

## Economics 2102: Homework 2

11 November, 2004

1.  $N$  bidders have IID private valuations  $v_i \sim F$  for the right to harvest a forest. In stage 1 this right is sold via an English auction. After the auction, in stage 2, there is then a resale market.  $M$  different bidders with IID private valuations  $u_i$  (with the same distribution as  $v_i$ ) are interested in purchasing the rights. Given the original winner can choose any second-stage mechanism, how should they bid in the original auction? How would revenue change if the auctioneer sold to all  $M + N$  bidders at once?

2. (a) There is one bidder with value  $v_1 \sim U[a, a + 1]$ , where  $a \geq 0$ . What is the optimal auction? Intuitively, why is the optimal reservation price increasing in  $a$ ?

(b) Now there is a second bidder with value  $v_2 \sim U[0, 1]$ , where agents' types are independent. What is the optimal auction?

3. A principal wishes to procure a number of units of a good from an agent with unknown marginal cost  $\theta \sim F$  on  $[0, 1]$ . The value to the agent from  $q$  units is  $V(q)$ , which is increasing and concave. Denote the monetary transfer by  $t$ .

(a) Specify the DRM and write down the agent's utility.

(b) Characterise the agent's utility under incentive compatibility in terms of an integral equation and a monotonicity constraint.

Define marginal cost to be

$$MC(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$$

(c) Write down the principal's problem in terms of  $MC(\theta)$ .

(d) Assuming  $MC(\theta)$  is increasing, what is the optimal quantity?

(e) If  $\theta$  were known by the principal, what would be the optimal quantity? Why does this differ from (d)?

4. A principal wishes to procure a number of units of a good from one of  $N$  agents with unknown marginal cost  $\theta_i \sim F$  on  $[0, 1]$ . First suppose the number of units demanded,  $q$ , is independent of the price the principal has to pay,  $p$ .

(a) Suppose the principal holds a second price auction. The agent who bids the lowest per-unit price,  $p_{(1)}$ , wins the auction and is paid the second highest per-unit price,  $p_{(2)}$ , for  $q$  units. How

will they bid?

(b) Now suppose the principal holds a first price auction. How will the agents bid?

Next, suppose demand is downward sloping. That is, if the principal has to pay a per-unit price of  $p$ , they demand  $q(p)$  units, where  $q'(p) \leq 0$ .

(c) Suppose the principal holds a SPA. How will agents bid? What will be the quantity supplied?

(d) Suppose the principal holds a FPA. Using Milgrom and Weber's ranking lemma on the two first order conditions, show that agents bid more aggressively than in part (b). That is, when demand is downward sloping, each type bids less. [Note: other methods of proof are perfectly acceptable].

(e) Show the expected price is less in a FPA than a SPA when demand is downward sloping. Intuitively, what's going on?

5. Each of  $N$  agents have a project which needs funding. The value they place on funding is  $\theta \sim F$  on  $[0, 1]$ . The SSHRC wants to fund the most worthwhile project, but cannot observe  $\theta$ . Agents write proposals which are time consuming: an agent who spends time  $t$  on a proposal gains utility  $u_i(\theta_i) = P_i \cdot \theta - t_i$ , where the project is funded with probability  $P_i$ . The SSHRC can only observe the time  $t_i$  each agent spends writing their proposal. Their aim is to maximise welfare which, since writing proposals is wasteful, is the same as maximising  $\sum_i u_i$ .

(a) Specify the problem as a DRM and write down the agents' utility.

(b) Characterise the agent's utility under incentive compatibility in terms of an integral equation and a monotonicity constraint.

(c) Suppose  $(1 - F(x))/f(x)$  is strictly decreasing in  $x$ . Show the SSHRC's optimal policy is to allocate the grant randomly.

6. A firm is bargaining with a union. The firm's gain from agreement is  $\theta \in \{\theta_L, \theta_H\}$ . If  $\theta$  were perfectly known suppose the parties split the surplus, wages would increase by  $\theta/2$  and the firm would gain  $\theta/2$ . At time 0 the firm can signal their type by committing to walk away from the table until time  $t$ . The common interest rate is  $r$ . What does the set of (separating and pooling) equilibria look like?