Theory Appendix

A Model Setup

- V - the value function of an unemployed worker at the beginning of the UI spell.
- s_t- search effort / normalized to the hazard of exiting unemployment at each point in time.
  - searching at a given intensity comes at a cost of $\psi_t(s_t)$ where $\psi_t(.)$ may be varying over time.
- S_t - the survival function for remaining in unemployment
  - The individual can choose search effort in a way that will determine the survival function. $S_t = \exp\left(-\int_0^t s_t dt\right)$
  - We can model this as the individual deciding over a survival function S (where S represents the whole function while S_t represents the value of the survival function at t) directly. A given survival function comes at a per period cost of search effort $\psi_t(S)$.
- b_t - Flow of UI benefits at point in time t
  - $b_t = b$ until P Potential Benefit Duration
  - $b_t = 0$ after P
- T - Time horizon
- $u(.)$ - the flow utility function when unemployed
- $v(.)$ - the flow utility function when employed (depends on net wage $w - \tau$)

The value function is given as:

$$\begin{aligned} V &= \int_0^T S_t u(b_t) + [1 - S_t] v(w - \tau) \, - S_t \psi_t(s_t) \, dt && (1) \\ &= \int_0^P S_t u(b) \, dt + \int_P^T S_t u(0) \, dt + \int_0^T [1 - S_t] v(w - \tau) \, dt - \int_0^T S_t \psi_t(s_t) \, dt && (2) \end{aligned}$$

Budget constraint of the government:

$$\begin{aligned} \int_0^B S_t b \, dt - \int_0^T [1 - S_t] \tau \, dt &= 0 && (3) \\ Bb - (T - D)\tau &= 0 \end{aligned}$$

therefore:

$$\tau = \frac{B}{T - D} b \quad (4)$$

$$\frac{d\tau}{db} = \frac{B}{T-D} + \frac{1}{T-D}\frac{dB}{db}b + \frac{B}{(T-D)^2}\frac{dD}{db}b \tag{5}$$
$$= \frac{B}{T-D}\left(1 + \varepsilon_{B,b} + \varepsilon_{D,b}\frac{D}{T-D}\right)$$

$$\frac{d\tau}{dP} = \left(\frac{1}{T-D}\left(\frac{dB}{dP} + \frac{B}{T-D}\frac{dD}{dP}\right)\right)b$$
$$= \left(\frac{1}{T-D}\left(\frac{dB}{dP}b + \frac{dD}{dP}\tau\right)\right) \tag{6}$$

Note that $B = \int_0^P S_t\,dt$ and therefore $\frac{dB}{dP} = S_P + \int_0^P \frac{dS_t}{dP}\,dt$ and $\frac{dB}{db} = \int_0^P \frac{dS_t}{db}\,dt$

Approximation with Constant Hazard

If the hazard of exiting from unemployment is constant ($s_t = s$), then unemployment duration follows an exponential distribution where all relevant statistics can be calculated from the search intensity s:

Survival function: $S_t = e^{-st}$, $D = \frac{1}{s}$. Furthermore:

$$B = \int_0^P S_t\,dt$$
$$= \int_0^P e^{-st}\,dt$$
$$= \frac{1-e^{-sP}}{s}$$
$$= \frac{1-S_p}{s}$$
$$= D[1-S_p]$$

$$S_P = e^{-sP}$$

Furthermore we can define:

$$\xi \equiv 1 - (1+Ps)e^{-Ps}$$

For the case of a constant hazard $\int_0^P \frac{dS_t}{dP}\,dt$ is proportional to the nonemployment effect: $\int_0^P \frac{dS_t}{dP}\,dt = \frac{dD}{dP}\xi$ and $\int_0^P \frac{dS_t}{db}\,dt = \frac{dD}{db}\xi$,

Proof:

$$\xi = 1 - (1+Ps)e^{-Ps}$$

$$
\begin{aligned}
\frac{dD}{dP} = \int_0^T \frac{dS_t}{dP}\, dt &= \int_0^T \frac{de^{-st}}{dP}\, dt \\
&= \int_0^T -te^{-st}\frac{ds}{dP}\, dt \\
&= \frac{ds}{dP}\int_0^T -te^{-st}\, dt \\
&= \frac{ds}{dP}\left(-\frac{1-(1+T\,s)e^{-Ts}}{s^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\int_0^P \frac{dS_t}{dP}\, dt &= \int_0^P \frac{de^{-st}}{dP}\, dt \\
&= \int_0^P -te^{-st}\frac{ds}{dP}\, dt \\
&= \frac{ds}{dP}\int_0^P -te^{-st}\, dt \\
&= \frac{ds}{dP}\left(-\frac{1-(1+P\,s)e^{-Ps}}{s^2}\right) \\
&= \left(\frac{dD}{dP}\frac{-s^2}{1-e^{Ts}(T\,s+1)}\right)\left(-\frac{1-e^{-Ps}(P\,s+1)}{s^2}\right) \\
&= \frac{dD}{dP}\frac{1-(1+P\,s)e^{-Ps}}{1-(1+T\,s)e^{-Ts}}
\end{aligned}
$$

For T going to infinity:

$$
\int_0^P \frac{dS_t}{dP}\, dt = \frac{dD}{dP}\left(1-(1+P\,s)e^{-Ps}\right) = \frac{dD}{dP}\xi
$$

Note that:

$$
\frac{dB}{dP} = S_P + \frac{dD}{dP}\xi
$$

and therefore:

$$
\xi = \frac{\frac{dB}{dP} - S_P}{\frac{dD}{dP}}
$$

And for $\frac{dB}{db}$ we get similarly:

$$
\frac{dB}{db} = \int_0^P \frac{dS_t}{db}\, dt = \frac{dD}{db}\xi
$$

Social Planner's problem:

$$
\max_{b,P,\tau} V(b,P,\tau)\, s.t.\, Budget\, Constraint \tag{7}
$$

However the budget constraint implicitely defines τ as a function of b, so we can instead maximize the unconstrained problem:

$$\max_{b,P} V(b, P, \tau(b)) \tag{8}$$

FOC for b

Using the envelope condition, the marginal effect of increasing UI benefits by $1 is given as:

$$\begin{aligned} \frac{dV}{db} &= \int_0^P S_t\, dt\, u'(b) - \int_0^T [1 - S_t]\, dt\, v' \frac{d\tau}{db} \\ &= B\, u'(b) - (T - D)\, v' \frac{d\tau}{db} \end{aligned} \tag{9}$$

Plugging in $\frac{d\tau}{db}$

$$\begin{aligned} \frac{dV}{db} &= B\, u'(b) - [T - D]\, v' \left[\frac{B}{T - D} \left(1 + \varepsilon_{B,b} + \varepsilon_{D,b} \frac{D}{T - D} \right) \right] \\ &= B\, [u'(b) - v'(w - \tau)] - v'(w - \tau) \left[\left(\varepsilon_{B,b} + \varepsilon_{D,b} \frac{D}{T - D} \right) B \right] \end{aligned}$$

Rescaling:

$$\frac{dV}{db} \frac{1}{v'(w - \tau)} = B\, \frac{u'(c_{u,t \le P}) - v'(c_e)}{v'(c_e)} - \left(\varepsilon_{B,b} + \varepsilon_{D,b} \frac{D}{T - D} \right) B$$

Or using marginal effects:

$$\frac{dV}{db} \frac{1}{v'} = B\, \frac{u'(c_{u,t \le P}) - v'(c_e)}{v'(c_e)} - \left(\frac{dB}{db} + \frac{dD}{db} \frac{B}{T - D} \right) b$$

$$\tau = \frac{B}{T - D} b$$

$$\frac{dV}{db} \frac{1}{v'} = B\, \frac{u'(c_{u,t \le P}) - v'(c_e)}{v'(c_e)} - \left(\frac{dB}{db} b + \frac{dD}{db} \tau \right)$$

$$\frac{dW}{db} \frac{1}{B\, v'(c_e)} = \underbrace{\frac{u'(c_{u,t \le P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\frac{\frac{dB}{db} b + \frac{dD}{db} \tau}{B}}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{10}$$

$$\underbrace{\frac{u'(c_{u,t \le P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\left(\eta_{B,b} + \eta_{D,b} \frac{D}{B} \frac{\tau}{b} \right)}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{11}$$

Using the constant hazard approximation:

$$\frac{dV}{db}\frac{1}{v'} = B\,\frac{u'(c_{u,t\leq P}) - v'(c_e)}{v'(c_e)} - \frac{dD}{db}\left(\xi b + \tau\right)$$

FOC for P

$$\begin{aligned}\frac{dV}{dP} &= S_P\left(u(b) - u(0)\right) - \int_0^T \left[1 - S_t\right] dt\, v'\frac{d\tau}{dP} \\ &= S_P\left(u(b) - u(0)\right) - \left(T - D\right) v'\frac{d\tau}{dP}\end{aligned}$$

$$\begin{aligned}\frac{dV}{dP} &= S_P\left(u(b) - u(0)\right) - v'\left(S_P b + \int_0^P \frac{dS_t}{dP}\,dtb + \frac{dD}{dP}\tau\right) \\ &= S_P\left(u(b) - u(0) - bv'\right) - v'\left(\int_0^P \frac{dS_t}{dP}\,dtb + \frac{dD}{dP}\tau\right)\end{aligned}$$

$$\frac{dV}{dP}\quad\tfrac{1}{v'} = \quad S_P\frac{u(b) - u(0) - bv'}{v'} - \left(\int_0^P \frac{dS_t}{dP}\,dtb + \frac{dD}{dP}\tau\right)$$

Using the constant hazard approximation we get:

$$\begin{aligned}\frac{dV}{dP}\quad\tfrac{1}{v'} &= \quad S_P\frac{u(b) - u(0) - bv'(w-\tau)}{v'(w-\tau)} - \left(\frac{dD}{dP}\xi b + \frac{dD}{dP}\tau\right) \\ &= \quad S_P\frac{u(b) - u(0) - bv'(w-\tau)}{v'(w-\tau)} - \frac{dD}{dP}\left(\xi b + \tau\right) \qquad (12)\end{aligned}$$

If we let define $\widetilde{u}'(c_{u,t>P}) \equiv \frac{1}{b}\int_0^b u'(k)dk = \frac{u(b)-u(0)}{b}$ as the average marginal utility between consumption levels of b and 0 for a UI exhaustee, we can write this as:

$$\frac{dV}{dP}\quad\tfrac{1}{v'} = \quad S_P b\frac{\widetilde{u}'(b) - v'(w-\tau)}{v'(w-\tau)} - \frac{dD}{dP}\left(\xi b + \tau\right) \qquad (13)$$

$$\frac{dW}{dP}\frac{1}{S(P)b\,v'(c_e)} = \underbrace{\frac{\widetilde{u}'(c_{u,t>P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\frac{\int_0^P \frac{dS_t}{dP}\,dtb + \frac{dD}{dP}\tau}{S(P)b}}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{14}$$

$$= \underbrace{\frac{\widetilde{u}'(c_{u,t>P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} \underbrace{- \frac{1}{S_P}\left(\int_0^P \frac{dS_t}{dP}\,dt + \frac{dD}{dP}\frac{\tau}{b}\right)}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{15}$$

Two equations to highlight:

FOC for b (Baily-Chetty Formula):

$$\frac{dV}{db}\frac{1}{v'} = \underbrace{\underbrace{B}_{\substack{\text{Mechanical increase} \\ \text{in spending}}} \underbrace{\frac{u'(b) - v'(w-\tau)}{v'(w-\tau)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}}}_{\text{Mechanical Transfer to Unemployed}} - \underbrace{\frac{dD}{db}(\xi b + \tau)}_{\text{Behavioral Cost}} \tag{16}$$

FOC for P (SWB Formula):

$$\frac{dV}{dP}\frac{1}{v'} = \underbrace{\underbrace{S_P b}_{\substack{\text{Mechanical increase} \\ \text{in spending}}} \underbrace{\frac{\widetilde{u}'(0) - v'(w-\tau)}{v'(w-\tau)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}}}_{\text{Mechanical Transfer to Unemployed}} - \underbrace{\frac{dD}{dP}(\xi b + \tau)}_{\text{Behavioral Cost}} \tag{17}$$

Calculating comparable discincentive effects from literature

FOC for b (Baily-Chetty Formula):

$$\frac{dV}{db}\frac{1}{v'B} = \underbrace{\frac{u'(c_{u,t\leq P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\frac{dD}{db}\frac{\xi b + \tau}{B}}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{18}$$

$$= \underbrace{\frac{u'(c_{u,t\leq P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\eta_{D,b}\frac{D}{B}\left(\xi + \frac{q}{\rho}\right)}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{19}$$

$$= \underbrace{\frac{u'(c_{u,t\leq P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\eta_{D,b}\frac{1}{1-S_P}\left(\xi + \frac{q}{\rho}\right)}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{20}$$

where q is the payroll tax rate ($\tau = qw$) and ρ is the replacement rate: $b = \rho w$ and we assume that pre-unemployment wages equal post unemployment wages.

FOC for B (SWB Formula):

$$\frac{dV}{dP}\frac{1}{v'S_P b} = \underbrace{\frac{\widetilde{u}'(c_{u,t>P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\frac{dD}{dP}\frac{\xi b + \tau}{S_P b}}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{21}$$

$$= \underbrace{\frac{\widetilde{u}'(c_{u,t>P}) - v'(c_e)}{v'(c_e)}}_{\substack{\text{Social Value} \\ \text{of \$1 add. transfer}}} - \underbrace{\frac{dD}{dP}\frac{1}{S_P}\left(\xi + \frac{q}{\rho}\right)}_{\substack{\text{Behavioral Cost} \\ \text{per \$1 add. Transfer}}} \tag{22}$$