Comprehensive Exam: Mathematical Economics

Date: 10th May, 2006

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Time: 10am–5pm.

Aids: Open Book, Calculators, Brain.

Good luck!

Question 1

There are $L \geq 2$ firms in a market, each of whom have n loyal customers with a willingness to pay of 1. In addition, there are m new customers up for grabs. Firms have constant marginal costs $c_i \in [0, 1]$ with distribution $F(c_i)$ and density $f(c_i)$.

(a) Assume firms can price discriminate and so charge their incumbents a price of 1 while offering new customers lower prices. Each firm simultaneously announces a price $p_i \leq 1$. New customers all go to the cheapest firm. Calculate the prices and profits of each firm as a function of its costs.

(b) Assume firms cannot price discriminate. Again, each firm announces a price $p_i \leq 1$. New customers go to the cheapest firm, while old customers buy from their current firm. Calculate the prices and profits of each firm as a function of its costs. What happens to prices as n/m varies?

Question 2

A seller wants to sell a good to one of two symmetric buyers. Buyer i gains utility $v_i x_i - t_i$, where v_i is his valuation, x_i is the probability he gets the good and t_i is his payment to the seller. The seller wishes to maximise expected payments.

A seller designs a mechanism $(x_i(v_1, v_2), t_i(v_1, v_2))$, $i \in \{1, 2\}$, where the allocation probability and payments are a function of the agents' reports. The mechanism must allocate the good to the highest valuation buyer if valuations are different, and to each buyer with probability $1/2$ if the valuations are the same. We consider only symmetric mechanisms, where payments depend on the agents' reports and not their identities. Denote $t_{ab} := t_1(v_a, v_b) = t_2(v_b, v_a)$.

Each buyer has one of two valuations, v_l or v_h, where $v_h > v_l$. The probability that the agents have valuations a, b is given by p_{ab}, where $a, b \in \{l, h\}$. We assume $p_{hh}p_{ll} > p_{hl}^2$, so valuations are positively correlated.

(a) The seller wants to design an ex–post individually rational (EPIR) and ex–post incentive compatible (EPIC) mechanism to maximise their expected revenue. Determine the optimal transfers and the expected utility of a high and low type.

(b) The seller now drops the EPIR and EPIC requirement. The mechanism only has to be interim individually rational (IR) and interim incentive compatible (IC). Show that the seller can fully extract from the buyers. [Hint: Choose $t_{hh} = v_h/2$ and $t_{hl} = v_h$.] Intuitively, why can the seller fully extract the buyers' rent?

(c) The seller is concerned the buyers may collude. Suppose that if the buyers collude, they choose a pair of reports that minimises the sum of the transfers they pay. Show that if the buyers collude in the mechanism from part (a), they pay a total of v_l. Show that if the buyers collude in the mechanism from part (b), they pay less than v_l.

(d) Show that any (IR) and (IC) mechanism where buyers pay at least v_l by colluding, gives them at least as much rent as the mechanism from part (a).