# The Economics of E-commerce and Technology 

Dynamic Pricing

## Peak Load Pricing

- Suppose a firm has zero marginal cost, with capacity K
- Broadband capacity, cell phone towers, hotel rooms
- Capacity costs z per unit to build.
- There are two periods (or two equally likely states)
> Period L demand is low, $\mathrm{P}_{\mathrm{L}}(\mathrm{q})$
- Period H demand is high, $\mathrm{P}_{\mathrm{H}}(\mathrm{q})$
- Firm chooses $q_{L}, q_{H}$ and $K$ to maximize profits

$$
\pi=q_{L} P_{L}\left(q_{L}\right)+q_{H} P_{H}\left(q_{H}\right)-z K \quad \text { subject to } q_{L}, q_{H} \leq K
$$

- Lagrangian: choose $\mathrm{q}_{L}, \mathrm{q}_{H}$ and K to maximize
$\mathrm{L}=\mathrm{q}_{\mathrm{L}} \mathrm{P}_{\mathrm{L}}\left(\mathrm{q}_{\mathrm{L}}\right)+\mathrm{q}_{H} \mathrm{P}_{\mathrm{H}}\left(\mathrm{q}_{\mathrm{H}}\right)-\mathrm{zK}+\lambda_{\mathrm{L}}\left[\mathrm{K}-\mathrm{q}_{\mathrm{L}}\right]+\lambda_{\mathrm{H}}\left[\mathrm{K}-\mathrm{q}_{H}\right]$


## Peak Load Pricing

- Solution
- FOCs for $q_{L}, q_{H}$ and $K: M R_{L}\left(q_{L}{ }^{*}\right)=\lambda_{L}, M R_{H}\left(q_{H}{ }^{*}\right)=\lambda_{H}, \mathrm{z}=\lambda_{\mathrm{L}}+\lambda_{H}$
, Optimal capacity: $\mathrm{K}^{*}=\mathrm{q}_{\mathrm{H}}{ }^{*}$
- Idea: Charge capacity when constraint binds.
- Two cases:
I. Constraint slack in period L (big difference in demands)

2. Constraint binds in period L (small difference in demands)

- Price in H higher for two reasons
(a) The demand is higher,
(b) Charging more of the capacity
- Examples: cheap evening calls and Christmas flights


## 1. Constraint Slack in Period L $\left(\mathrm{q}_{\mathrm{L}}{ }^{*}<\mathrm{q}_{\mathrm{H}}{ }^{*}\right)$

- Optimal quantities: $\mathrm{MR}_{\mathrm{L}}\left(\mathrm{q}_{\mathrm{L}}{ }^{*}\right)=0, \mathrm{MR}_{\mathrm{H}}\left(\mathrm{q}_{{ }^{*}}{ }^{*}\right)=\mathrm{z}$



## 2. Constraint Binds in Period L $\left(\mathrm{q}_{\mathrm{L}}{ }^{*}<\mathrm{q}_{\mathrm{H}}{ }^{*}\right)$

- Optimal quantities: $\mathrm{q}_{\mathrm{L}}^{*}=\mathrm{q}_{H}{ }^{*}, \mathrm{MR}_{\mathrm{L}}\left(\mathrm{q}_{\mathrm{L}}{ }^{*}\right)=\lambda_{\mathrm{L}}, \mathrm{MR}_{\mathrm{H}}\left(\mathrm{q}_{H}{ }^{*}\right)=\mathrm{z}-\lambda_{\mathrm{L}}$



## Revenue Management

- A firm has K tickets to sell
- Airline seats, hotel rooms, advertising slots
- Customers arrive over time
- Customers have value $v$ unknown to firm
- How should firm set prices over time? If lower price:
(a) sell to marginal agents today
(b) make less revenue from inframarginal agents
(c) lose opportunity to sell tomorrow


## Revenue Management: Example

- Example: one item to sell $(K=I)$
- There are N customers with $\mathrm{v} \sim \mathrm{U}[0, \mathrm{I}]$
- Last customer
- Choose $\mathrm{P}_{\mathrm{N}}$ to maximize $\Pi_{\mathrm{N}}=($ prob sell $) \times$ price $=\left(I-\mathrm{P}_{\mathrm{N}}\right) \mathrm{P}_{\mathrm{N}}$.
, Solution: $\mathrm{P}_{\mathrm{N}}{ }^{*}=0.5$, yielding $\Pi_{\mathrm{N}}{ }^{*}=0.25$.
- Dynamic programming: suppose $\mathrm{n}^{\text {th }}$ customer arrives
* Choose $p_{n}$ to maximize $\Pi_{n}=\left(I-p_{n}\right) p_{n}+p_{n} \Pi_{n+1}$.

Solution: $\mathrm{P}_{\mathrm{n}}$ *=0.5[I+ $\left.\Pi_{\mathrm{n}+1}\right]$, yielding $\Pi_{\mathrm{n}} *=0.25\left[1+\Pi_{\mathrm{n}+1}\right]^{2}$

- Working backwards with 5 customers:

|  | $5^{\text {th }}$ | $4^{\text {th }}$ | $3^{\text {rd }}$ | $2^{\text {nd }}$ | $1^{\text {st }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Price, $p_{n}^{*}$ | 0.5 | 0.63 | 0.70 | 0.74 | 0.78 |
| Profit, $\Pi_{n}^{*}$ | 0.25 | 0.39 | 0.48 | 0.55 | 0.60 |

## Durable Goods and Price Commitment

- Apple is thinking how to price the iPhone
* In the first year it sells to high value customers
- Then lowers price to sell to low value customers
- Problem: Customers anticipate price will fall
- Customer delay purchases until price falls
- Monopolist competes with future selves
- Model applies to durable goods
- Software, Xbox,Art
- Model applies to durable services
- Movies, information goods.


## Durable Goods: Example

- N customers have $\mathrm{v}=30, \mathrm{~N}$ customers have $\mathrm{v}=10$.
- Suppose there are two periods, with discount rate $\delta$
- If commit to one price, charge $p=30$, profit $\Pi=30 \mathrm{~N}$.
- Suppose sell to high agents in period I
- Charge $\mathrm{p}_{2}=10$ and sell to low agents in period 2.
- High agents anticipate price will fall and may wait
* Charge at most $p_{1}=30-20 \delta$, for high agents to buy in period I , Total profits $\Pi=(30-20 \delta) \mathrm{N}+\delta(10) \mathrm{N}=(30-10 \delta) \mathrm{N}$
- Firm suffers because it cannot commit
- Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.


## Durable Goods: Solutions

- Solution I: Destroy the mould (e.g. artist)
b Without mould cannot create quantity in second period
- Solution 2: Reputation (e.g. Xbox games)
- Develop reputation for not dropping prices
- Solution 3: Renting (e.g. Xerox)
* Good no longer "durable", so sell static monopoly quantity each period
- Solution 4: Best-price provision (e.g. iPhone)
- If firm lowers price then customers get rebate
- Firm never any incentive to lower price below monopoly price since lose money in rebates


## Behavior Based Pricing and Commitment

- Suppose a firm sells to customers multiple times
- Purchasing behavior in early period tells firm about values
- Firm tempted to condition price on past behavior
- Problem: Customers anticipate "ratchet effect"
b Customers delay purchases to get lower prices later
- Monopolist competes with her future selves
- Applications
- Online sites with cookies, magazine subscriptions, cable TV


## Behavior Based Pricing: Example

- N customers have $\mathrm{v}=30, \mathrm{~N}$ customers have $\mathrm{v}=10$.
- Suppose there are two periods, with discount rate $\delta$
b If cannot see past behavior, charge $\mathrm{p}=30$, profit $\Pi_{0}=30(1+\delta) \mathrm{N}$.
- Suppose sell to high agents in period I
- Charge $P_{2}=10$ if did not buy in period I
- Charge $P_{2}=30$ if bought in period I (ratchet effect)
- If customers myopic charge $\mathrm{p}_{\mathrm{I}}=30$
, Total profits $\Pi_{M}=30 \mathrm{~N}+\delta(30+10) \mathrm{N}=(30+40 \delta) \mathrm{N}>\Pi_{0}$
- If customers forward looking, anticipate price fall if don't buy
- Charge at most $\mathrm{P}_{\mathrm{I}}=30-20 \delta$, for high agents to buy in period I
, Total profits $\Pi_{\mathrm{F}}=(30-20 \delta) \mathrm{N}+\delta(30+10) \mathrm{N}=(30+20 \delta) \mathrm{N}<\Pi_{0}$
- Firm suffers because it cannot commit
- Firm cannot resist lowering price in period 2 , exerting a negative externality on its former self.


## Why are there Introductory Discounts?

- Behavioral-based pricing view
- Firms can't resist giving discount to people who don't purchase
- These discounts hurt the firm if
(a) Consumers are forward looking
(b) Consumers get annoyed
- Introductory discounts may be good idea
- Network effects (see network slides)
- Overcome switching costs (see lockin slides)
- Encourage customer experimentation (next slide)


## Customer Experimentation

- Product is "experience good"
- Don't know taste until tried it
- Customers have value $v=30$ or $v=10$ with equal prob.
- Optimal pricing: niche market strategy
- Period I, charge price $\mathrm{P}_{\mathrm{I}}=20$, and everyone buys
- Period $\mathrm{t} \geq 2$, charge price $\mathrm{P}_{\mathrm{t}}=30$, and high value agents buy
- Customers have value $v=30$ or $v=20$ with equal prob.
- Optimal pricing: mass market strategy
- Period I, charge price $\mathrm{P}_{\mathrm{I}}=25$, and everyone buys
- Period $\mathrm{t} \geq 2$, charge price $\mathrm{p}_{\mathrm{t}}=20$, and everyone buy


## Firm Experimentation

- How does a firm price when it does not know demand?
- Firm wishes to sell a unique good.
, Customers enter each period (not forward looking)
- Each buyer has the same value, v, unknown to firm
- Optimal policy: start price high and lower slowly.
- Solve through backwards induction.
- Rate of decrease depends on firm's patience.
- What if have good each period to sell?
- Price may go up or down.
- But should move prices around to experiment.
- Experimentation very easy online
- Run A/B tests. Seems scientific, but can be misleading.


## Sticky Prices

- For single firm, choose price so $d \pi\left(p^{*}\right) / d p=0$.
- Suppose there is inflation and cost k of changing prices
, E.g. Physical cost of printing menu
- No reason to change price when almost optimal.
b Costs k, but only second order gain.
- Optimal policy is called $(\mathrm{S}, \mathrm{s})$ rule.
- When real price hits $p_{L}<p^{*}$ then increase to $p_{H}>\mathrm{p}^{*}$.
- Similar story when demand subject to shocks.
- If demand has transitory increase, bring
- If demand has permanent increase, also increase S .

