# The Economics of E-commerce and Technology

Dynamic Pricing

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## Peak Load Pricing

- Suppose a firm has zero marginal cost, with capacity K
  - Broadband capacity, cell phone towers, hotel rooms
  - Capacity costs z per unit to build.
- There are two periods (or two equally likely states)
  - Period L demand is low,  $p_L(q)$
  - Period H demand is high,  $p_H(q)$
- Firm chooses  $q_L$ ,  $q_H$  and K to maximize profits

$$\pi = q_L p_L (q_L) + q_H p_H (q_H) - zK$$
 subject to  $q_L, q_H \le K$ 

Lagrangian: choose q<sub>1</sub>, q<sub>H</sub> and K to maximize

$$L = q_{L}P_{L}(q_{L}) + q_{H}P_{H}(q_{H}) - zK + \lambda_{L}[K-q_{L}] + \lambda_{H}[K-q_{H}]$$

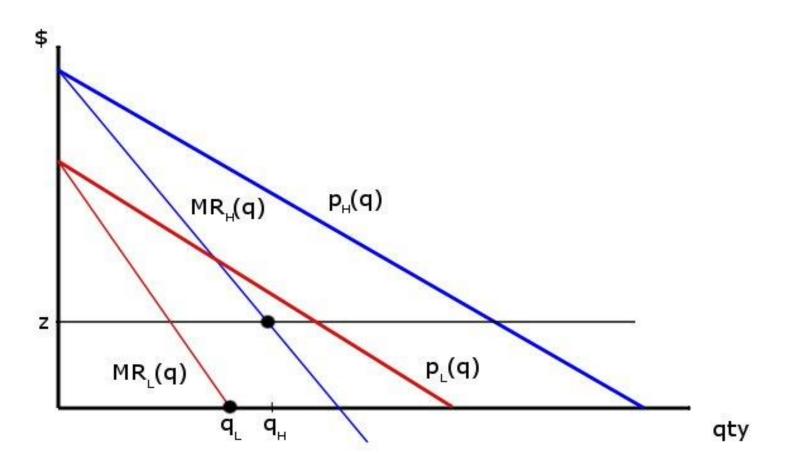
## Peak Load Pricing

#### Solution

- ► FOCs for  $q_L, q_H$  and K:  $MR_L(q_L^*) = \lambda_L$ ,  $MR_H(q_H^*) = \lambda_H$ ,  $z = \lambda_L + \lambda_H$
- Optimal capacity:  $K^*=q_H^*$
- ▶ Idea: Charge capacity when constraint binds.
- Two cases:
  - I. Constraint slack in period L (big difference in demands)
  - 2. Constraint binds in period L (small difference in demands)
- Price in H higher for two reasons
  - (a) The demand is higher,
  - (b) Charging more of the capacity
- Examples: cheap evening calls and Christmas flights

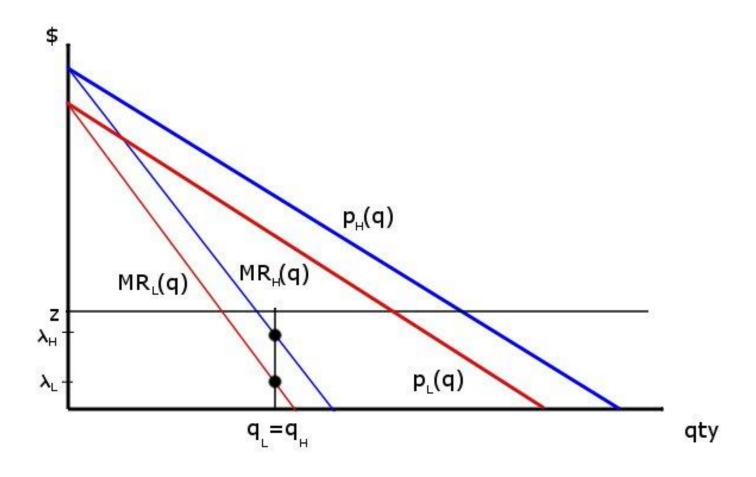
## 1. Constraint Slack in Period L (q<sub>L</sub>\*<q<sub>H</sub>\*)

▶ Optimal quantities:  $MR_L(q_L^*)=0$ ,  $MR_H(q_H^*)=z$ 



## 2. Constraint Binds in Period L $(q_L^* < q_H^*)$

• Optimal quantities:  $q_L^* = q_H^*$ ,  $MR_L(q_L^*) = \lambda_L$ ,  $MR_H(q_H^*) = z - \lambda_L$ 



## Revenue Management

- A firm has K tickets to sell
  - Airline seats, hotel rooms, advertising slots
- Customers arrive over time
  - Customers have value v unknown to firm
- ▶ How should firm set prices over time? If lower price:
  - (a) sell to marginal agents today
  - (b) make less revenue from inframarginal agents
  - (c) lose opportunity to sell tomorrow

## Revenue Management: Example

- Example: one item to sell (K=I)
  - ▶ There are N customers with  $v\sim U[0,1]$
- Last customer
  - ▶ Choose  $p_N$  to maximize  $\Pi_N$ = (prob sell) x price =  $(I-p_N)p_N$ .
  - Solution:  $p_N$ \*=0.5, yielding  $\Pi_N$ \*=0.25.
- Dynamic programming: suppose n<sup>th</sup> customer arrives
  - Choose  $p_n$  to maximize  $\Pi_n = (I p_n)p_n + p_n\Pi_{n+1}$ .
  - ▶ Solution:  $p_n^*=0.5[I + \Pi_{n+1}]$ , yielding  $\Pi_n^*=0.25[I + \Pi_{n+1}]^2$
- Working backwards with 5 customers:

	5 <sup>th</sup>	4 <sup>th</sup>	3 <sup>rd</sup>	2 <sup>nd</sup>	st
Price, p <sub>n</sub> *	0.5	0.63	0.70	0.74	0.78
Profit, $\Pi_n^*$	0.25	0.39	0.48	0.55	0.60

### Durable Goods and Price Commitment

- Apple is thinking how to price the iPhone
  - In the first year it sells to high value customers
  - Then lowers price to sell to low value customers
- Problem: Customers anticipate price will fall
  - Customer delay purchases until price falls
  - Monopolist competes with future selves
- Model applies to durable goods
  - Software, Xbox, Art
- Model applies to durable services
  - Movies, information goods.

## Durable Goods: Example

- N customers have v=30, N customers have v=10.
- $\blacktriangleright$  Suppose there are two periods, with discount rate  $\delta$ 
  - If commit to one price, charge p=30, profit  $\Pi$  =30N.
- Suppose sell to high agents in period I
  - ▶ Charge  $p_2=10$  and sell to low agents in period 2.
- High agents anticipate price will fall and may wait
  - ▶ Charge at most  $p_1$ =30-20 $\delta$ , for high agents to buy in period I
  - ▶ Total profits  $\Pi = (30-20\delta)N + \delta(10)N = (30-10\delta)N$
- Firm suffers because it cannot commit
  - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.

### **Durable Goods: Solutions**

- Solution I: Destroy the mould (e.g. artist)
  - Without mould cannot create quantity in second period
- Solution 2: Reputation (e.g. Xbox games)
  - Develop reputation for not dropping prices
- Solution 3: Renting (e.g. Xerox)
  - Good no longer "durable", so sell static monopoly quantity each period
- Solution 4: Best-price provision (e.g. iPhone)
  - If firm lowers price then customers get rebate
  - Firm never any incentive to lower price below monopoly price since lose money in rebates

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## Behavior Based Pricing and Commitment

- Suppose a firm sells to customers multiple times
- Purchasing behavior in early period tells firm about values
  - Firm tempted to condition price on past behavior
- Problem: Customers anticipate "ratchet effect"
  - Customers delay purchases to get lower prices later
  - Monopolist competes with her future selves
- Applications
  - Online sites with cookies, magazine subscriptions, cable TV

## Behavior Based Pricing: Example

- N customers have v=30, N customers have v=10.
- $\blacktriangleright$  Suppose there are two periods, with discount rate  $\delta$ 
  - If cannot see past behavior, charge p=30, profit  $\Pi_0$  =30(1+ $\delta$ )N.
- Suppose sell to high agents in period I
  - ▶ Charge  $p_2$ =10 if did not buy in period 1
  - ▶ Charge  $p_2$ =30 if bought in period I (ratchet effect)
- ▶ If customers myopic charge  $p_1$ =30
  - ▶ Total profits  $\Pi_{M}$  = 30N+  $\delta$ (30+10)N = (30+40 $\delta$ )N >  $\Pi_{0}$
- If customers forward looking, anticipate price fall if don't buy
  - ▶ Charge at most  $p_1$ =30-20 $\delta$ , for high agents to buy in period I
  - ▶ Total profits  $\Pi_F = (30-20\delta)N + \delta(30+10)N = (30+20\delta)N < \Pi_0$
- ▶ Firm suffers because it cannot commit
  - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.

## Why are there Introductory Discounts?

#### Behavioral-based pricing view

- Firms can't resist giving discount to people who don't purchase
- These discounts hurt the firm if
  - (a) Consumers are forward looking
  - (b) Consumers get annoyed

### Introductory discounts may be good idea

- Network effects (see network slides)
- Overcome switching costs (see lockin slides)
- Encourage customer experimentation (next slide)

## Customer Experimentation

- Product is "experience good"
  - Don't know taste until tried it
- $\triangleright$  Customers have value v=30 or v=10 with equal prob.
  - Optimal pricing: niche market strategy
  - ▶ Period I, charge price  $p_1$ =20, and everyone buys
  - ▶ Period t≥2, charge price  $p_t$ =30, and high value agents buy
- Customers have value v=30 or v=20 with equal prob.
  - Optimal pricing: mass market strategy
  - Period I, charge price  $p_1=25$ , and everyone buys
  - ▶ Period t≥2, charge price  $p_t$ =20, and everyone buy

## Firm Experimentation

- How does a firm price when it does not know demand?
  - Firm wishes to sell a unique good.
  - Customers enter each period (not forward looking)
  - Each buyer has the same value, v, unknown to firm
- Optimal policy: start price high and lower slowly.
  - Solve through backwards induction.
  - ▶ Rate of decrease depends on firm's patience.
- What if have good each period to sell?
  - Price may go up or down.
  - But should move prices around to experiment.
- Experimentation very easy online
  - Run A/B tests. Seems scientific, but can be misleading.

## Sticky Prices

- ▶ For single firm, choose price so  $d\pi(p^*)/dp=0$ .
- Suppose there is inflation and cost k of changing prices
  - E.g. Physical cost of printing menu
- No reason to change price when almost optimal.
  - Costs k, but only second order gain.
- Optimal policy is called (S,s) rule.
  - When real price hits  $p_L < p^*$  then increase to  $p_H > p^*$ .
- Similar story when demand subject to shocks.
  - If demand has transitory increase, bring
  - If demand has permanent increase, also increase S.

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