta^*$, $X(\theta_b)$ is maximized by setting $P = \theta_l$.

This is the first part of *Lemma 3* in FLT and is not shown here.

Next, I show that for θ^* sufficiently low, the proposer cannot do better in equilibrium by any other strategy than he could do by splitting θ_l according to the Baron Ferejohn payments.

Claim 2. For θ_b sufficiently low, offering $P = \theta_l$ to be divided according to the Baron-Ferejohn payments is part of any equilibrium strategy.

To prove this claim I show that whenever $P = \theta_l$ maximizes the aggregate payoff (X) to the creditors, I can always find a plan which dominates any plan with aggregate payment $P^* > \theta_l$.

Proof of Claim

Suppose not. Consider a candidate equilibrium plan with aggregate payment $P^* > \theta_l$. This aggregate payment implies a set of payments $\{p_j^*\}_{j=1}^n$ to the individual creditors and a type $\theta_1 > \theta_l$ that accepts P^*. Instead of offering the firm P^* and the creditors $\{p_j^*\}_{j=1}^n$, the proposer receives a higher payoff by giving the firm a plan with a lower aggregate payment, $P = \theta_l$, and the individual creditors a share of P equal to their expected payoff associated with the original offer and consuming the residual. If the original offer passes the requisite votes, the new offer also passes the requisite vote and approval/ veto decision. Because of the previous claim, the proposer prefers the new plan to the original plan.

□

Next I complete the original proposition with a lemma that the upper bound on belief θ^* is reached in finite time.

Lemma 3. For any initial belief, μ, there exist a period T such that all types have settled in at most T periods of bargaining. That is within $T - 1$ periods the upper bound on beliefs has fallen below θ^*.

The above lemma is demonstrated in Lemma 2 in FLT. Hence the proof is omitted.

The lemma completes the proof of Proposition (3). □

Proof of Proposition (2)

For ease of exposition, I assume all relevant functions are differentiable. Consider Program (1).

Replace the constraint (2) with

$$p_i + \frac{(n-1)}{2} p_j \leq P(\theta_a)$$

where:

$$P(\theta_a) = (1-\delta)\theta_a + \delta C(\theta_a)$$

For multipliers,λ, ξ, ν, the Lagrangian for this problem is:

$$p_i\left(\frac{F(\theta_b)-F(\theta_a)}{F(\theta_b)}\right)+\delta\frac{1}{n}\frac{X(\theta_a)}{F(\theta_b)}+\lambda\left(p_i+\frac{n-1}{2}p_j-P(\theta_a)\right) \tag{38}$$

$$+\xi\left(p_j\frac{(F(\theta_b)-F(\theta_a))}{F(\theta_b)}+\delta\frac{1}{n}\frac{X(\theta_a)}{F(\theta_b)}-\frac{1}{n}X(\theta_b)\right)+\nu(\theta_a-\theta_l) \tag{39}$$

The F.O.C.'s for an interior optimum for this problem are

$$\frac{(F(\theta_b)-F(\theta_a))}{F(\theta_b)}=-\lambda \tag{40}$$

$$-\lambda\frac{n-1}{2}=\xi\frac{(F(\theta_b)-F(\theta_a))}{F(\theta_b)} \tag{41}$$

$$(p_i+\xi p_j)\frac{(-f(\theta_a))}{F(\theta_b)}+\delta\frac{1}{n}\frac{X_1(\theta_a)}{F(\theta_b)}-\lambda P'(\theta_a)+\xi\delta\frac{1}{n}\frac{X_1(\theta_a)}{F(\theta_b)}=0 \tag{42}$$

Next consider the problem in Program (2).

For the proper choice of $P(\theta_b)$ and $X(\theta_b)$ the proposer's problem is equivalent to:

$$V(\theta_b)=\left(\max_{\theta_a}P(\theta_a)\left(F(\theta_b)-F(\theta_a)\right)+\bar{\delta}(n,\delta)X(\theta_a)\right)$$

The F.O.C. for an interior optimum for this problem is:

$$P(\theta_a)(-f(\theta_a))+P'(\theta_a)(F(\theta_b)-F(\theta_a))+\delta\left(\frac{1}{2}+\frac{1}{2n}\right)X'(\theta_a) \tag{43}$$

Hence any solution to Program 2 must satisfy (43).

Substituting (40) into (41) gives ξ

$$\xi=\frac{n-1}{2}$$

From here, substituting ξ and λ into (42) yields (43) and the F.O.C.'s that determine $g(\theta_b)$ for the two models match.

Thus the belief updating functions g are identical at every period if the C and X functions for the two problems are identical. It is a simple result that identical g functions for different n's force the C and X functions to be identical. These relationships imply that any (g, C, X) associated with a solution to **Program 1** is a solution to **Program 2**.

□

Proof of Theorem 1

I prove this theorem by deriving the only possible equilibrium recursively from the last period of bargaining. I show that in every period the proposer has a well defined maximization problem with a unique solution. Subscripts refer to additional periods of bargaining not including the current period. I conjecture that the game ends in the current period.

The Zero Iteration

From previous work, for all θ_b, let

$$g_0(\theta_b) = \theta_l \tag{44}$$
$$C_0(\theta_b) = \theta_l \tag{45}$$
$$X_0(\theta_b) = \theta_l F(\theta_b) \tag{46}$$

Note this is the unique outcome to the game given the initial conjecture.

From here I conjecture that all bargaining ends in two periods. Note if the game requires two periods to end, then in the final period, the payment in the final period is $C_0(\theta_b)$. The expected revenues to the creditor in the final period are $X_0(\theta_b)$.

The First Iteration

Given C_0, and X_0; for all θ_b, let

$$g_1(\theta_b) = \arg\max_{\theta_a} \left((1-\delta)\theta_a + \delta C_0(\theta_a)\right)\left(F(\theta_b) - F(\theta_a)\right) + \bar{\delta} X_0(\theta_a) \tag{47}$$
$$C_1(\theta_b) = (1-\delta) g_1(\theta_b) + \delta C_0\left(g_1(\theta_b)\right) \tag{48}$$
$$X_1(\theta_b) = C_1(\theta_b)\left(F(\theta_b) - F\left(g_1(\theta_b)\right)\right) + \delta X_0(g_1(\theta_b)) \tag{49}$$

The maximum theorem gives that g_1 is a UHC correspondence. An argument in Ausubel and Deneckere (1989) (Proposition 4.3) gives that g_1 is single valued, except possibly in the initial period over a countable set. Hence C_1 and X_1 are continuous functions and are uniquely defined. I note that having completed the first iteration, I can place a restriction on the highest upper bound such that bargaining lasts one additional period.

$$\theta_1 = \sup_{\theta_b}\{\theta_b : g_1(\theta_b) = \theta_l\} \tag{50}$$

Lemma 2 requires $\theta_1 > 0$ and the equilibrium outcome has been uniquely determined for all $\theta_b < \theta_1$.

Given that t iterations have been completed and the equilibrium outcome has been uniquely determined for all $\theta_b < \theta_t$, I can perform the $t+1$ iteration:

The $t+1$ Iteration

Given C_t, and X_t; for all θ_b, let

$$g_{t+1}(\theta_b) = \arg\max_{\theta_a} \left((1-\delta)\theta_a + \delta C_t(\theta_a)\right)\left(F(\theta_b) - F(\theta_a)\right) + \bar{\delta} X_t(\theta_a) \tag{51}$$
$$C_{t+1}(\theta_b) = (1-\delta) g_{t+1}(\theta_b) + \delta C_t\left(g_{t+1}(\theta_b)\right) \tag{52}$$
$$X_{t+1}(\theta_b) = C_{t+1}(\theta_b)\left(F(\theta_b) - F\left(g_{t+1}(\theta_b)\right)\right) + \delta X_t\left(g_{t+1}(\theta_b)\right) \tag{53}$$

Again unless $\theta_b = \theta_h$, the solution to the above problem is unique. Once this iteration is complete, I have uniquely described equilibrium behavior for all upper bounds on beliefs $\theta_b \leq \theta_{t+1}$ where given θ_t, θ_{t+1} is defined as:

$$\theta_{t+1} = \sup_{\theta_b}\{\theta_b : g_{t+1}(\theta_b) \leq \theta_t\} \tag{54}$$

Since the bargaining lasts a finite number of periods, after at most T iterations, $(g_T, C_T, X_T,)$ is the solution to the simpler functional equation for a given value of θ_h. Since the argument in Ausubel and Deneckere (1989) does not apply in the initial period, g_T may not be single valued. But the monotonicity of the problem guarantees that g_T is single valued except at most a countable set, which completes the existence and uniqueness results.

□

Proof of Lemma 1

Consider an upper bound on beliefs θ_h, that is small enough such that for either n the game ends in no more than two periods of bargaining. Hence the proposer with such a belief solves:

$$g_1(\theta_b) = \arg\max_{\theta_a} \left((1-\delta)\theta_a + \delta\theta_l\right)\left(F(\theta_h) - F(\theta_a)\right) + \bar{\delta}\theta_l F(\theta_a) \tag{55}$$

Consider an upper bound on beliefs, θ_N, such that θ_N equals the supremum of all upper bounds on beliefs for which $g_N(\theta_N) = \theta_l$ when $n = N$. It follows that:

$$(1-\delta)\left(F(\theta_N) - F\left(\theta_l\right)\right) - \theta_l f\left(\theta_l\right) = -\bar{\delta}_N \theta_l f\left(\theta_l\right)$$

where N is an argument in $\bar{\delta}_N$ When the F.O.C. is evaluated for $n = 1$ it becomes,

$$(1-\delta)\left(F(\theta_N) - F(\theta_l)\right) - \theta_l f\left(\theta_l\right) > -\bar{\delta}_1 \theta_l f\left(\theta_l\right)$$

Hence, for $n = 1$ the solution to the two period problem is strictly greater than θ_l. (Though not shown, this can be seen by examining the F.O.C evaluated at θ_N instead of θ_l for $n = 1$ where the inequality is flipped.)

And, for all θ_h in a neighborhood of θ_1, the lemma holds.

□

Proof of Proposition 6

To update notation, let X be a function of θ_h, n and also δ. First I begin with a well known property of these models which is the basis of the proof.

Property 1. Revenues satisfy Coase Conjecture

Let $\delta \to 1$. Then, for any θ_h, and any bankruptcy system:

$$X(\theta_h, 1, \delta) \downarrow 0$$

The proof of the above property is in GSW and is not shown here due to its length.

It is convenient to rewrite the problem to find a more manageable measure of welfare. I prefer to minimize the deadweight or social losses. Hence I define a new term, $SL(\theta_h, n, \delta)$, which is the lost surplus due to delay for a bankruptcy system with an upper bound of θ_h and a linear belief updating function g with slope $\theta_a(n, \delta)$.

$$SL(\theta_h, n, \delta) = \sum_t (1-\delta)\delta^t \int_0^{\theta_{at}(\theta_h(n,\delta))} \theta f(\theta) d\theta$$

Since f is uniform this equals:

$$= \sum_{t=1}^{\infty} (1-\delta)\delta^t \frac{(\theta_{at}(\theta_h(n,\delta)))^2}{2}$$

$$= \frac{1}{2}\sum_{t=1}^{\infty} (1-\delta)\delta^t (\theta_a(n,\delta))^{2t} \theta_h^2$$

$$SL(\theta_h, n, \delta) = \frac{(1-\delta)\theta_a(n,\delta)^2\theta_h^2}{2(1-\delta\theta_a(n,\delta)^2)} \tag{56}$$

I require one more item of notation before I can state the original problem in a form I prefer. The revenues creditors achieve both inside and outside bankruptcy are $R(\theta_h, n, \delta)$ where θ_h is the highest type in bankruptcy, n the number of creditors who bargain with the firm and δ the discount factor.

$$R(\theta_h, n, \delta) = X(\theta_h, n, \delta) + \theta_h (1 - F(\theta_h))$$

The choice of system that maximizes ex-ante welfare can be found by solving a related problem. In the UK, for any n the revenues in bankruptcy are identical. Hence what is indeterminant is whether $I - X(\theta_h, 1)$ is supplied by creditor one or by the other $n-1$ creditors. Hence without loss of generality, I can assume this finance is supplied by the receiver. Thus the problem which defines ex-ante welfare is identical in the UK system to the US system with one creditor. Thus fix n as either 1 or $N > 1$. Consider the following problem with the US law.

$$\theta_h, n \in \{1, N\} = \arg\min SL(\theta_h, n, \delta) \tag{57}$$

$$\text{such that } R(\theta_h, n, \delta) = I \tag{58}$$

If N solves the above problem, the US system maximizes welfare. If not welfare is equivalent in the two systems.

Now given this alternative form of the problem, I show that any $N > 1$ such that the above problem is well defined solves the problem for δ sufficiently high.

To prove the proposition, fix $N > 1$ and I and pick ϵ arbitrarily small.

Let $\theta_h(N,\delta)$ and $\theta_h(1,\delta)$ be the values of θ_h associated with the N and one creditor systems at I. Note both terms increase in δ. Also choose δ such

$$\theta_h(N,\delta) - \theta_h(1,\delta) < \epsilon$$

Such a choice is possible since for high discount factors, the one creditor system produces arbitrarily small revenues in bankruptcy. Also note that for δ sufficiently high, $\theta_h(1,\delta)$ is bounded away from zero. Next I transform the SL function so that it does not converge to zero as δ converges to one. Let $SLB(\theta_h, n, \delta) = \sqrt{\frac{SL(\theta_h,n,\delta)}{1-\delta}}$

Note immediately that SLB is continuous over the relevant range of every variable.

Next I use identity (21) to bound $SLB(\theta_h, 1, \delta) - SLB(\theta_h, N, \delta)$ from below by using the implied differences in the belief updating function. Note the β which solves equation (21) is continuous in δ. Let $\theta_a = \beta$ and write it as a function of N and δ. Take the limit of $\theta_a(n,\delta)$ as δ goes to one. For $n = 1$ this limit is one. For $n = N$, this limit is strictly bounded away from 1. Consider:

$$M(\delta^*) = \inf_{\delta \geq \delta^*} \frac{\theta_a(1,\delta)}{\sqrt{1-\delta\theta_a(1,\delta)^2}} - \frac{\theta_a(N,\delta)}{\sqrt{1-\delta\theta_a(N,\delta)^2}}.$$

M is continuous and converges to ∞. For δ^* sufficiently high this number must be bounded away from zero.

Also let $c \geq sup \frac{\theta_a(N,\delta)}{\sqrt{1-\delta\theta_a(N,\delta)^2}}$ which is bounded from above. Also choose c to be finite.

Consider

$$SLB\left(\theta_h(1,\delta), 1, \delta\right) - SLB\left(\theta_h(1,\delta), N, \delta\right)$$

which is

$$\geq M(\delta)(\theta_h(1,\delta)) - c * (\theta_h(N,\delta) - \theta_h(1,\delta))$$

For ϵ sufficiently small (δ sufficiently close to one), the first term is bounded away from zero, whereas the second term converges to zero. Hence,

$$SLB(\theta_h, 1, \delta) - SLB(\theta_h, N, \delta) > 0$$

which proves the proposition.

□

10 Appendix C: Brief Detail on Data

The only numbers that are original to my research are the numbers on delays in bankruptcy. The numbers for US delays are from *Bankruptcy DataSource* which covers firms of assets of more than 50 million dollars. The data comes from a sample of approximately 200 firms. Firms that were not successfully reorganized were removed from the sample. The two events from the which the duration is taken are the firm's entrance in to Chapter 11 and the confirmation of the plan. The later date is very close to the formal conclusion of bankruptcy, but is available more frequently. All firms in the sample exited bankruptcy between January 1995 December 2001. The firms in the sample are attached as Table 1. This number closely tracks what other authors have found. See in particular, White (1996b) and Altman and Eberhart (1999).

The British data is taken from the ICC Directory of UK Companies. This source consists of firms that exited Receivership between August 1997 and November 2003. The relevant events for the duration number are the ”Appointment of the Receivership” and the ”Notice of Ceasing to Act as A Receiver or a Manager.” I took a subsample[24] of approximately 400 firms from the data. Firms that failed to successfully reorganize[25] were removed from the sample and the time between the two events for the remaining firms is attached as Table 2.

[24]The randomization device chosen was to select firms that used the abbreviation Ltd. in their title instead of the full word Limited.

[25]I do not consider firms sold as a going concern to be successfully reorganized. I adopt this convention to be consistent with the theory in this paper.

Figure 1: Timelines

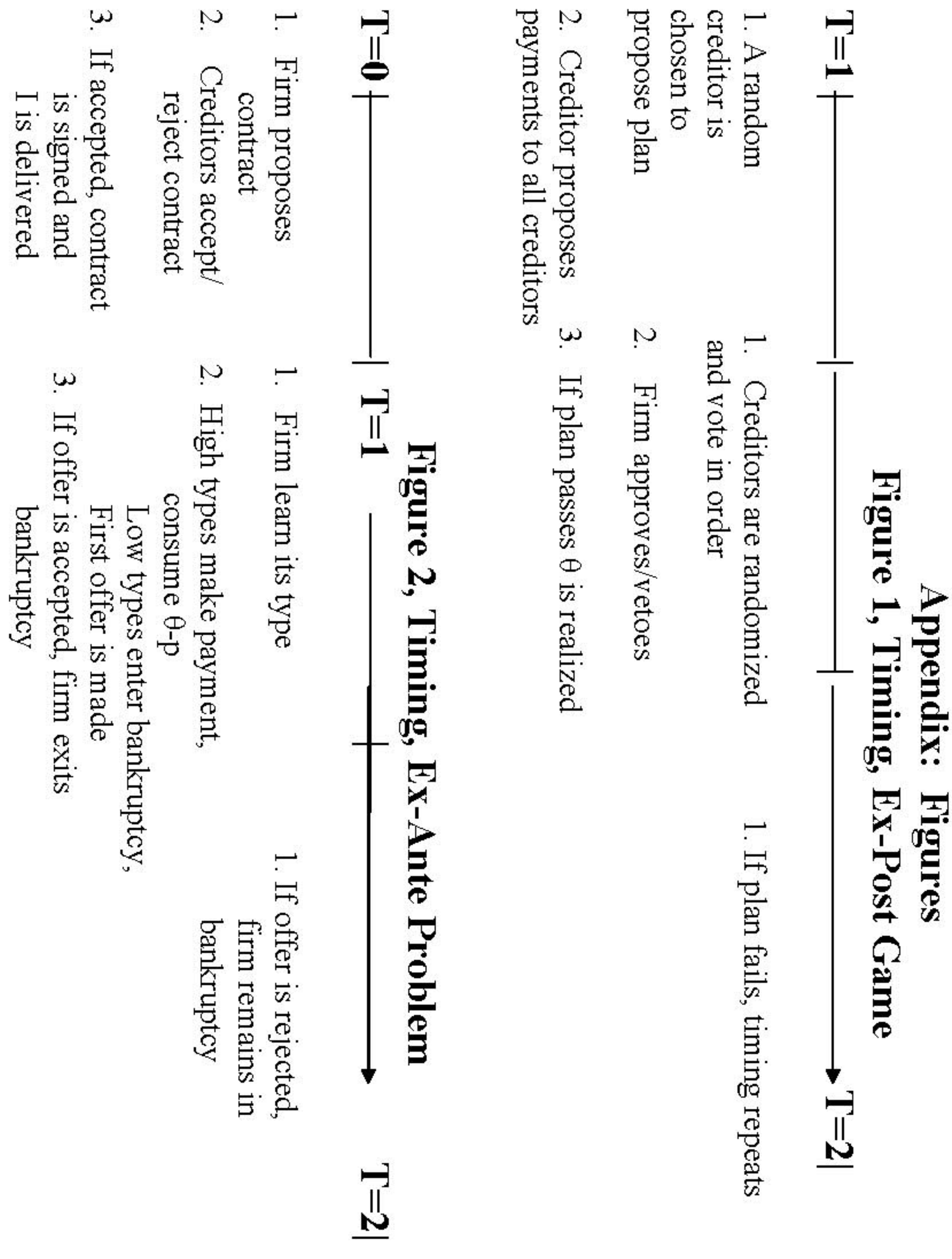

Figure 2: Example 1: No Gap Case

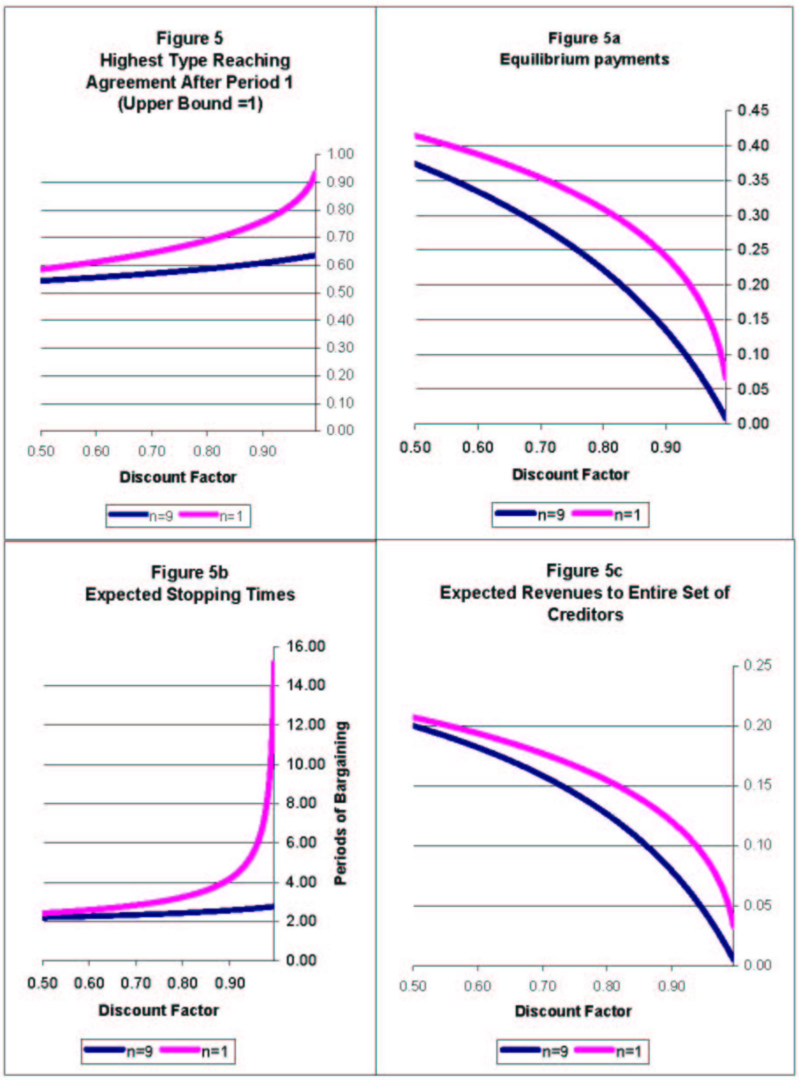

Figure 3: Example 2: Gap Case

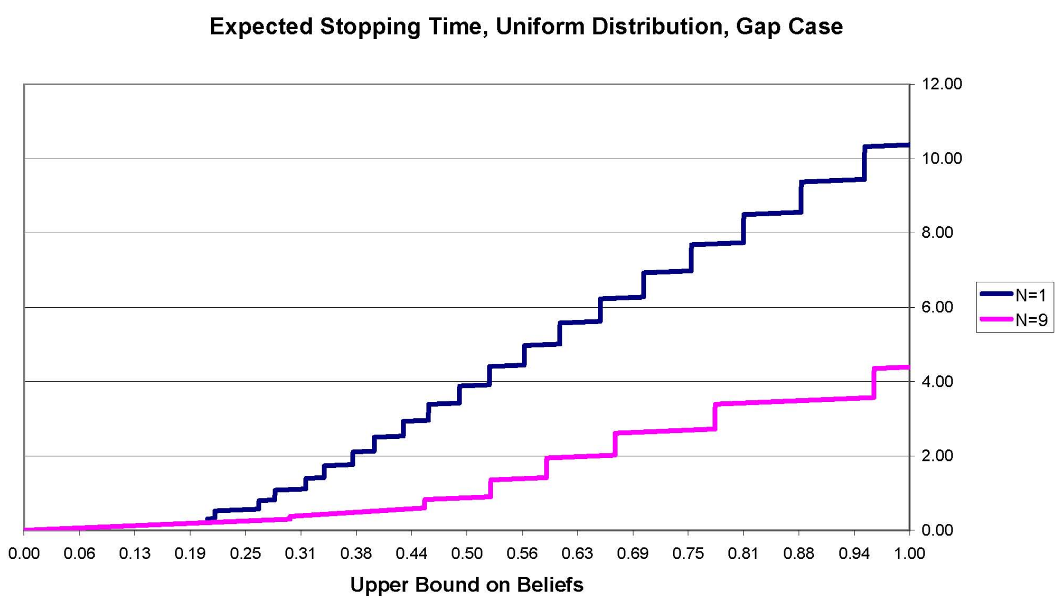

Figure 4: US Data on Durations

Table 1

Length of US Procedures

US Firms	Months In Negotiation		
Adience	4	Marvel Entertainment	18
Anchor Glass	4	Grossman Inc	20
Penn Traffic	4	Leaseway	20
Republic Health	4	Sun Healthcare	23
First Wave Marine	5	Amalgamated	24
LA Gear	5	Americaold	26
Marcade Group	6	Farley	26
US Leather	6	Victory Markets	26
Americtruck	7	First Executive	28
Creditrust	7	Hechinger	28
First Merchant	7	Lermont & Hauspice	28
Crowley, Maher & Company	8	Standard Brad Painting	28
Kitty Hawk	8	Circle K	33
Leasing Solutions	8	America West	36
Wireless One	9	Olympia & Yort	39
Global Ocean	10	City Bank of Texas	47
50 % off Store	12	F.C. I.	56
Brothers Gourment Coffee	14	Edison Brothers	65
Grand Union Co	15	Best Products	78
Lomas	15	First Republic Bank	90
Stage Stores	15	Mean	21.5
Alexanders	16	St. Deviation	19
Harvard	16		
American Carriers	17		
Big V	17		
Brendles	17		
Northwestern Steel & Wire	17		
Marvel Entertainment	18		

*Non Random Sample from Bankruptcy Datasource
Marks the difference between filing and confirmation date
for firms that exited bankruptcy no sooner than 1990

Figure 5: UK Data on Durations

Table 2

Length of UK Procedures

UK Firm	Months Spent in Negotiations						
MVT	3	Blackspar	31	ML Group	47	FIL Group	86
Hargraves	4	Landis	31	Chipwood	48	Chequers	86
Pebberville	4	NECA Holding	31	Campbell & A.	50	Connaught Com.	88
Franco	6	Palmgrade	31	Hey & Croft	50	Newspace Grp	89
TT	6	Stewart Mech	31	Int. Food	51	BIMEC Ind.	95
Daffryn	11	Image	32	Essex	53	Land and Urban	99
WTC	12	Finecare	33	Energy	54	Broadwell Land	100
DSB	14	MEI Ind	33	Abbycraft	55	Result	104
Floorspec	15	Harlequin	34	Toriamai	56	Navigations	108
Heller	15	London& Co	35	Rothwell	57	McLaughlin & Harvey	110
IWEC	19	Whitton	35	Upperfield	58	Advanced Business	112
Beaumont	21	Britt Timber	36	Lornging Forn	59	Coloroll Group	115
AO Real	22	Beaconpoint	37	London Wall	60	Orlinworth	117
DDG	22	Cardiff	37	Colorvision	63	Hughes Food Group	127
Leading	23	Metraking	37	LI Realisation	65	Pennant Properties	142
Lledo	23	Car Group	38	Eurocrush	66	Rockwood Holdings	149
Tyron	25	Prince	39	Sale Tilney	71	Mean	52
Beach Home	26	Total Office Grp	40	Kendell	71	Standard Deviation	32
Chadwick Web	26	Tring	40	HAL	71		
NFF	26	Wildwood G C	40	Binary R.	71		
Four Seasons	27	Priorywood	45	BCI Europe	71		
Reading L.	28	Blackwall	46	Tysons	75		
Worthfine Fr.	28	Pavilion Leisure	46	DTP Real.	75		
Gogas	29	Miskin Group	46	Donelon Tyson	75		
Archway	30	Rider Fenn & Co	46	Equity General	79		
Versailes	30	Abelmart	47	Omega Group	83		

Source ICC Directory of UK Companies