Economics 380: Homework 3

1 December, 2005

1. Consider the repeated Bertrand game studied in class, but with N symmetric firms. Show that we require $\delta \geq (N-1)/N$ to sustain cooperation.

2. A firm has two plants. The cost of producing q_i in plant $i \in \{1,2\}$ is given by increasing, convex $c_i(q_i)$. Demand is given by $p(q)$, where $q = q_1 + q_2$. Assuming it is optimal for both plants to produce, derive the first order conditions for q_1 and q_2 in the firm's maximisation problem.

3. [Harder] Two Bertrand competitors are trying to cooperate. Mass 1 of customers have known valuation v. The firms have marginal costs c_1 and c_2, where $v > c_2 > c_1$.
(a) What are prices and profits under Bertrand competition?
(b) If we thought of these firms as two plants belonging to one firm, what would be the monopoly price p^M and joint profits π^M?
(c) Suppose the firms agree to cooperate, with firm 1 pricing at p^M and firm 2 setting a higher price. Firm 1 pays 2 transfer t each period, while they revert to Nash (i.e. Bertrand) if either deviates. How much can firm 2 get from deviating? What condition is required to stop firm 2 deviating, and setting a lower price? What condition is required to stop firm 1 deviating, and refusing to pay t? Show these two inequalities imply that cooperation is only feasible if and only if $\delta \geq 1/2$?
(d) Suppose transfers are not feasible. Firms 1 and 2 agree to set price p^M but only produce q_1^* and q_2^* each, where $q_1^* + q_2^* = 1$, inducing profits $q_1^*(p^M - c_1)$ and $q_2^*(p^M - c_2)$. Again, this is enforced by Nash reversion. What conditions are required to stop the firms deviating? Show that when $\delta = 1/2$ at least one of the firms will deviate. Why is cooperation harder to sustain than in part (c)?

4. Double marginalisation cannot really be an important issue in the economy, since it's easy to design contracts that sidestep the problem. Discuss.

5. Two Bertrand competitors face demand curve $p = a - q$ and have costs 0. The firms agree to price at $p^* \in (0, a/2)$, split the profits and to sustain it using Nash–reversion (i.e. returning to price 0) if either firm defects. What condition on the common discount factor, δ, is required to sustain cooperation?

6. When you get admitted into Harvard this sends a positive signal to the job market. If

education is unproductive, what use would there be to actually completing the degree?

7. Alcohol prices should be higher in the UK (where the government taxes alcohol and supermarket sells it) than Toronto (where the government sells alcohol via the LCBO). In the UK, the government and the supermarkets take a slice of the pie; in Toronto, there's only the government to worry about. Discuss.