

ALLOCATION OF A STOCK SUBJECT TO A CONSTRAINT

by

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**Discussion Paper Number 16
January/1972**

**Preliminary Report on Research in Progress
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I

The explanations of pricing tend to be either very general as in standard price theory or very specific as in econometric practice. Economic theorists except for those interested in simulation have not reacted to the possibilities of the kind of approach associated with the computer. Price theory uses general categories; cases described by a continuous function usually reduce to a single categorical explanation. It may make more sense to seek to formulate a richer repertory of explanations, including as special cases those already known to be of interest as well as those flowing from combinations of parameters that might potentially hold interest. In other words, formalization of a problem might seek on methodological grounds not to simplify but to enrich.

II

As a case in point, consider a discriminating legal monopolist who is not free to set whatever price he wishes. It appears that if we treat such a monopolist as a wealth-maximizer subject to a constraint on his price selection, then we find that the monopolist will be led to a number of differing types of behavior depending on rather trivial differences in the economic environment in which he functions. This suggests that observed differences in the behavior of discriminating legal monopolists may not spring from a variety of objectives or sheer irrationality

¹I am indebted to helpful comments by Meyer Burstein and Ronald Britto.

but from a uniform objective implemented in differing environments. The analysis might also lead us to conjecture that we should find more cases of discrimination where legal monopolists are franchised private firms rather than statutory bodies. If these conclusions stand up they seem a rather worthwhile return from modifying the standard approach to price discrimination.

III

As a concrete example of such a monopolist, consider an airport parking franchise, such as at Los Angeles International Airport, with two classes of customers, those meeting friends and those who are either taking short vacations or are commuters wishing to park their cars for longer periods. Suppose further that the parking franchise prices what the two markets will bear so long as it does not set a price which the public reacts to as "unfair", leading them to complain in considerable numbers to the Airport authority. The California State Department of Parks and Recreation sells booklets to campers consisting of ten tickets with each ticket good for one night's camping on a first-come, first-served basis. The price of the booklet is \$10. At the same time it sells reserved camping spots at prices ranging from \$1.50 to \$4.00.² It seems clear enough that the Department views itself faced by two classes of consumers and discriminates between them as classes. (It might also be

²It also sells unreserved camping spots singly. Booklets may be viewed as a price cut to the unreserved camping spot purchaser although some income may be generated by those who buy booklets, misestimating their demand. However, this is likely to be a once-for-all, i.e., a consumer is unlikely to buy a second book if he did not use the first one fully.

considered as discriminating within a class, i.e., the reserved camping spot purchasers). It seems unlikely that prices are set without consideration of nonmarket consequences. We could view the Department as a wealth-maximizer subject to the consideration that it does not set prices which produce trouble for itself. Trouble might be defined as some given volume of letters to Assemblymen or as the difficulties the Department finds itself subjected to when State policing costs rise when the prices of camping spots reach a certain level. When campers feel prices are "unfair" they simply steal camping spots, and if property rights are to be maintained this calls for additional policing costs. A municipal theatre management with two classes of consumers, which feels it cannot price any seat above a certain level because of reactions of its competitors might be another example.

Basically recurring stocks of short-run property rights (e.g., one buys the property right to a theatre seat for a given performance) which may or may not be optimal with respect to short run wealth maximization have to be priced in all of these cases. It will turn out that the analysis is less useful for a single stock, for example a salmon packer who packing a season's catch of salmon of uniform grade, for a discount and a quality market subject to some upper limit on price.³ An additional

³Different managers in the situations described obviously arrive at different estimations of what prices they can get away with. Those desiring a "quiet life" must generally feel themselves more constrained. It might be objected that "trouble" should be treated a continuous variable, i.e., the higher the price the more trouble occasioned. Nonetheless, at some point of consumer dissatisfaction, franchises are removed. Hence the existence of a price constraint need imply no more than that at some point the probability of a wealth loss due to unfavorable consumer reactions rises sharply, (i.e., the function is discontinuous and has a step in it).

complication in that example, or in the example of a cinema, is that trouble comes by way of competitive reactions.

IV

To analyze the case, recall the airport franchise example. Assume that marginal costs are zero; all parking structures and lots are completed, and labor and management are hired on an annual basis. Sufficient labor is available on contract to handle a volume of traffic up to and including completely packed lots and structures. The costs of denying parking spots to the public are also zero. These can be sealed off with equipment already purchased. The authority faces two linear demand functions with $a_2 > 1/2 a_1$. (This latter assumption will be varied later). The authority charges what the traffic will bear up to a price which it feels will cause trouble for itself, e.g., responses that might influence its continued possession of the franchise.

The price which constrains wealth-maximization can be at various levels. Four levels are of interest. In addition, the original stock of parking lots relative to market demand can vary. (In other words, we do not assume an optimal decision on producing the stock.) Three cases are of interest in this connection. This generates twelve possible initial situations in which the monopolist might find himself. An examination of this case shows that in some of the situations there are more than one possible response depending on other circumstances than those detailed above. In fact, there are twenty-four possible sets of conditions; the monopolistic response is similar in some cases to more than one set of conditions. Diagram 1 and Table 1 show the initial twelve situations.

These situations are then analyzed with Diagrams 2 - 5 which disclose the additional situations. The responses of the monopolist in the twenty four situations is summarized in Table 2. Next the assumption that $a_2 > 1/2 a_1$ is changed. There are three possible alternative cases: $a_2 = a_1$; $a_2 = 1/2 a_1$; and $a_2 < 1/2 a_1$. Discrimination does not occur in the first case for when marginal receipts are equal then prices are equal.⁴ The latter two cases generate their own family of situations and these are summarized in Diagrams 6 and 7 and Tables 3 to 6.

There seems no good reason to expect any one of the numerous situations indicated to prevail rather than another. With respect to the price constraint, there seems no reason why a sense of "fairness" should be linked to the property of elasticity of a market demand function. Hence no case should prevail relative to another given the information available here. Monopolists or decision-makers for a cartel are faced with disposal of stocks which can be treated as having a size which is accidental.⁵ Authorities which construct camping grounds or parking structures do misestimate demand. Furthermore, the period of production particularly if it is affected by political considerations (e.g., voting of a bond issue) may be quite long, so that persistence of a disequilibrium in the size of a stock over time may be quite long. With respect to the relative values of the a-coefficient in the demand function, again it seems difficult to

⁴ If $p_1 = a - bq$ and $p_2 = a - cq$, then $MR_1 = a - 2bq$ and $MR_2 = a - 2cq$. But if $MR_1 = MR_2$ then $bq = cq$. Hence $p_1 = p_2$.

⁵ If attention is broadened to all disposable stocks available to monopolists one could certainly find cases of stocks not optimally produced. A famous example which points to a family of cases is the American stockpile of quinine sold to an international cartel.

make any prior judgment about likelihood of particular cases. The coefficient merely tells us, if we separate markets into a quality and a discount market (or first-class seats and second-class seats) at what prices consumers will start buying in the respective markets. The difference ($a_1 - a_2$) gives some indication of how much price must be cut for lower income groups to buy upper income style goods perhaps, but such a difference will clearly vary in various markets. In sum, in the absence of definite empirical information, there seems no good reason to expect any particular case to emerge. Hence the logical space created by the various change of assumptions might be considered an empirical space likely to be filled up with actual cases. One final word: the monopolist in the rather bewildering array of cases which follows, behaves in a rather simple manner. It is a plausible manner and a consistent one. Basically, he tries to equate marginal receipts and given discontinuities in the marginal receipts functions then he sells his stock so that each successive unit gets the best return. He will not sell at a price which will cause him trouble.

Diagram 1 displays the first set of possible price constraints and Table 1 classifies the situations the monopolist might face given those constraints.

If we analyze these cases, then the characteristics of the monopolist's response displayed in Table 2 will be observed. The details of the analysis leading to the contents of the table are given in Appendix One.

Diagram 1. Four possible price constraints when $a_2 > 1/2 a_1$

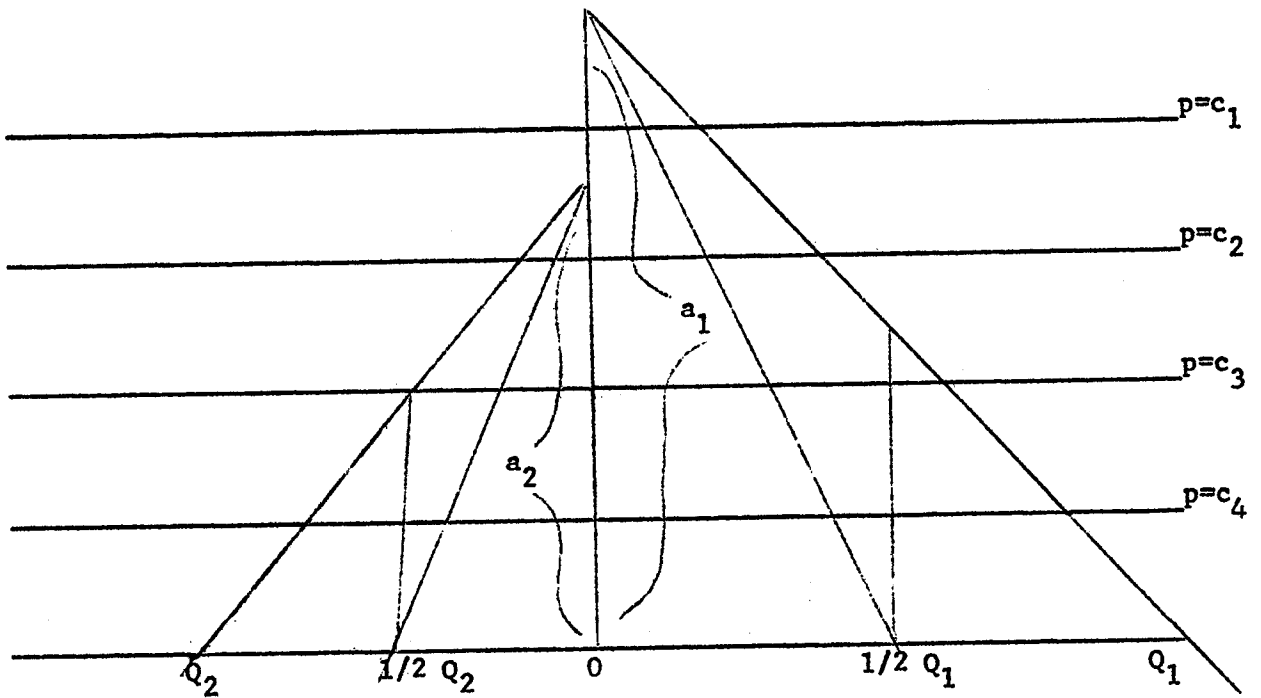


Table 1. A classification of the situations the monopolist faces when $a_2 > 1/2 a_1$ ⁶

Value of constraint C	Size of stock S				
$a_1 > c_1 \geq a_2$	Situation 1				
$a_2 > c_2 \geq 1/2 a_1$	4				
$a_1 > c_3 > 1/2 a_2$	7a, 7b, 7c				
$1/2 a_2 \geq c_4 > 0$	10a, 10b, 10c				
$S > 1/2(Q_1 + Q_2)$	2				
$S = 1/2(Q_1 + Q_2)$	5				
$S < 1/2(Q_1 + Q_2)$	3a, 3b, 3c	6a, 6b, 6c	9a, 9b, 9c	12	

⁶The subcases a, b and c will be defined in the pages following. Nothing emerges if we set $c_1 = 0$ or $c_1 = a_1$, of course. Similarly, previously, the case $a_2 = 0$ was not of interest. These cases are excluded because discriminating monopoly would not emerge.

Table 2. Features of the monopolist's response to the differing situations

Situation	Outputs in both markets are determinate	Price Discrimination exists	Marginal Receipts are equated	There is Excess Demand	There is unsold stock	Output is greater than when no constraint exists in	
						Market one	Market two
1.	True	True	True	False	True	False	False
2.	True	True	True	False	False	False	False
3a.	False	False	False	True	False	False	False
3b.	True	True(*)	False	False	False	False	False
3c.	True	True	True	False	False	False	False
4.	True	True	True	False	True	False	False
5.	True	True	True	False	False	False	False
6a.	False(**)	False	True	True(*)	False	False	False
6b.	True	True	False	False	False	False	False
6c.(***)	True	True	True	False	False	False	False
7a.	True	True	False	False	False	True	False
7b.	True	True	False	False	True	True	False
7c.	False	False	True	True	False	True	False
8a.	True	True	False	False	False	True	False
8b.	True	False	True	False	False	True	False
8c.	False	False	True	True	False	?	?
9a.	True	True	False	False	False	True	False
9b.	True	False	True	False	False	True	False
9c.	False	False	True	True	False	?	?
10a.	True	False	True	False	True	True	True(*)
10b.	True	False	True	False	False	True	True(*)
10c.	False	False	True	True	False	?	?
11.	False	False	True	True	False	?	?
12.	False	False	True	True	False	?	?

(*) except in the "equals" case, then "false".

(**) except in the "equals" case, then "true".

(***) This case does not emerge when $c_2 = 1/2 a_1$. It should be noted that if we treat "equals" cases as separate cases then this generates another 5 situations.

Price discrimination occurs in only half of the cases above. Thus if this analysis were to be felt to be applicable, then we should not be disturbed to find one airport authority using a single price in a situation which looks similar to another in which discrimination actually occurs. In eight cases excess demand occurs, and in four cases there will be excess stock. In some cases, although this is rare in the "discount" market, the sense of "fairness" which leads to the consumer constraining the monopolist does work to the consumer's benefit. But generally this is not the case even for the buyer in the "quality" market. It should be observed that the relative frequency of true or false judgments are partly a function of the logical space created by the original assumptions, but this does not render the whole analysis meaningless. Finally, it should be noted there is another way of looking at the problem. If we see a situation in which we expect price discrimination but do not find it, then we should not automatically conclude that the monopolist is a poor judge of his own interest. We do not get price discrimination, if this analysis has any relevance, precisely because the monopolist is sensitive to the preservation of his monopoly right.

If the assumption that $a_2 > 1/2 a_1$ is changed, two broad sets of results emerge when $a_2 = 1/2 a_1$ and $a_2 < 1/2 a_1$. The monopolist who faces the two alternative general demand conditions is less likely to discriminate. When $a_2 = 1/2 a_1$ seventeen cases emerge. Only three involve new situations as Table 3 discloses. (An appropriate diagram (Diagram 6) is included in Appendix Two.

Table 3. A classification of the situations the monopolist faces when $a_2 = 1/2 a_1$

Size of stock S.	Value of constraint C	$1/2 a_1 = c_1 = a_2$	$1/2 a_1 > c > 1/2 a_2$	$1/2 a_2 \geq c > 0$
$s > 1/2(Q_1 + Q_2)$		13	7a, 7b, 7c	10a, 10b, 10c
$s = 1/2(Q_1 + Q_2)$		14	8a, 8b, 8c	11
$s < 1/2(Q_1 + Q_2)$		15	9a, 9b, 9c	12

The observable results of the new situations are given in Table 4.

Table 4. Additional truth values for new situations in Table 3

Situation	Outputs in both markets are determinate	Price Discrimination exists	Marginal Receipts are equated	There is Excess Demand	There is unsold stock	Output is greater than when no constraint exists	
						in Market one	in Market two
13.	False	False	False	False	True	False	False
14.	False	False	False	False	False	False	False
15.	False	False	False	True	False	False	False

It should be noted that the monopolist in this case discriminates only in four out of seventeen situations. When $a_2 < 1/2 a_1$, twenty cases emerge, three of them new as Table 5 discloses. (For the appropriate diagram see Appendix Two.)

Table 5. A classification of the situations the monopolist faces when $a_2 < 1/2 a_1$

Size of stock S	Value of constraint C			
	$1/2 a_1 = c_1$	$1/2 a_1 > c_2 \geq a_2$	$a_2 > c_3 > 1/2 a_2$	$1/2 a_2 \geq c_4 > 0$
$s > 1/2(Q_1 + Q_2)$	13	16	7a, 7b, 7c	10a, 10b, 10c
$s = 1/2(Q_1 + Q_2)$	14	17	8a, 8b, 8c	11
$s < 1/2(Q_1 + Q_2)$	15	18	9a, 9b, 9c	12

Situations 16-18 are not identical to 13-15 as the truth tables show.

Table 6. Additional Truth Values for new situations in Table 4

Situation	Outputs in both markets are determinate	Price Discrimination exists	Marginal Receipts are equated	There is Excess Demand	There is unsold stock	Output is greater than when no constraint exists in	
						Market one	Market two
16.	False	False	False	False	True	True	False
17.	False	False	False	False	False	True	False
18.	False	False	False	True	False	True	False

It should be noted that in this case the monopolist discriminates only in four out of twenty situations.

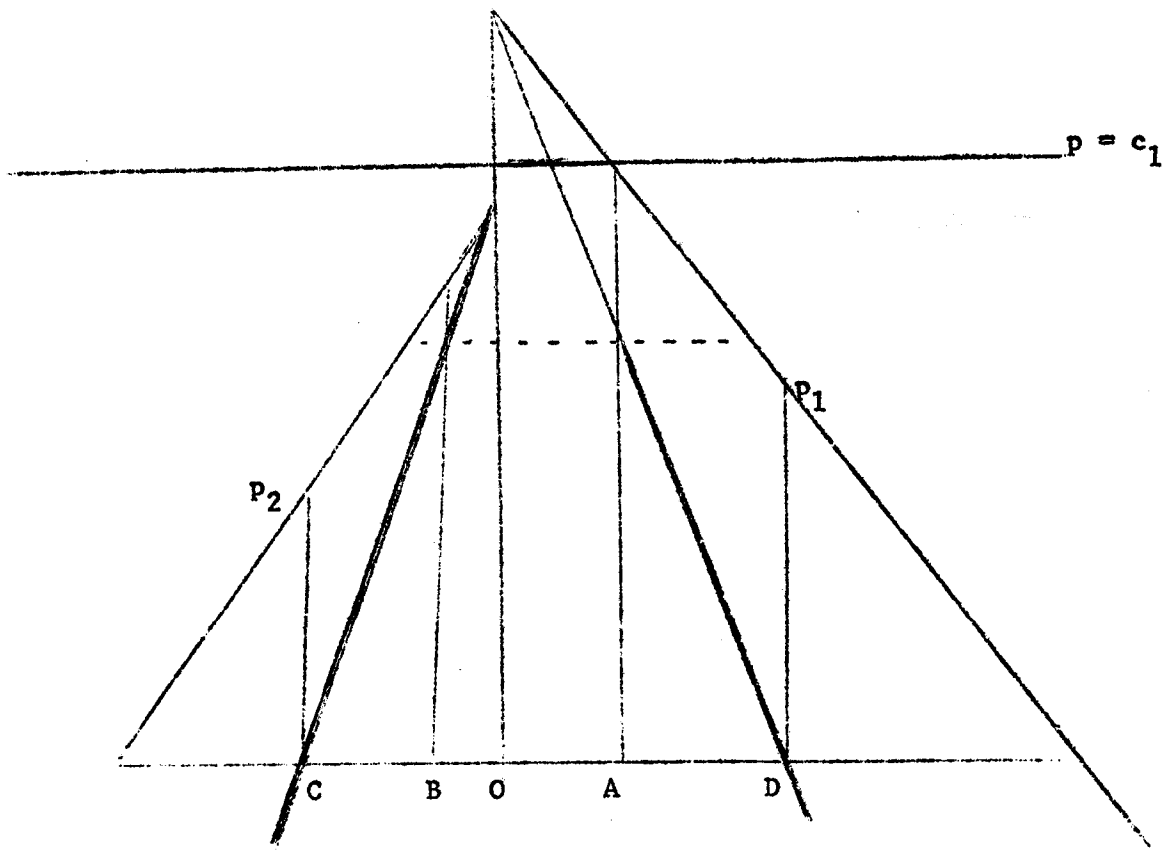
V.

If franchises are extended to private firms we might conjecture that such firms are more aggressive in behavior than other types of sellers (e.g., a public authority.) If this is the case, they will be less concerned about a constraint, or they will view it as being higher than less aggressive sellers. This implies they will discriminate more.⁷

⁷ I have no particular knowledge of airports, etc. but one example of the relative behavior of a private firm and a public authority has struck me very forcibly. ARA which had a franchise for all automatic machines at UCLA lost this franchise due to student protests. Service is now provided by a student organization and it is very much more responsive to students' demands (i.e., for kosher food, organically grown food, and "better service.") I would conjecture that all of these activities were not provided by ARA because this would reduce profits to them. Clearly they were for more ready to tolerate dissatisfaction with their policies than ASCULA is.

Appendix 1. Analysis of Situations when $a_2 > 1/2 a$.

Diagram 2. Situations where $a_1 > c_1 \geq a_2$



Situation 1. In this diagram as for the subsequent ones, the appropriate marginal receipts functions for making decisions are in heavy black lines. This is the normal case of price discrimination and two prices emerge,

P_1 and P_2 . At these prices excess stock will exist. In the case that the monopolist or cartel feels the existence of unsold stocks is a "bad" then such stocks will be destroyed or held off the market in some way.

Situation 2. In this case, the discriminatory prices will exactly allocate the stock.

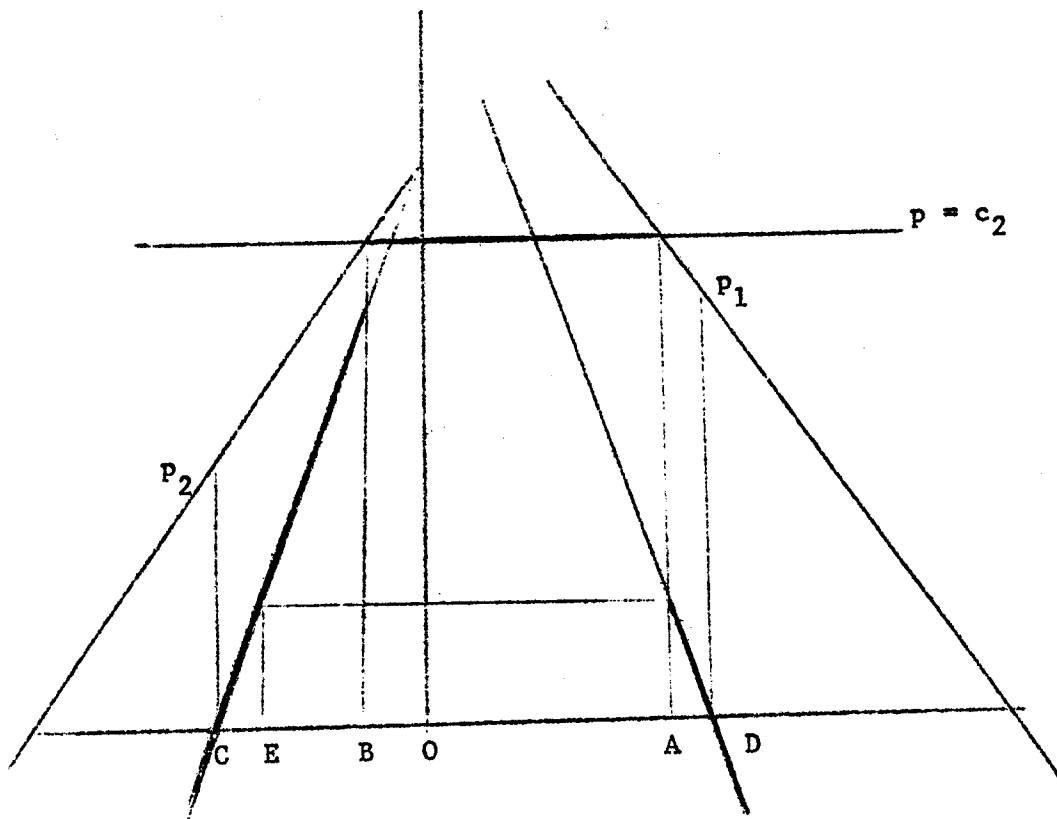
Situation 3. There are three alternatives in this situation depending on the size of the stock to be sold.

Situation 3a. If $S < OA$, then the monopolist will face excess demand at $p = c_1$. Output might, for example, be allocated on a first come, first served basis.

Situation 3b. If $OA \leq S \leq OA + OB$, then the monopolist sells OA in the first market at $p = c_1$, and sells the rest, if any, in market two at the market clearing price.

Situation 3c. If $OA + OB < S \leq OC + OD$, then the monopolist equates marginal receipts.

Diagram 3. Situations where $a_2 > c_2 \geq 1/2 a_1$



Situation 4. As for situation 1.

Situation 5. As for situation 2.

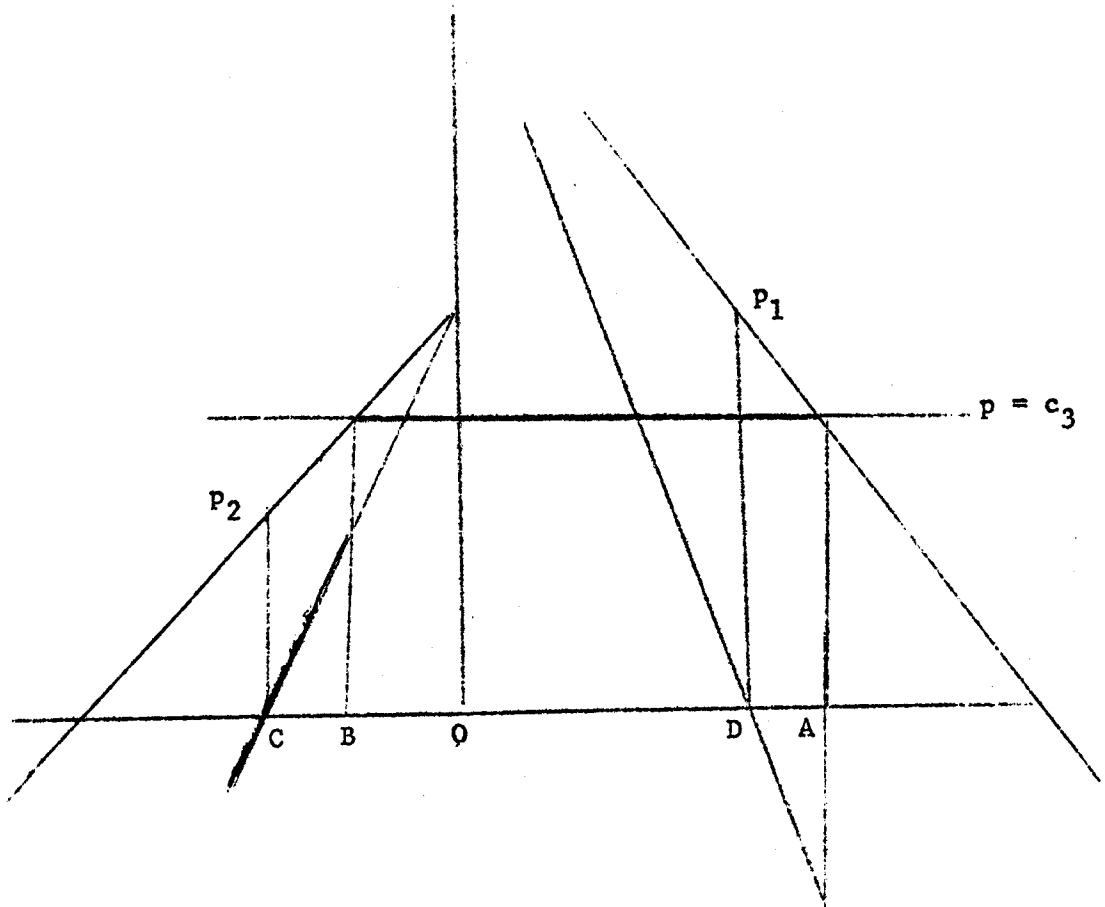
Situation 6. There are three alternatives in this situation depending on the size of the stock to be sold.

Situation 6a. If $S \leq OB + OA$, then stock must be allocated to the two markets at $p = c_2$. This might be on a first come, first served basis, except when $S = OB + OA$.

Situation 6b. If $OE + OA > S > OB + OA$, then the monopolist sells OA at p_1 and sells the rest at market clearing prices in market two.

Situation 6c. If $OC + OD > S \geq OE + OA$, then the monopolist equates marginal receipts. This case does not emerge if $c_2 = 1/2 a_1$.

Diagram 4. Situations where $a_1 > c_3 > 1/2 a_2$



Situation 7. There are three alternatives.

Situation 7a. If $S > OB + OA$, then the monopolist will sell at $p = c_3$ in market one and use market clearing prices in market two so long as $S \leq OC + OA$.

Situation 7b. If $S \geq OC + OA$, then the monopolist will sell at c_3 and p_2 . Excess stock exists or is destroyed.

Situation 7c. If $S \leq OB + OA$, then the price is c_3 . There will be excess demand.

Situation 8. There are three alternatives.

Situation 8a. If $S > OB + OA$, then the monopolist will sell at c_3 in market one and use market clearing prices in market two.

Situation 8b. If $S = OB + OA$, then price c_3 is established.

Situation 8c. If $S < OB + OA$, then price c_3 is established. There will be excess demand.

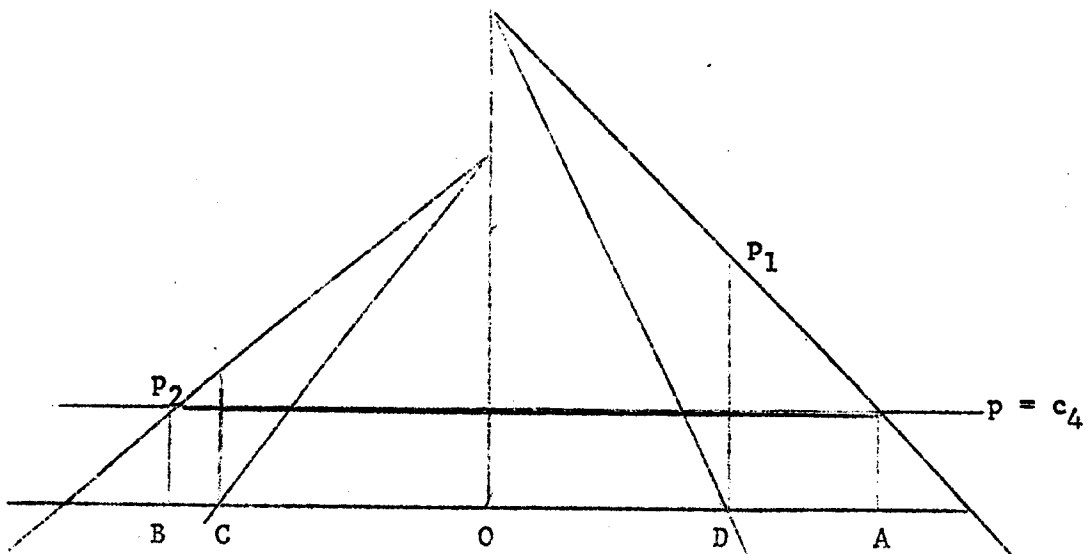
Situation 9. There are three alternatives.

Situation 9a. If $S > OB + OA$, then the monopolist will sell at c_3 in market one and use the market clearing price in market two.

Situation 9b. If $S = OB + OA$, then c_3 is established.

Situation 9c. If $S < OB + OA$, then c_3 is established. There will be excess demand.

Diagram 5. Situations where $1/2 a_2 \geq c_4 > 0$



Situation 10. There are three alternatives.

Situation 10a. If $S > OA + OB$, then $p = c_4$. There will be excess stock.

Situation 10b. If $S = OA + OB$, then $p = c_4$.

Situation 10c. If $S < OA + OB$, then $p = c_4$. There will be excess demand.

Situation 11. The price will be c_4 . There will be excess demand.

Situation 12. The price will be c_4 . There will be excess demand.

Appendix Two. Diagrams appropriate to assumptions $a_2 = 1/2 a_1$ and $a_2 < 1/2 a_1$

Diagram 6. Three possible price constraints when $a_2 = 1/2 a_1$

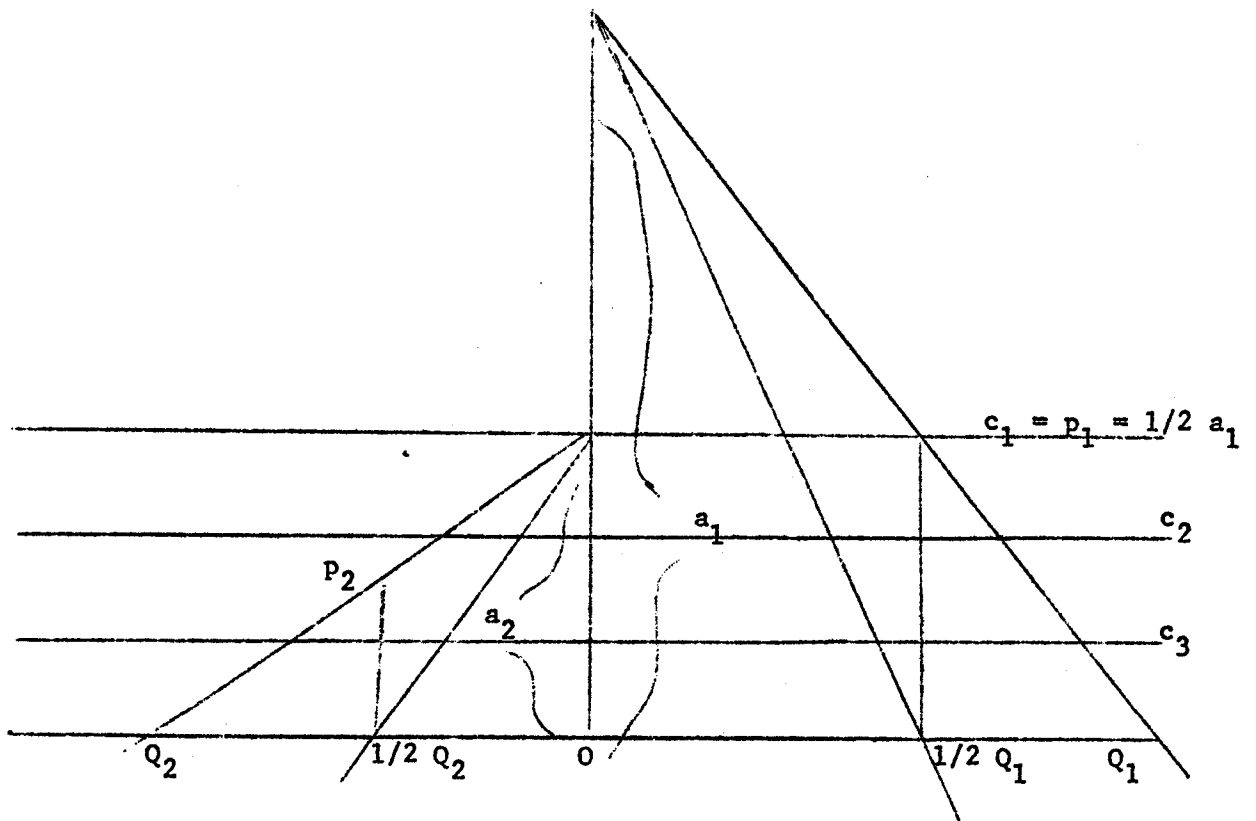
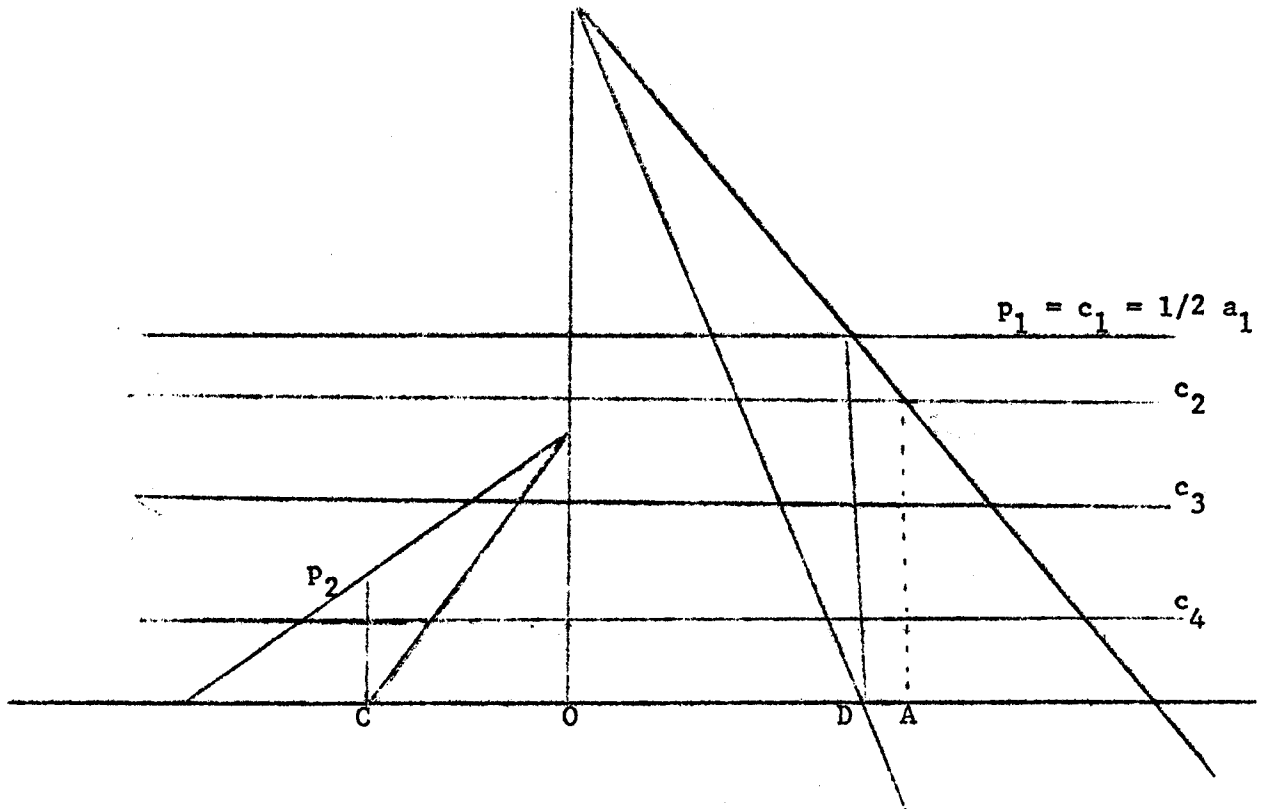


Diagram 7. Four possible price constraints when $a_2 < 1/2 a_1$



Analysis of cases 16-18 depend on when the stock available is greater, equal or less than OA .