Economics 2102: Homework 1

30 September, 2004

1. Suppose N bidders have independently distributed private valuations $v_i \sim U[0,1]$. Derive the FPA equilibrium bidding strategy from first principals.

2. Suppose N bidders have values $(v_1, \ldots, v_N)$, and these valuations are common knowledge. How should the agents bid in a FPA?

3. Suppose I auction \$1 among two bidders in an all–pay auction. What is the symmetric nash equilibrium? Are there asymmetric equilibria?

4. Assume all bidders have IID private valuations $v_i \sim F(v)$ with support $[0,1]$. Suppose the good is sold via an all–pay auction.
(a) Derive the symmetric equilibrium bidding strategy directly.
(b) Derive the symmetric equilibrium bidding strategy via revenue equivalence.

5. Assume all bidders have independent private valuations $v_i \sim F(v)$ with support $[0,1]$. Derive the symmetric equilibrium bidding strategy for the war of attrition.

6. Two bidders receive independent signals $x_i \sim U[0,1]$. Agent $i's$ value for the good is $v_i = 3x_i + x_j$.
(a) Compute the bidding strategy in an English auction.
(b) Compute the bidding strategy in a SPA.
(c) Directly solve for the FPA bidding strategy.
(d) Use the revenue equivalence theorem to solve for the FPA bidding strategy.

7. Assume all bidders have IID private valuations $v_i \sim F(v)$ with support $[\underline{V}, \overline{V}]$. Define marginal revenue as

$$MR(v) = v - \frac{1 - F(v)}{f(v)}$$

(a) Show that $E[MR(v)] = \underline{V}$.
(b) In terms of marginal revenues, what is the revenue from 2 bidders with no reservation price?
(c) Let the sellers valuation be v_0. In terms of marginal revenue, what is the revenue from 1 bidder and a reservation price?
(d) Assume $\underline{V} \geq v_0$, i.e. all bidders are "serious". How is revenue affected if one bidder is swapped for a reservation price?