# Competitive Strategy: Week 6 Dynamic Pricing 

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## Capacity Choice

- Consider building a stadium for the olympics.
- Demand is given by $p(q)=a-q$.
- Firm chooses capacity $K$.
- Capacity costs $c$ per unit.
- After capacity built the marginal cost is zero
- Profit maximisation problem

$$
\max _{q, K} p(q) q-c K \quad \text { s.t. } \quad q \leq K
$$

- Set capacity equal to quantity, $K=q$. Hence $\max _{q}(p(q)-c) q$.
- Standard monopoly problem: set $M R(q)=c$,
- With linear demand $q=(a-c) / 2$ and $p=(a+c) / 2$.


## Peak-Load Pricing

- Suppose there are two periods: High and Low demand
- Demand given by $a_{i}-q$ where $a_{i} \in\left\{a_{L}, a_{H}\right\}$
- Profit maximisation problem:

$$
\max _{q_{1}, q_{2}, K}\left(a_{L}-q_{L}\right) q_{L}+\left(a_{H}-q_{H}\right) q_{H}-c K \quad \text { s.t. } \quad q_{L}, q_{H} \leq K
$$

- Case 1: Suppose capacity binds in high period only.
- Solution: $q_{L}=a_{L} / 2$ and $K=q_{H}=\left(a_{H}-c\right) / 2$.
- Prices: $p_{L}=a_{L} / 2$ and $p_{H}=\left(a_{H}+c\right) / 2$
- Requires $q_{L} \leq K$, i.e. $a_{H}-a_{L} \geq c$.
- Key idea: Charge capacity when capacity constraint binds.


## Peak-Load Pricing cont.

- Case 2: Suppose capacity binds in both periods.
- Solution: $K=q_{L}=q_{H}=\left(a_{H}+a_{L}-c\right) / 4$.
- Prices: $p_{L}=\left(3 a_{L}-a_{H}-c\right) / 4$ and $p_{H}=\left(3 a_{H}-a_{L}-c\right) / 4$.
- Requires $q_{L} \leq a_{L} / 2$, i.e. $a_{H}-a_{L} \leq c$
- Examples
- Discounted electricity prices at midnight
- Happy hours at bars
- $\$ 1$ baseball tickets on Wednesday
- Cheap seaside hotel rooms in March.
- Matinee pricing at cinemas
- Cheap cell phone calls in the afternoon


## Yield Management

- Assumptions:
- Customers are arriving over time
- Have capacity constraint for total number who are served.
- Examples: airlines, hotels, the superbowl, package holidays.
- Tradeoff:
- Sell cheap seat today
- Retain option value of seat.


## Yield Management cont.

- Two types of customers
- Some willing to pay full fare $p_{F}$
- Some only willing to pay discounted prices $p_{D}$
- There are $q$ seats left on the plane.
- Baseline: charge full price $p_{F}$ to all customers.
- Let $s$ be probability plane sells out.
- Let $n$ be probability next customer is low value.
- If charge next customer $p_{D}$ what happens?
- Gain revenue $p_{D}$.
- Lose revenue $(n s+(1-n)) p_{F}$.
- Each period $s$ rises (falls) if do (do not) make sale.


## Durable Goods Monopoly and Declining Prices

- Consider the problem of Xerox
- There is a demand for Xerox copiers
- Initially sell to high valuation customers
- Next year sell to customers with lower valuations
- Problem: Customers anticipate prices will fall
- Customer delay purchases until price falls
- Monopolist competes with future selves
- The Coase Conjecture
- When the good is infinitely durable the monopolist will have no market power
- Price instantly falls to marginal cost


## Durable Goods: Two-Period Model

- Agents have values $\theta \sim U[0,1]$. Zero cost. Discount rate $\delta$.
- Suppose sell to $\left[\theta_{1}, 1\right]$ in period 1 .
- Profit in period 2 is $\pi_{2}=\left(\theta_{1}-p_{2}\right) p_{2}$
- Optimal price $p_{2}^{*}=\theta_{1} / 2$
- Type $\theta_{1}$ is indifferent between buying in periods 1 and 2 . Hence

$$
\left(\theta_{1}-p_{1}\right)=\delta\left(\theta_{1}-p_{2}\right)
$$

Rearranging, $p_{1}=(1-\delta / 2) \theta_{1}$

- Total profit from both periods,

$$
\pi=\left(1-\theta_{1}\right)\left(1-\frac{\delta}{2}\right) \theta_{1}+\delta\left(\frac{\theta_{1}}{2}\right)^{2}
$$

## Durable Goods: Two-Period Model

- The firm chooses $\theta_{1}$ to maximise $\pi$. The FOC yields

$$
\theta_{1}^{*}=\left(\frac{1-\delta / 2}{2-\delta / 2}\right)
$$

Thus $\theta_{1}^{*}$ decreases in $\delta$.

- Substituting and rearranging, total profit is

$$
\pi=\frac{(2-\delta)^{2}}{4(4-\delta)}
$$

Profit decreases in $\delta$. If $\delta=1$, then $\pi=1 / 12$. If $\delta=0$ then $\pi=1 / 4$ as in static monopoly.

- Key: firm can't commit not to reduce price.
- General result: if firm can commit to any price path, the best they can do is $p_{1}=p_{2}=1 / 2$ (static monopoly).


## Durable Goods Monopoly: Solutions

- What does this model apply to?
- Classic durable goods (e.g. cars)
- Durable goods with resale (e.g. prams)
- Durable services (e.g. movies)
- Solution 1: Reputation (e.g. record companies).
- Solution 2: Renting (e.g. Xerox)
- Each period sell static monopoly quantity.
- Solution 3: Best-price provision (e.g. Chrysler)
- If firm lowers price then customers get rebate.
- Firm never any incentive to lower price below monopoly price since lose money in rebates.


## Durable Goods and Holdup

- Durable goods firms face two types of commitment problems:
- They wish to lower prices to keep making sales (see above)
- They can holdup their customers.
- Example 1: Servicing and supplying accessories.
- After buy car owner still needs parts if it breaks.
- Customers usually don't sign contract over part prices.
- Firm has holdup customers and increase parts prices.
- Solution: licence manufacturing of parts, or use standard parts, to keep prices competitive.
- Example 2: New models
- When Apple launches iPod nano, value of iPod falls.
- Firm has excessive incentive to introduce new products.


## Experimentation

- Firm wishes to sell a unique good (e.g. one of a kind dress).
- At time $t$ charge $p(t)$
- Each period a buyer chooses to buy or not.
- Each buyer has the same value $v$
- Firm does not know the valuation.
- Optimal policy: start price high and lower slowly.
- Solve through backwards induction.
- What if have good each period to sell?
- Price may go up or down.
- But should move prices around to experiment.


## Inventories

- Why need inventories?
- Input inventories: ordering has fixed costs.
- Fix prices. A random number of sales, $q_{t}$, occur in period $t$.
- Firm should adopt (S,s) rule. When inventories, $I_{t}$, fall below $s$ then put in order to bring $I_{t}$ back to $S$.
- If order only possible once a month then use yield management within the month.
- Output inventories: smoothing production.
- Suppose firm likes to keep production constant (e.g. convex costs)
- Transitory demand increase: build up inventories and raise prices.
- Permanent demand increase: increase production.


## Assignment

- Read "Hooked on Discounts", The Economist, 9th July 2005.
- Why did GM slash prices?
- What was the immediate impact on sales?
- What do you think will happen to sales next quarter? Will GM's market share be less than $41 \%$ ? Will it be less than $33 \%$ ?
- Is GM's strategy a good idea?

