## Economics 380: Suggested Solutions 3

30 March, 2005

1. A firm does not deviate

$$
\pi^{M} \leq \frac{1}{1-\delta} \frac{\pi^{M}}{N}
$$

Now rearrange.
2. The first order conditions are given by $M R\left(q_{1}+q_{2}\right)=M C_{1}\left(q_{1}\right)=M C_{2}\left(q_{2}\right)$.
3. (a) Under Bertrand, $p=c_{2}$. Profits are $\pi_{1}=c_{2}-c_{1}$ and $\pi_{2}=0$.
(b) Under monopoly pricing, $p^{M}=v$ and $\pi_{M}=v-c_{1}$.
(c) To stop firm 2 deviating we require

$$
\left(v-c_{2}\right) \leq \frac{1}{1-\delta} t
$$

To stop firm 1 deviating we require

$$
\left(v-c_{1}\right)+\frac{\delta}{1-\delta}\left(c_{2}-c_{1}\right) \leq \frac{1}{1-\delta}\left(v-c_{1}-t\right)
$$

Putting these together,

$$
(1-\delta)\left(v-c_{2}\right) \leq t \leq \delta\left(v-c_{2}\right)
$$

Hence we require $\delta \geq 1 / 2$.
(d) To stop firm 1 deviating we require

$$
\begin{equation*}
\left(v-c_{1}\right)+\frac{\delta}{1-\delta}\left(c_{2}-c_{1}\right) \leq \frac{1}{1-\delta} q_{1}^{*}\left(v-c_{1}\right) \tag{1}
\end{equation*}
$$

To stop firm 2 deviating we require

$$
\left(v-c_{2}\right) \leq \frac{1}{1-\delta} q_{2}^{*}\left(v-c_{2}\right)
$$

If firm 2 is indifferent between deviating and not, $q_{2}^{*}=1-\delta$, and $q_{1}^{*}=\delta$. Substituting, into (1) and rearranging, cooperation requires

$$
\delta \geq \frac{v-c_{1}}{2 v-c_{1}-c_{2}}>\frac{1}{2}
$$

Intuitively, efficiency is higher in part (c), and so there is more to gain from cooperating.
4. If you bid 84 , you'll win with $43 \%$ chance, yielding profit 6.976 .
5. A good answer would hit the following points.
(a) Explain double marginalisation.
(b) Explain what contractual devices can sidestep double marginalisation. For example, twopart tariffs.
(c) Say what's wrong with these contractual solutions. With two-part tariffs, we require a lot of information and need to stop arbitrage. Two-part tariffs are also unwieldy: can you imagine going to a two-part tariff style supermarket?
(d) Practically there is evidence that it's a problem. There are many real world examples, such as the hudson bay case.

