Economics 2102: Homework 1

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Question 1

An agent with concave utility $u(w)$ faces random wealth w with mean $\overline{w}$. The risk premium π can be defined by $u(\overline{w} - \pi) = E[U(w)]$.

(a) By taking a first–order Taylor expansion of the left hand side and a second–order expansion of the right–hand side, show that

$$\pi \approx \frac{1}{2} r(\overline{w}) \text{Var}(w)$$

where $r(\overline{w}) = -u''(\overline{w})/u(\overline{w})$ is the coefficient of absolute risk aversion.

(b) Approximate the π by taking second order expansions of both sides. Can you come up with a justification for the asymmetric expansions in part (a)?

Question 2

A principal employs $N+1$ agents with exponential utility $u(w) = -\exp(-rw)$ and outside options $\overline{u}$. The performance of agent i is given by

$$z_i = e_i + x_i + x_c$$

where (x_i, x_c) are independent and normally distributed with variance $\text{Var}(x_i)$ and $\text{Var}(x_c)$ respectively. Assume the principal offers a linear contract

$$w_i = \alpha_i + \beta_i (z_i - \sum_{j \neq i} \gamma_j z_j)$$

Efforts $\{e_i\}_i$ induce profit $\sum_i P(e_i)$ for the principal.

Solve for the optimal contract $(\alpha_i, \beta_i, \{\gamma_j\}_{j \neq i})$. Interpret the coefficients $\{\gamma_j\}_{j \neq i}$. What implications does this have for the incentives of car salesmen?

Question 3

An agent has increasing, concave utility $u(\cdot)$. They start with wealth W_0 and may have an accident costing x of her wealth. Assume x is publicly observable. The agent has access to a perfectly competitive market of risk–neutral insurers who offer payments $R(x)$ net of any insurance premium. The distribution of x is as follows

$$f(0,a) = 1 - p(a) \quad (1)$$
$$f(x,a) = p(a)g(x) \quad \text{for} \quad x > 0 \quad (2)$$

where $\int g(x)dx = 0$. The agent can affect the probability of an accident through their choice of a. The cost is given by increasing convex function, $\psi(a)$. The function $p(a)$ is decreasing and convex.

(a) Suppose there is no insurance market. What action $\hat{a}$ does the agent take?

(b) Suppose a is contractible. Describe the first–best payment schedule $R(x)$ and the effort choice, a^*. Can you compare $\hat{a}$ and a^* when $p(a)$ is small? [Hint: mean value theorem]. What is the intuition?

(c) Suppose a is not contractible. Describe the second–best payment schedule $R(x)$. What does the specification in equation (1) buy us?

(d) Interpret the second–best payment schedule. [Hint: I mentioned it briefly in the lecture]. Would anything change if the agent could hide an accident? (i.e. when $x > 0$ they could report $x = 0$).

Question 4

A principal employs an agent with utility $u(t)-g(a)$ and reserve utility $\overline{u}$, where $u(t)$ is increasing and concave. The agent takes an action $a \in \{L, H\}$, where $g(L) < g(H)$. This induces a distribution over output, $f(x|a)$ which satisfies MLRP. Assume the agent has *limited liability*, i.e. $t \geq 0$.

The principal's utility is $x-t$. After x is revealed they may launch an investigation and observe the agent's action, at cost $m > 0$. A contract thus specifies the probability of investigation

$\pi(x) \in [0,1]$, the transfer if there is no investigation $t(x) \geq 0$, and the transfer if there is an investigation $s_a(x) \geq 0$ and action a was taken.

The principal makes a TIOLI offer to the agent. Assume she wants to implement $a = H$.

(a) Write down the principal's profit–maximisation problem subject to (IC) and (IR).

(b) Formulate the Lagrangian for the problem.

(c) What is the shape of the payments $s_L(x)$ and $s_H(x)$? Provide an interpretation.

(d) What is the shape of $t(x)$, when no investigation occurs?

(e) What if the form of the optimal investigation policy $\pi(x)$? Is the principal more likely to investigate when output is high or low? Interpret your findings.

(f) Why did we assume limited liability? What is the optimal scheme is transfers are allowed to be negative?

Question 5

Consider Holstrom's model of moral hazard in teams. N agents work in a team with joint output $x(a_i, \ldots, a_N)$, where a_i is the effort of agent i.

(a) Show that by introducing a principal (agent $N+1$) who does not participate in the production process, we can sustain an efficient effort profile as a Nash equilibrium using a differentiable balanced–budget output–sharing rule, i.e. $\sum_i t_i(x) = x$ $(\forall x)$.

(b) Suppose the principal can collude with one agent (call her agent k). That is, the colluders secretly write a side contract based on x to increase their joint payoff (other agents are unaware of the side contract). Show the scheme in (a) is susceptible to collusion.

(c) Suppose we restricted ourselves to differentiable output–sharing schemes that are invulnerable to collusion. Show that it is impossible to sustain the efficient effort profile.

An alternative solution to the problem is monitoring. Let $x_j(a_i, \ldots, a_N)$, $j = 1, \ldots, m$ be a series of output measures summing to total output, $\sum_j x_j(a) = x(a)$. Assume all functions

$x_j(a)$ are weakly differentiable and nondecreasing. The output sharing rules $t_i(x_1, \ldots, x_N)$ are differentiable, nondecreasing and balance the budget, $\sum_i t_i(x_1, \ldots, x_N) = x(a)$.

(d) Derive the first order conditions for the agents' equilibrium effort choices.

(e) An accounting system is *sufficient* if one can implement the efficient effort levels. Show that a sufficient accounting system must have at least N measures. [Hint: Use the fact that $\partial x_j(a)/\partial a_i \leq \partial x(a)/\partial a_i$ $(\forall j)$.]