Dynamic Pricing

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Capacity Choice

- Consider building a stadium for the olympics.
- Demand is given by \( p(q) = a - q \).
- Firm chooses capacity \( K \).
  - Capacity costs \( c \) per unit.
  - After capacity built the marginal cost is zero
- Profit maximisation problem
  \[
  \max_{q,K} p(q)q - cK \quad \text{s.t.} \quad q \leq K
  \]
- Set capacity equal to quantity, \( K = q \). Hence \( \max_q (p(q) - c)q \).
  - Standard monopoly problem: set \( MR(q) = c \),
  - With linear demand \( q = (a - c)/2 \) and \( p = (a + c)/2 \).
Peak–Load Pricing

- Suppose there are two periods: High and Low demand
  - Demand given by $a_i - q$ where $a_i \in \{a_L, a_H\}$

- Profit maximisation problem:

  $$
  \max_{q_1, q_2, K} (a_L - q_L)q_L + (a_H - q_H)q_H - cK \quad \text{s.t.} \quad q_L, q_H \leq K
  $$

- Case 1: Suppose capacity binds in high period only.
  - Solution: $q_L = a_L/2$ and $K = q_H = (a_H - c)/2$.
  - Prices: $p_L = a_L/2$ and $p_H = (a_H + c)/2$
  - Requires $q_L \leq K$, i.e. $a_H - a_L \geq c$.

- Key idea: Charge capacity when capacity constraint binds.

Peak–Load Pricing cont.

- Case 2: Suppose capacity binds in both periods.
  - Solution: $K = q_L = q_H = (a_H + a_L - c)/4$.
  - Prices: $p_L = (3a_L - a_H - c)/4$ and $p_H = (3a_H - a_L - c)/4$.
  - Requires $q_L \leq a_L/2$, i.e. $a_H - a_L \leq c$

- Examples
  - Discounted electricity prices at midnight
  - Happy hours at bars
  - $1$ baseball tickets on Wednesday
  - Cheap seaside hotel rooms in March.
  - Matinee pricing at cinemas
  - Cheap cell phone calls in the afternoon
Yield Management

- Assumptions:
  - Customers are arriving over time
  - Have capacity constraint for total number who are served.
- Examples: airlines, hotels, the superbowl, package holidays.
- Tradeoff:
  - Sell cheap seat today
  - Retain option value of seat.

Yield Management cont.

- Two types of customers
  - Some willing to pay full fare $p_F$
  - Some only willing to pay discounted prices $p_D$
- There are $q$ seats left on the plane.
- Baseline: charge full price $p_F$ to all customers.
  - Let $s$ be probability plane sells out.
  - Let $n$ be probability next customer is low value.
- If charge next customer $p_D$ what happens?
  - Gain revenue $p_D$.
  - Lose revenue $(ns + (1 - n))p_F$.
- Each period $s$ rises (falls) if do (do not) make sale.
Durable Goods Monopoly and Declining Prices

- Consider the problem of Xerox
- There is a demand for Xerox copiers
  - Initially sell to high valuation customers
  - Next year sell to customers with lower valuations
- Problem: Customers anticipate prices will fall
  - Customer delay purchases until price falls
  - Monopolist competes with future selves
- The Coase Conjecture
  - When the good is infinitely durable the monopolist will have no market power
  - Price instantly falls to marginal cost

Durable Goods: Two-Period Model

- Agents have values \( \theta \sim U[0, 1] \). Zero cost. Discount rate \( \delta \).
- Suppose sell to \([\theta_1, 1]\) in period 1.
  - Profit in period 2 is \( \pi_2 = (\theta_1 - p_2)p_2 \)
  - Optimal price \( p_2^* = \theta_1/2 \)
- Type \( \theta_1 \) is indifferent between buying in periods 1 and 2. Hence
  \[
  (\theta_1 - p_1) = \delta(\theta_1 - p_2)
  \]
  Rearranging, \( p_1 = (1 - \delta/2)\theta_1 \)
- Total profit from both periods,
  \[
  \pi = (1 - \theta_1)(1 - \frac{\delta}{2})\theta_1 + \delta \left(\frac{\theta_1}{2}\right)^2
  \]
Durable Goods: Two-Period Model

- The firm chooses $\theta_1$ to maximise $\pi$. The FOC yields
  \[ \theta_1^* = \left( \frac{1 - \delta/2}{2 - \delta/2} \right) \]

  Thus $\theta_1^*$ decreases in $\delta$.

- Substituting and rearranging, total profit is
  \[ \pi = \frac{(2 - \delta)^2}{4(4 - \delta)} \]

  Profit decreases in $\delta$. If $\delta = 1$, then $\pi = 1/12$. If $\delta = 0$ then $\pi = 1/4$ as in static monopoly.

- Key: firm can’t commit not to reduce price.

- General result: if firm can commit to any price path, the best they can do is $p_1 = p_2 = 1/2$ (static monopoly).

Durable Goods Monopoly: Solutions

- What does this model apply to?
  - Classic durable goods (e.g. cars)
  - Durable goods with resale (e.g. prams)
  - Durable services (e.g. movies)

- Solution 1: Reputation (e.g. record companies).

- Solution 2: Renting (e.g. Xerox)
  - Each period sell static monopoly quantity.

- Solution 3: Best-price provision (e.g. Chrysler)
  - If firm lowers price then customers get rebate.
  - Firm never any incentive to lower price below monopoly price since lose money in rebates.
Durable Goods and Holdup

- Durable goods firms face two types of commitment problems:
  - They wish to lower prices to keep making sales (see above)
  - They can holdup their customers.

- Example 1: Servicing and supplying accessories.
  - After buy car owner still needs parts if it breaks.
  - Customers usually don’t sign contract over part prices.
  - Firm has holdup customers and increase parts prices.
  - Solution: licence manufacturing of parts, or use standard parts, to keep prices competitive.

- Example 2: New models
  - When Apple launches iPod nano, value of iPod falls.
  - Firm has excessive incentive to introduce new products.

Experimentation

- Firm wishes to sell a unique good (e.g. one of a kind dress).
  - At time $t$ charge $p(t)$

- Each period a buyer chooses to buy or not.
  - Each buyer has the same value $v$
  - Firm does not know the valuation.

- Optimal policy: start price high and lower slowly.
  - Solve through backwards induction.

- What if have good each period to sell?
  - Price may go up or down.
  - But should move prices around to experiment.
Inventories

• Why need inventories?

• Input inventories: ordering has fixed costs.
  – Fix prices. A random number of sales, $q_t$, occur in period $t$.
  – Firm should adopt $(S,s)$ rule. When inventories, $I_t$, fall below $s$ then put in order to bring $I_t$ back to $S$.
  – If order only possible once a month then use yield management within the month.

• Output inventories: smoothing production.
  – Suppose firm likes to keep production constant (e.g. convex costs)
  – Transitory demand increase: build up inventories and raise prices.
  – Permanent demand increase: increase production.

Assignment

• Read “Hooked on Discounts”, The Economist, 9th July 2005.
• Why did GM slash prices?
• What was the immediate impact on sales?
• What do you think will happen to sales next quarter? Will GM’s market share be less than 41%? Will it be less than 33%?
• Is GM’s strategy a good idea?