# Economics 2102: Homework 1

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## Question 1

An agent chooses to execute a project at some time $\tau \in \{1, \ldots, T\}$. The agent receives revenue $\theta$ and faces costs $\{c_t\}_t$, known at time 0. The agent thus chooses $\tau$ to maximise

$$u(\theta, \tau) = (\theta - c_\tau)\delta^\tau$$

where $\delta \in (0, 1)$. How does the optimal choice of execution time vary in $\theta$? Can you provide an economic intuition for your answer?

## Question 2

The following normal–linear model is regularly used in applied models. Given action $a \in \Re$, output is $q = a + x$, where $x \sim N(0, V)$. The cost of effort is $g(a)$ is increasing and convex. The agent's utility equals $u(w(q) - g(a))$, while the principals is $q - w(q)$. Suppose the agent's outside option is $u(0)$.

We make two large assumptions. First, the principal uses a linear contract:

$$w(q) = \alpha + \beta q$$

Second, the agent's utility is CARA, i.e., $u(w) = -e^{-w}$.

(a) Suppose $w \sim N(\mu, \sigma^2)$. Denote the certainty equivalent of $w$ by $\bar{w}$, where

$$u(\bar{w}) = E[u(w)]$$

Show that $\bar{w} = \mu - \sigma^2/2$.

(b) Suppose effort is unobservable. The principal's problem is

$$\begin{aligned} \max_{w(q),a} \; & E[q - w(q)] \\ \text{s.t.} \quad & E[u(w(q) - g(a))|a] \geq u(0) \\ & a \in \text{argmax}_{a' \in \Re} E[u(w(q) - g(a'))|a'] \end{aligned}$$

Using the first order approach, characterise the optimal contract $(\alpha, \beta, a)$. [Hint: write utilities in terms of their certainty equivalent.]

## Question 3

An agent has increasing, concave utility $u(\cdot)$. They start with wealth $W_0$ and may have an accident costing $x$ of their wealth. Assume $x$ is publicly observable. The agent has access to a perfectly competitive market of risk–neutral insurers who offer payments $R(x)$ net of any insurance premium. The distribution of $x$ is as follows

$$f(0, a) = 1 - p(a) \tag{1}$$

$$f(x, a) = p(a)g(x) \quad \text{for} \quad x > 0 \tag{2}$$

where $\int g(x)dx = 1$. The agent can affect the probability of an accident through their choice of $a$. The cost is given by increasing convex function, $\psi(a)$. The function $p(a)$ is decreasing and convex.

(a) Suppose there is no insurance market. What action $\hat{a}$ does the agent take?

(b) Suppose $a$ is contractible. Describe the first–best payment schedule $R(x)$ and the effort choice, $a^*$.

(c) Suppose $a$ is not contractible. Describe the second–best payment schedule $R(x)$. What does the specification in equation (1) buy us?

(d) Interpret the second–best payment schedule. Would the agent ever have an incentive to hide an accident? (i.e. report $x = 0$ when $x > 0$).

## Question 4

A principal employs an agent with utility $u(t) - g(a)$ and reserve utility $\overline{u}$, where $u(t)$ is increasing and concave. The agent takes an action $a \in \{L, H\}$, where $g(L) < g(H)$. This induces a distribution over output, $f(x|a)$, which satisfies MLRP. Assume the agent has *limited liability*, i.e. $t \geq 0$.

The principal's utility is $x - t$. After $x$ is revealed they may launch an investigation and observe the agent's action at cost $m > 0$. A contract thus specifies the probability of investigation, $\pi(x) \in [0, 1]$; the transfer if there is no investigation, $t(x) \geq 0$; and the transfer if there is an investigation and the principal uncovers action $a$, $s_a(x) \geq 0$.

The principal makes a TIOLI offer to the agent. Assume she wants to implement $a = H$.

(a) Write down the principal's profit–maximisation problem subject to (IC) and (IR).

(b) Formulate the Lagrangian for the problem.

(c) What is the shape of the payments $s_L(x)$ and $s_H(x)$? Provide an interpretation.

(d) What is the shape of $t(x)$, when no investigation occurs?

(e) What if the form of the optimal investigation policy $\pi(x)$? Is the principal more likely to investigate when output is high or low? Interpret your findings.

(f) Why did we assume limited liability? What is the optimal scheme is transfers are allowed to be negative?

## Question 5

Consider Holstrom's model of moral hazard in teams. $N$ agents work in a team with joint output $x(a_i, \ldots, a_N)$, where $a_i$ is the effort of agent $i$ and $g(a_i)$ is is the increasing, convex cost function.

(a) Show that by introducing a principal (agent $N+1$) who does not participate in the production process, we can sustain an efficient effort profile as a Nash equilibrium using a differentiable balanced–budget output–sharing rule, i.e. $\sum_i t_i(x) = x$ $(\forall x)$.

(b) Suppose the principal can collude with one agent (call her agent $k$). That is, the colluders secretly write a side contract based on $x$ to increase their joint payoff (other agents are unaware of the side contract). Show the scheme in (a) is susceptible to collusion.

(c) Suppose we restricted ourselves to differentiable output–sharing schemes that are invulnerable to collusion. Show that it is impossible to sustain the efficient effort profile.

An alternative solution to the problem is monitoring. Let $x_j(a_i, \ldots, a_N)$, $j = 1, \ldots, M$ be a series of output measures summing to total output, $\sum_j x_j(a) = x(a)$. Assume all functions $x_j(a)$ are weakly differentiable and nondecreasing. The output sharing rules $t_i(x_1, \ldots, x_N)$ are differentiable, nondecreasing and balance the budget, $\sum_i t_i(x_1, \ldots, x_N) = x(a)$.

(d) Derive the first order conditions for the agents' equilibrium effort choices.

(e) An accounting system is *sufficient* if one can implement the efficient effort levels. Show that a sufficient accounting system must have at least $N$ measures. [Hint: Use the fact that $\partial x_j(a)/\partial a_i \leq \partial x(a)/\partial a_i$ $(\forall j)$.]

## Question 6

Suppose a risk neutral principal employs a risk averse agent. The two parties both sign a contract stating wage profile $w(q)$. The agent then chooses action $a \in A$ at cost $g(a)$.

Payoffs are as follows. The agent gets

$$u(w - g(a))$$

where $g(a)$ is increasing and convex. Utility is strictly increasing and strictly concave. The principal gets

$$q - w$$

The principal has reservation profit 0; the agent has reservation utility $u(0)$.

First, suppose the principal makes a TIOLI offer to the agent.

(a) Assume the effort $a$ is observable. Set up and solve the principal's optimal contract.

(b) Assume effort $a$ is not observable. Set up the principal's problem.

Next, suppose the agent makes a TIOLI offer to the principal.

(c) Assume the effort $a$ is observable. Show that the optimal contract induces the same effort as when the principal proposes the contract.

(d) Assume effort $a$ is not observable. Set up the agent's problem. Next, suppose that utility is CARA, i.e. $u(w) = -\exp(-w)$. Show that the optimal contract induces the same effort as when the principal proposes the contract (part (b)).