

Economics 380: Homework 3

30 March, 2005

1. Consider the repeated Bertrand game studied in class, but with N symmetric firms. Show that we require $\delta \geq (N - 1)/N$ to sustain cooperation.

2. A firm has two plants. The cost of producing q_i in plant $i \in \{1, 2\}$ is given by increasing, convex $c_i(q_i)$. Demand is given by $p(q)$, where $q = q_1 + q_2$. Assuming it is optimal for both plants to produce, derive the first order conditions for q_1 and q_2 in the firm's maximisation problem.

3. Two Bertrand competitors are trying to cooperate. Mass 1 of customers have known valuation v . The firms have marginal costs c_1 and c_2 , where $v > c_2 > c_1$.
 - (a) What are prices and profits under Bertrand competition?
 - (b) If we thought of these firms as two plants belonging to one firm, what would be the monopoly price p^M and joint profits π^M ?
 - (c) Suppose the firms agree to cooperate, with firm 1 pricing at p^M and firm 2 setting a higher price. Firm 1 pays 2 transfer t each period, while they revert to Nash (i.e. Bertrand) if either deviates. How much can firm 2 get from deviating? What condition is required to stop firm 2 deviating, and setting a lower price? What condition is required to stop firm 1 deviating, and refusing to pay t ? Show these two inequalities imply that cooperation is only feasible if and only if $\delta \geq 1/2$?
 - (d) Suppose transfers are not feasible. Firms 1 and 2 agree to set price p^M but only produce q_1^* and q_2^* each, where $q_1^* + q_2^* = 1$, inducing profits $q_1^*(p^M - c_1)$ and $q_2^*(p^M - c_2)$. Again, this is enforced by Nash reversion. What conditions are required to stop the firms deviating? Show that when $\delta = 1/2$ at least one of the firms will deviate. Why is cooperation harder to sustain than in part (c)?

4. You are bidding in a first-price private value auction with value 100. Looking at 40 similar auctions the winning bids are: 64 76 63 75 96 65 98 87 83 86 99 87 67 98 102 83 98 90 104 70 88 92 60 84 103 77 82 82 97 64 87 96 92 94 61 98 62 99 92 79. You wish to choose a bid b to maximise your profit $(v - b)Pr(\text{win})$. What should you bid? [I advise you to use Excel or a similar program. I use Quattro, where the command @PERCENTRANK will give you the probability a bid b will win.]

5. Double marginalisation cannot really be an important issue in the economy, since it's easy to design contracts that sidestep the problem. Discuss.