The Information Economy

Dynamic Prices

Peak Load Pricing

- Suppose a firm has zero marginal cost, with capacity K
 - Broadband capacity, cell phone towers, number of tickets
 - Capacity costs z per unit to build.
- There are two periods (or two equally likely states)
 - Period L demand is low, $p_L(q)$
 - Period H demand is high, $p_H(q)$
- Firm chooses q_L , q_H and K to maximize profits

$$\pi = q_L p_L (q_L) + q_H p_H (q_H) - zK$$
 subject to $q_L, q_H \le K$

Lagrangian: choose q_L , q_H and K to maximize

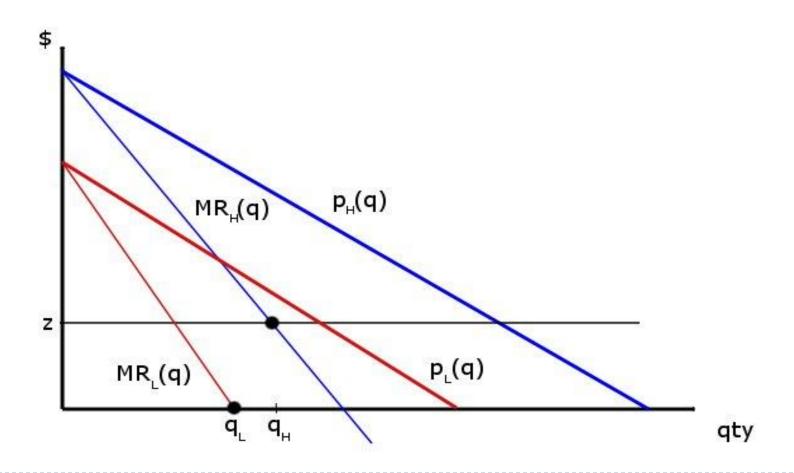
$$L = q_{L}P_{L}(q_{L}) + q_{H}P_{H}(q_{H}) - zK + \lambda_{L}[K-q_{L}] + \lambda_{H}[K-q_{H}]$$

Peak Load Pricing

- Solution
 - ▶ FOCs for q_L, q_H and K: $MR_L(q_L^*) = \lambda_L$, $MR_H(q_H^*) = \lambda_H$, $z = \lambda_L + \lambda_H$
 - Optimal capacity: K*=q_H*
- ▶ Idea: Charge capacity when constraint binds.
- Two cases:
 - I. Constraint slack in period L (big difference in demands)
 - 2. Constraint binds in period L (small difference in demands)
- Price in H higher for two reasons
 - (a) The demand is higher,
 - (b) Charging more of the capacity
- Examples: cheap evening calls and Christmas flights

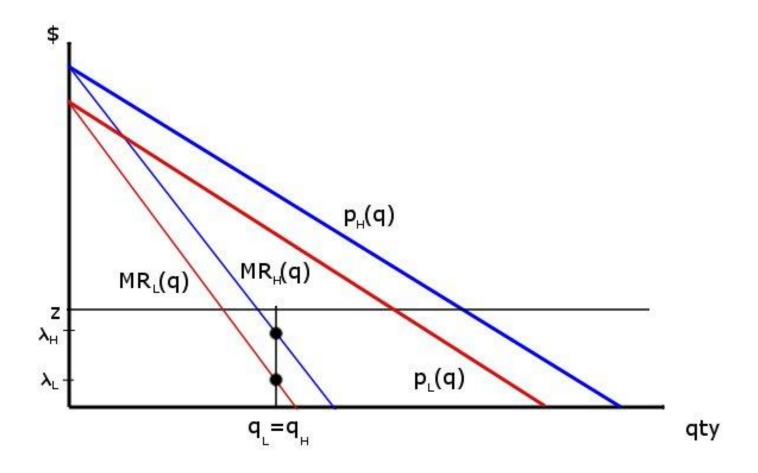
1. Constraint Slack in Period L (q_L*<q_H*)

▶ Optimal quantities: $MR_L(q_L^*)=0$, $MR_H(q_H^*)=z$



2. Constraint Binds in Period L $(q_L^* < q_H^*)$

▶ Optimal quantities: $q_L^* = q_H^*$, $MR_L(q_L^*) = \lambda_L$, $MR_H(q_H^*) = z - \lambda_L$



Revenue Management

- A firm has K tickets to sell
 - Airline seats, hotel rooms, advertising slots
- Customers arrive over time
 - Customers have value v unknown to firm
- How should firm set prices over time?
 - If lower price then:
 - (a) sell to marginal agents today
 - (b) make less revenue from inframarginal agents
 - (c) lose opportunity to sell tomorrow

Revenue Management: Example

- Example: one item to sell (K=I)
 - ▶ There are N customers with $v\sim U[0,1]$
- Last customer
 - ▶ Choose p_N to maximize Π_N = (prob sell) x price = $(I-p_N)p_N$.
 - Solution: p_N *=0.5, yielding Π_N *=0.25.
- Dynamic programming: suppose nth customer arrives
 - Choose p_n to maximize $\Pi_n = (I p_n)p_n + p_n\Pi_{n+1}$.
 - ▶ Solution: $p_n^*=0.5[I+\Pi_{n+1}]$, yielding $\Pi_n^*=0.25[I+\Pi_{n+1}]^2$
- Working backwards with 5 customers:

	5 th	4 th	3rd	2 nd	st
Price, p _n *	0.5	0.63	0.70	0.74	0.78
Profit, Π_n^*	0.25	0.39	0.48	0.55	0.60

Durable Goods and Price Commitment

- Apple is thinking how to price the iPhone
 - In the first year it sells to high value customers
 - Then lowers price to sell to low value customers
- Problem: Customers anticipate price will fall
 - Customer delay purchases until price falls
 - Monopolist competes with future selves
- Model applies to durable goods
 - Software, Xbox, Art
- Model applies to durable services
 - Movies, information goods.

Durable Goods: Example

- N customers have v=30, N customers have v=10.
- \blacktriangleright Suppose there are two periods, with discount rate δ
 - If commit to one price, charge p=30, profit Π =30N.
- Suppose sell to high agents in period I
 - ▶ Charge p_2 =10 and sell to low agents in period 2.
- High agents anticipate price will fall and may wait
 - ▶ Charge at most p_1 =30-20 δ , for high agents to buy in period I
 - ▶ Total profits $\Pi = (30-20\delta)N + \delta(10)N = (30-10\delta)N$
- ▶ Firm suffers because it cannot commit
 - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.

Durable Goods: Solutions

- Solution I: Destroy the mould (e.g. artist)
 - Without mould cannot create quantity in second period
- Solution 2: Reputation (e.g. record companies)
 - Develop reputation for not dropping prices
- Solution 3: Renting (e.g. Xerox)
 - Good no longer "durable", so sell static monopoly quantity each period
- Solution 4: Best-price provision (e.g. iPhone)
 - If firm lowers price then customers get rebate
 - Firm never any incentive to lower price below monopoly price since lose money in rebates

Behavior Based Pricing and Commitment

- Suppose a firm sells to customers multiple times
- Purchasing behavior in early period tells firm about values
 - Firm tempted to condition price on past behavior
- Problem: Customers anticipate "ratchet effect"
 - Customers delay purchases to get lower prices later
 - Monopolist competes with her future selves
- Applications
 - Online sites with cookies, magazine subscriptions, cable TV

Behavior Based Pricing: Example

- \triangleright N customers have v=30, N customers have v=10.
- \blacktriangleright Suppose there are two periods, with discount rate δ
 - If cannot see past behavior, charge p=30, profit Π_0 =30(1+ δ)N.
- Suppose sell to high agents in period I
 - ▶ Charge $p_2=10$ if did not buy in period I
 - ▶ Charge p_2 =30 if bought in period I (ratchet effect)
- ▶ If customers myopic charge p_1 =30
 - ▶ Total profits Π_{M} = 30N+ δ (30+10)N = (30+40 δ)N > Π_{0}
- If customers forward looking, anticipate price fall if don't buy
 - ▶ Charge at most p_1 =30-20 δ , for high agents to buy in period I
 - ▶ Total profits $\Pi_F = (30-20\delta)N + \delta(30+10)N = (30+20\delta)N < \Pi_0$
- ▶ Firm suffers because it cannot commit
 - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.

Why are there Introductory Discounts?

Behavioral-based pricing view

- Firms can't resist giving discount to people who don't purchase
- These discounts hurt the firm if
 - (a) Consumers are forward looking
 - (b) Consumers get annoyed

Introductory discounts may be good idea

- Network effects (see network slides)
- Overcome switching costs (see lockin slides)
- Encourage customer experimentation (next slide)

Customer Experimentation

- Product is "experience good"
 - Don't know taste until tried it
- ▶ Customers have value v=30 or v=10 with equal prob.
 - Optimal pricing: niche market strategy
 - ▶ Period I, charge price p_1 =20, and everyone buys
 - ▶ Period t≥2, charge price p_t =30, and high value agents buy
- Customers have value v=30 or v=20 with equal prob.
 - Optimal pricing: mass market strategy
 - Period I, charge price $p_1=25$, and everyone buys
 - ▶ Period t≥2, charge price p_t =20, and everyone buy

Learning the Demand Curve

- How does a firm price when it does not know demand?
 - Firm wishes to sell a unique good.
 - Customers enter each period (not forward looking)
 - Each buyer has the same value, v, unknown to firm
- Optimal policy: start price high and lower slowly.
 - Solve through backwards induction.
 - Rate of decrease depends on firm's patience.
- What if have good each period to sell?
 - Price may go up or down.
 - But should move prices around to experiment.
- Experimentation very easy online

Inventories

- Need inventories because
 - Fixed costs to order
 - Take time for delivery to arrive
- Firm should adopt (S,s) rule.
 - If inventories fall below s then bring back to S.
- Demand shocks
 - If demand has transitory increase, bring inventory back to S
 - If demand has permanent increase, also increase S.
- If it take time to place new order, then use revenue management to find optimal prices.