THE INFORMATIONAL EFFICIENCY

OF

MONETARY EXCHANGE

(Revised)

By

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ABSTRACT

When a sequential bilateral trading arrangement is introduced into the
description of a Walrasian exchange economy, the issue of the existence of a
competitive equilibrium (CE) allocation can no longer be divorced from the
dynamic problem of execution. To capture some of the costs of execution, a
limit, varying with the number of individuals, is placed on the number of
trading opportunities. The resulting model demonstrates that, without a
medium of exchange, if each individual knows only CE prices (which means he
may calculate his own but not others' excess demands), the CE allocation could
but would not be reached.

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The gains from exchange may be extended if individuals are not required at each trade to balance the value of purchases and sales. But self-interest, the motive force of trade, does not move individuals to realize these gains. Suppose that X has an excess demand for ten apples and an excess supply of ten oranges. It may be efficient for X to receive the ten apples from Y, who has an excess supply, and later give his ten oranges to Z, who has an excess demand. In the actual execution, what is to prevent X from asking for eleven apples from Y, justifying his claim by saying he will supply an equal number of oranges, or give up only nine oranges to Z, saying that he took only nine apples? The purpose of this paper is to show that the essential property of money is to discourage the making of such inconsistent claims without also discouraging efficient patterns of trade.

This is "old stuff"; but it cannot be incorporated into the standard theory of value.[1] To illustrate, consider a paradox contained in Robert Clower's (1967) proposal to introduce money into the standard theory. His suggestion was to add to the existing budget constraint the injunction that current purchases be financed by sale of money only, not by current supplies of other commodities. This modification contradicts the belief that the introduction of money improves the allocation of resources. Because it is an additional constraint, it is at best not binding; and if binding, it will narrow the set of permissible exchanges compared to its barter counterpart. Clower's constraint makes no sense in the Walrasian model of exchange, but for a perfectly sensible reason. It does no good to append what is a trading constraint to a model which ignores trade.

In the standard theory, equilibrium is said to exist when (W) the sum of individual excess demands is zero for each commodity. What (W) defines is

a Walrasian equilibrium of prices. Nevertheless, from the individuals' points of view, they are in disequilibrium as long as we do not have (A) all individual excess demands are zero for each commodity.

Should the economy reach the state defined by (W), will it then go to (A)? Interpretations of the standard theory say that the Walrasian auctioneer, after announcing equilibrium prices, expedites demands and supplies. If there is one theme which distinguishes the present treatment from the standard theory, it is that exchange is a do-it-yourself affair. Individuals will not exchange with "the Market;" they will exchange with each other. This elementary logistical consideration is the basis upon which I shall construct an argument for monetary exchange.[2]

I. Summary

I shall assume that trade occurs between pairs of individuals so that the advantages of multilateral exchange must be obtained through a sequence of bilateral trades. During a unit interval of time, an individual meets with only one other so that during the interval an individual's trades are limited by his own and his trading partner's current endowments. When one pair meets, other pairs are also making contact, so that exchange occurs as a sequence of simultaneous bilateral trades.[3]

In the comparison of money and barter trading arrangements, no explicit account is taken of the physical or psychic costs of exchange. The single criterion is the number of periods it takes to accomplish the task of going from (W) to (A). Any decrease in the number is good and any increase is bad. Obviously a crude picture of the costs of exchange, it may be tolerated because the object is only an exposition of the role of money not a general theory of trade.[4]

In the analysis of the problem of going from (W) to (A), I shall focus on

the following three properties of trading sequences: (i) their technical feasibility, (ii) their informational feasibility, and (iii) their equilibrium properties, or what might be called their "behavioral feasibility." Property (i) defines the restrictions on a sequence of exchanges imposed by the fact that trade occurs between pairs. Property (ii) requires (i) and the restriction that a pair cannot base its trading decision on information available only to other pairs. For example, a pair cannot make its decisions depend on the full details of excess demands among all other pairs. Property (iii) requires (ii) and the restriction that each individual have no incentive to depart from the sequence. In a barter economy, if trades leading from (W) to (A) do not satisfy bilateral balance (BB) - where the value of purchases and sales are equated at each bilateral encounter - those trades will not form an equilibrium sequence.

We shall see that those trades which are technically and/or informationally feasible and which also minimize the number of periods in going from (W) to (A) will not satisfy BB. Imposition of BB does not preclude equilibrium in the sense of (A); it only means that it will take longer. But this time is not well spent because there is no technical or informational constraint underlying it. Additional time is required because individuals do not feel constrained to balance their budgets over a sequence of trades if they are not so compelled at each trade. Any device which would encourage such constraint could be substituted for the added time. Money is such a device.

I shall assume throughout that individuals know equilibrium exchange rates. Because we are accustomed to thinking of equilibrium in the sense of (W), this assumption may appear to be disquietingly strong. In general, it is; but not for the purposes of understanding monetary exchange. The line of reasoning adopted here permits me to assert that if we cannot find a role for money when

equilibrium prices are known, we shall not find one when they are unknown. There can hardly be a speculative demand for the medium of exchange without a transactions demand and this transactions demand does not depend on price uncertainty.

II. A Model of a Trading Economy

There are three components of the model: (i) the set of possible tastes and endowments and their corresponding competitive equilibrium allocations; (ii) the logistical description of the trading arrangement; and (iii) the pattern of information describing what individuals know and do not know at each trading opportunity.

A Family of Exchange Economies

Individual i is initially endowed with a non-negative quantity w_{ic} of commodity $c = 1,\ldots,m$. The complete list of his initial endowments is the m-vector $w_i = (w_{i1},\ldots,w_{im})$. If i, whose tastes are represented by $u_i(\cdot)$, were asked to exchange his initial endowment for any other (m-vector) x_i satisfying the constraint $p \cdot x_i = p \cdot w_i$, where $p = (p_1,\ldots,p_m)$ is the vector of prices, let his answer be the bundle $a_i = (a_{i1},\ldots,a_{im})$, $a_{ic} \geq 0$, $c = 1,\ldots,m$ and

(1) $$u_i(a_i) = \max u_i(x_i), \quad p \cdot a_i = p \cdot x_i = p \cdot w_i.^{5}$$

At prices p, let i be described by the pair of vectors (a_i, w_i) and let the collection of individuals $i = 1,\ldots,n$ be described by the pair of matrices (A,W), where a_i and w_i are the ith rows of the nxm matrices A and W, respectively. The pair (A,W) forms a competitive equilibrium (CE) if the aggregate demand for each commodity is equal to the aggregate supply,

(2) $$\sum_i a_{ic} = \sum_i w_{ic}, \quad c = 1,\ldots,m.$$

The matrix A <u>is the</u> CE <u>allocation</u> for the CE price vector p and matrix of initial endowments W. The problem of going from equilibrium in the sense of (W) to equilibrium in the sense of (A) is now reduced to the problem of going from the matrix W to the matrix A.

I shall deal only with collections of individuals described by non-negative matrices (A,W) satisfying

(3) $$n = m$$

and

(4) $$\sum_i a_{ic} = \sum_c a_{ic} = \sum_i w_{ic} = \sum_c w_{ic} = 1.$$

The set of all such pairs of nxn matrices whose row and column sums are unity, call $\mathcal{U}$. If (4) is to satisfy (1) for every member of $\mathcal{U}$, any $p = (r, r,...,r)$, $r > 0$, must be a CE price vector. This will mean that if i knows he is in an economy belonging to $\mathcal{U}$, he knows equilibrium exchange rates.

In the space of all possible economies, the set $\mathcal{U}$ occupies only a small corner. I choose to deal with it because $\mathcal{U}$ exhibits the salient features of the general case. The assumption, in (3), that the number of commodities is equal to the number of individuals is, strictly speaking, not essential to our results and can be shown to follow <u>a fortiori</u> if m > n. However, the trading arrangement I shall postulate allows each individual's trading opportunities to increase with the size of the population so that as n/m increases the logistical problems of exchange disappear.

The assumption that individual endowments are of the same size ($\sum_c w_{ic} = 1$, $i = 1,...,n$) is significant only insofar as it makes clear that there is no individual whose initial endowment is, for <u>all</u> (A,W) $\epsilon \mathcal{U}$, large enough for him to act as a central distributor supplying everyone else's excess demands.[6]

The set $\mathcal{U}$ exhibits a similar feature with respect to the insufficiency of endowments to permit a medium of exchange.

About the origin and use of money, Adam Smith said:

> In order to avoid the inconvenience of such situations, every
> prudent man in every period of society, after the first establish-
> ment of the division of labour, must naturally have endeavoured to
> manage his affairs in such a manner, as to have at all times by him
> besides the peculiar produce of his own industry, a certain quantity
> of some one commodity or other, such as he imagined few people would
> be likely to refuse in exchange for the produce of their industry. [p. 22]

The purpose of this paper is to isolate the function of a medium of exchange and I shall proceed by analyzing the difficulties when one does not have "a certain quantity of some one commodity." There is no commodity whose initial value is, for all $(A,W) \in \mathcal{U}$, a significant fraction of the value of each individual's planned purchases.[7] This will allow me to bring out more clearly that the essential feature of monetary exchange has its origin in the trading arrangement and not in the nature of the money commodity.[8]

The first result, which serves as an introduction to the problem, is

PROPOSITION 1: For almost all $(A,W) \in \mathcal{U}$, if the collection of n individuals is divided into any two groups consisting of k and n-k individuals, $1 \le k \le n - 1$, who cannot trade with each other, the CE allocation cannot be obtained.[9]

If $n > 2$, there is little hope of finding a double coincidence of wants. In fact, everyone will have to depend on everyone else if the CE allocation is to be realized.

How Traders Meet

I shall assume that the sequence of pair-wise meetings is parametric

rather than a subject for choice. From his point of view, each individual

seems to collide every so often with someone else.

Let $\pi = \{\pi^t\}$, $t = 1,\ldots,\tau$, be a sequence of permutations of the integers

$i = 1,\ldots,n$ such that for all i and t,

$$(5) \qquad \pi^t(i) = j \quad \text{if and only if} \quad \pi^t(j) = i.$$

The permutation π^t determines who meets whom at t — i.e., $\pi^t(i) = j$ means i and

j are trading partners at t. The final period, after which all trading

ceases, is $t = \tau$.

Let $\{W^t\}$, $t = 1,\ldots,\tau + 1$, be a sequence of matrices with non-negative

elements where $W^t = (w_{ic}^t)$ is the matrix of endowments at the beginning of t.

Suppose $\pi = \{\pi^t\}$ describes the sequence of meetings; then $\{W^t\}$ is technically

feasible for π, if for all i and t and $\pi^t(i) = j$,

$$(6) \qquad w_i^{t+1} + w_j^{t+1} = w_i^t + w_j^t.$$

This says that an individual can add to his current endowment only by sub-

tracting from the current endowment of his current trading partner and that

commodity totals are not changed in the process of trade, just redistributed.

If the sequence $\{W^t\}$ also satisfies

$$(7) \qquad p \cdot (w_i^{t+1} - w_i^t) = 0,$$

for all i and t, where p is the CE price vector, then bilateral balance (BB)

obtains. At every bilateral encounter, the value of what is given up is equal

to the value of what is received. Clearly, BB restricts the set of trades

beyond the demands of technical feasibility.

Let us agree to say that the CE allocation *is technically feasible for* π if, for all $(A,W) \in \Omega_1$, there exists a sequence $\{W^t\}$, technically feasible for π, with $W^1 = W$ and $W^{\tau+1} = A$. This means that the sequence π is not biased; it permits all possible CE configurations to be realized. Denote by Π_τ the set of all such π of length τ.

What is the minimum value of τ for which the CE allocation is technically feasible? Proposition 1 tells us that everyone must be "connected" to everyone else but this connection need not be direct since (6) permits indirect or middleman trade.

PROPOSITION 2: *If* $n = 2^k$, $k = 1, 2, \ldots$, *the minimum number of periods for which the CE allocation is technically feasible is* $\tau = k$ $(\tau = \log_2 n)$.[10] To demonstrate, note that it is true for $k = 1$, and assume it is true for $k = q$. This means that any group consisting of 2^q individuals can be connected in q periods so that two groups each consisting of 2^q individuals can be connected in period q+1 if every member of the one group is assigned to a member of the other. Since $2 \cdot 2^q = 2^{q+1}$, Proposition 2 is proved.

Call such a π which satisfies Proposition 2 an *indirect trading sequence* and a trading economy which makes use of it an *indirect exchange model*. It should be pointed out that

PROPOSITION 3: *If trades must satisfy BB, the CE allocation is not technically feasible for the indirect exchange model.*[11]

Suppose every individual is permitted to meet every other directly. Obviously, the CE allocation will be technically feasible. This will require $n(n-1)/2$ bilateral meetings and assuming n is even, will take, at a minimum, $\tau = n-1$ periods. Call such a sequence which allows everyone to meet everyone else in a minimum number of periods a *direct exchange sequence* and a trading economy which makes use of it a *direct exchange model*. An interesting feature

of this model is

PROPOSITION 4: *If trades must satisfy* BB, *the* CE *allocation is technically feasible for the direct exchange model.*[12]

Propositions 3 and 4 say that it takes longer to reach the CE allocation if BB is imposed. Alternatively, the temporal advantages of the indirect exchange model are incompatible with BB. This will not, by itself, offer a basis for monetary exchange. Other considerations intrude.

How Trades Are Made

I shall make a distinction between (a) the decision a pair of traders make based on the information they reveal to each other and (b) the decision as to what information to reveal. It will be assumed that (a) is taken out of the hands of the pair and given to a fictional third party, a broker, who makes the trading decision solely on the basis of what each member of the pair tells him. We may suppose that the broker uses his unlimited ingenuity and computational capacity to help the pair reach their CE allocations. His only constraint is that he knows no more than what the pair tells him. Until Section IV, the decision (b) will be ignored by assuming that there is no distinction between what individuals know and what they reveal.[13]

At the beginning of period t, those features of the economy which cannot be changed constitute the state of the economy. They are the initial configuration of tastes and endowments, the order in which pairs will meet, and the trades which have been made up to t. Denote a typical state by $\hat{S} = (\hat{A}, \hat{W}^1, \ldots, \hat{W}^t; \hat{\pi})$. The set of all possible states at t is

$$(8) \quad \hat{S}^t = \{\hat{S}^t : (\hat{A}, \hat{W}^1) \in \hat{\mathcal{U}} \text{ and } \{\hat{W}^k\}, k = 1, \ldots, t \text{ is technically feasible for some}$$

$$\hat{\pi} \in \Pi_\tau\}$$

Suppose the actual state is $S^t = (A, W^1, \ldots, W^t; \pi)$. Let $I_i^t(S^t)$ be the information i has at t about the actual state. It will be assumed throughout that

(9) $I_i^t(S^t) = \{\hat{S}^t \in \hat{}^t : \hat{a}_i = a_i, \hat{w}_i^k = w_i^k, \hat{\pi}^k(i) = \pi^k(i), k = 1, \ldots, t\}$.

If you were to ask i what he knows about the state of the economy, according to (9) he would say: "I have no idea. All I know is that it belongs to the set of possible states, and I can therefore tell you what CE prices are, and what I would want if I had to balance my budget at those prices, and that I have made certain trades as indicated by $\{w_i^k\}$, $k = 1, \ldots, t-1$, leading to my current position w_i^t."[14]

Each member of a pair of trading partners tells what he knows to the broker who then decides what trades they should make. The different trading pairs have different brokers who cannot communicate so the situation is much the same as if the pairs decided themselves what to trade.

As a formalization of this story, let $\rho_i^t(I_1^t(S^t), \ldots, I_n^t(S^t)) = w_i^{t+1}$, $i = 1, \ldots n$, be a trading rule, changing w_i^t into w_i^{t+1}, which depends on the actual state of the economy and what individuals know about it. I shall say that $\rho = \{\rho_i^t\}$ is an <u>informationally feasible trading rule</u> if for all i and t, and $\pi^t(i) = j$,

(10) $\rho_i^t + \rho_j^t = w_i^t + w_j^t$

and, for all $\hat{S}^t \in I_i^t(S^t) \cap I_j^t(S^t)$,

(11) $\rho_i^t(I_1^t(\hat{S}^t), \ldots, I_n^t(\hat{S}^t)) = \rho_i^t(I_1^t(S^t), \ldots, I_n(S^t))$.

Condition (10) says the rule must be technically feasible. If the pair (i,j)

were to share their information they could determine that the actual state was in the set $I_i^t \cap I_j^t$ and nothing more. Condition (11) says that the trading decision must respect this ignorance, which is to say that each pair's trading decision cannot be made contingent on the tastes and trading histories of other pairs.[15]

Once a trading rule is selected, the course of the economy is uniquely determined by its initial state. Given $S^1 = (A, W^1; \pi)$, (9) determines $\{I_i^1(S^1)\}$, the input into the trading rule $\{\rho_i^1\}$, which determines W^2 and therefore $S^2 = (A, W^1, W^2; \pi), \ldots$, etc. To summarize this recursive relation, let us say that if the initial state is $(A, W; \pi)$ and the trading rule is ρ, the end result is $g_i[\rho | (A, W; \pi)] = w_i^{\tau+1}$, $i = 1, \ldots, n$.

Now, the CE allocation is informationally feasible if there exists an informationally feasible trading rule $\rho = \{\rho_i^t\}$ such that for all $(A, W) \in \mathcal{U}$ and $\pi \in \Pi_\tau$, $g_i[\rho | (A, W; \pi)] = a_i$, $i = 1, \ldots, n$. To illustrate this definition, consider the first period trading decision for any pair (i,j). If the CE allocation is informationally feasible, their broker has a sure-fire method for putting them on a path leading to their CE allocation no matter what the values of a_k and w_k, $k \neq i,j$.

III. Informational Aspects of Trade

In this section, the consequences of informational feasibility are explored. First, we have

PROPOSITION 5: The CE allocation is not informationally feasible in the indirect exchange model.

To demonstrate, take $n = 4$. Let the trading partners be assigned as follows: individual 1's trading partners in the first and second periods are $\pi^1(1) = 2$ and $\pi^2(1) = 3$ while 4 is the partner of 3 in the first period and of 2 in the second - $\pi^1(3) = \pi^2(2) = 4$. It is readily verified that this

sequence allows each individual to trade directly or indirectly with everyone else. From Proposition 2, this is the minimum number of periods since $\tau = \log_2 4 = 2$. Assume that initial endowments are given by the identity matrix - i.e., $w_{ic} = 1$, if $i = c$, and $w_{ic} = 0$ if $i \neq c$, where $i,c = 1, 2, 3, 4$. This will simplify the demonstration, but it is not essential.

To go from W to A it is necessary that the exchange between individuals 1 and 2 be such that 1 begin the second period with $w_1^2 = (w_{11}^2, w_{12}^2, w_{13}^2, w_{14}^2)$ where

$$(12) \qquad w_{1c}^2 = \begin{cases} 1 - (a_{2c} + a_{4c}), & \text{if } c = 1 \\ (a_{1c} + a_{3c}), & \text{if } c = 2 \\ 0, & \text{otherwise.} \end{cases}$$

Individuals 1 and 2 do not know the tastes of 3 and 4, given by the vectors a_3 and a_4. According to (11), this means that for all possible values of a_3 and a_4, 1 and 2 must make the same trade. Now, whatever trade they make, they will have made the right decision for at most one pair (a_3, a_4) and will have made the wrong decision in all other cases. Therefore, the CE allocation is not informationally feasible.

Remark. The rule (12) is compatible with BB only if the configuration of tastes satisfies $a_{21} + a_{41} = a_{12} + a_{32}$ - i.e., almost never.

I have assumed that individuals know only equilibrium prices and their own tastes and endowments. They do not know each other's excess demands and trading decisions are bound by this ignorance. But, to take advantage of indirect exchange, individuals must act as middlemen passing excess supplies in just the right sequence of intermediary trading to the final demander. What is the right sequence depends on the entire configuration of excess demands as well as

on the order in which all pairs will meet. Proposition 5 brings out an obvious and basic point: the informational requirements for indirect trade go beyond a knowledge of prices.

Of course, the restrictions imposed on $\{I_i^t\}$ are rather harsh. I have taken the information available to an individual in the standard theory where he does not have to do his own trading and inquired as to its sufficiency where he does. From here, it would be possible to go on to find the minimum information compatible with the result that the CE allocation is informationally feasible in the indirect exchange model. It has already been determined from Propositions 3 and 4 that the required trades will not satisfy BB. Therefore, the argument could be made that only with money would the individuals reveal what they knew. I shall not follow this course because the informational demands for the indirect exchange model appear to be complicated and are certainly exhorbitant. The same argument can be more easily elaborated with the direct exchange model.[16]

PROPOSITION 6: If trades must satisfy BB, the CE allocation is not information-ally feasible in the direct exchange model.[17]

The reasons for Proposition 6 are similar to those underlying Proposition 5. In the class of economies $\mathcal{U}$, no commodity is in sufficient supply to serve merely as a balancing item; and it will not do to pay one's debts in just any commodity or commodities. If the CE allocation is to be achieved under BB, commodities used as payment by i in his trade with j must also be those which j can pass on to k,..., etc., so that they pass in just the right sequence and end up in just the right hands. But this involves the individuals in indirect trade whose informational demands they cannot meet.

Proposition 6 points to a feature of the medium of exchange distinguishing it from a standard I.O.U. When money is used, the parties to the transaction are admitting their inability to predict who and how the account will be settled.

Proposition 6 is important because of its relation to

PROPOSITION 7: **When BB is not imposed, the CE allocation is informationally feasible in the direct exchange model.**

The trading rule which demonstrates Proposition 6 is: for all i and t, $\pi^t(i) = j$, and $\rho_i^t = (\rho_{i1}^t, \ldots, \rho_{im}^t)$, let ρ_{ic}^t, $c = 1, \ldots, m$ be such that

$$(13) \quad \rho_{ic}^t = \begin{cases} w_{ic}^t + \min\left[|a_{ic} - w_{ic}^t|, |a_{jc} - w_{jc}^t|\right], & \text{if } (a_{ic} - w_{ic}^t) \geq 0 \text{ and } (a_{jc} - w_{jc}^t) < 0. \\[2mm] w_{ic}^t - \min\left[|a_{ic} - w_{ic}^t|, |a_{jc} - w_{jc}^t|\right], & \text{if } (a_{ic} - w_{ic}^t) < 0 \text{ and } (a_{jc} - w_{jc}^t) \geq 0. \\[2mm] w_{ic}^t, & \text{otherwise.} \end{cases}$$

The rule described by (13) is an example of what Ross Starr (1972) has called excess demand diminishing (EDD) trades. They follow the proscription "never engage in any trade which changes the sign of your excess demand." If you start out as a buyer of a commodity, do not accept more than you planned to purchase; and, if you start out as a seller of a commodity, do not give more than you planned to sell. The proscription is designed to prevent indirect or middleman trade.

It is clear that (13) satisfies the technical feasibility condition (6) and is informationally feasible since it requires only a knowledge of the trading pair's current excess demands, $(a_i - w_i^t)$ and $(a_j - w_j^t)$. It is a straightforward matter to show that (13) must always result in the CE allocation if everyone is able to meet everyone else.

The merits of EDD trades are substantial. In terms of information, they are extremely economical and they lead, in the direct exchange (but not the indirect exchange!) model, to the CE allocation. They have demerits as well.

1. **EDD trades do not satisfy BB.** This is to be expected in light of Proposition

6. For emphasis, I shall add that they almost never satisfy BB (if $n > 2$).
To illustrate, consider the case of $W = I$, the identity matrix. Then, for all
$(A, I) \in \lambda$, EDD trades satisfy BB if and only if A is symmetric.[18]

2. <u>EDD trades do not form a utility increasing sequence</u>. Whenever $p \cdot (w_i^{t+1} - w_i^t) < 0$,
so that sales exceed purchases, we must admit the possibility that $u_i(w_i^{t+1}) < u_i(w_i^t)$.
Similarly, whenever $p \cdot (w_i^{t+1} - w_i^t) > 0$, so that purchases exceed sales, we may have
$u_i(w_i^{t+1}) > u_i(w_i^t)$; and, if some of one's purchases are made before any of one's
sales, we may have $u_i(w_i^t) > u_i(a_i)$! This agrees with everyday experience. If
you did not have to pay for your purchases your utility would be above what it
otherwise is.

3. <u>EDD trades are unpredictable</u>. If an individual has a positive (negative)
excess demand at the start of t, he cannot tell how much of it will be fulfilled
(taken) during the period. To know this, he would have to know the entire
configuration of initial excess demands as well as who met whom before t. This
means that during the course of trade, an individual cannot determine whether
he is on a path leading to his CE allocation or some other point. Only at
$t = \tau + 1$ does he know where he ends up. Suppose i does not end up at a_i
because, to jump ahead, someone other than i misrepresented himself. Now, i
knows this was not his fault, but he cannot determine from his trading positions,
$w_i^1, w_i^2, \ldots, w_i^{\tau+1}$, who was responsible. He cannot, for example, surmise that if
$\pi^t(i) = j$, and $p \cdot (w_i^{t+1} - w_i^t) < 0$, that j overstated his demands. It may have been
that at period s, when $\pi^s(i) = k$, and $p \cdot (w_i^{s+1} - w_i^s) > 0$, that individual k under-
stated his supplies.

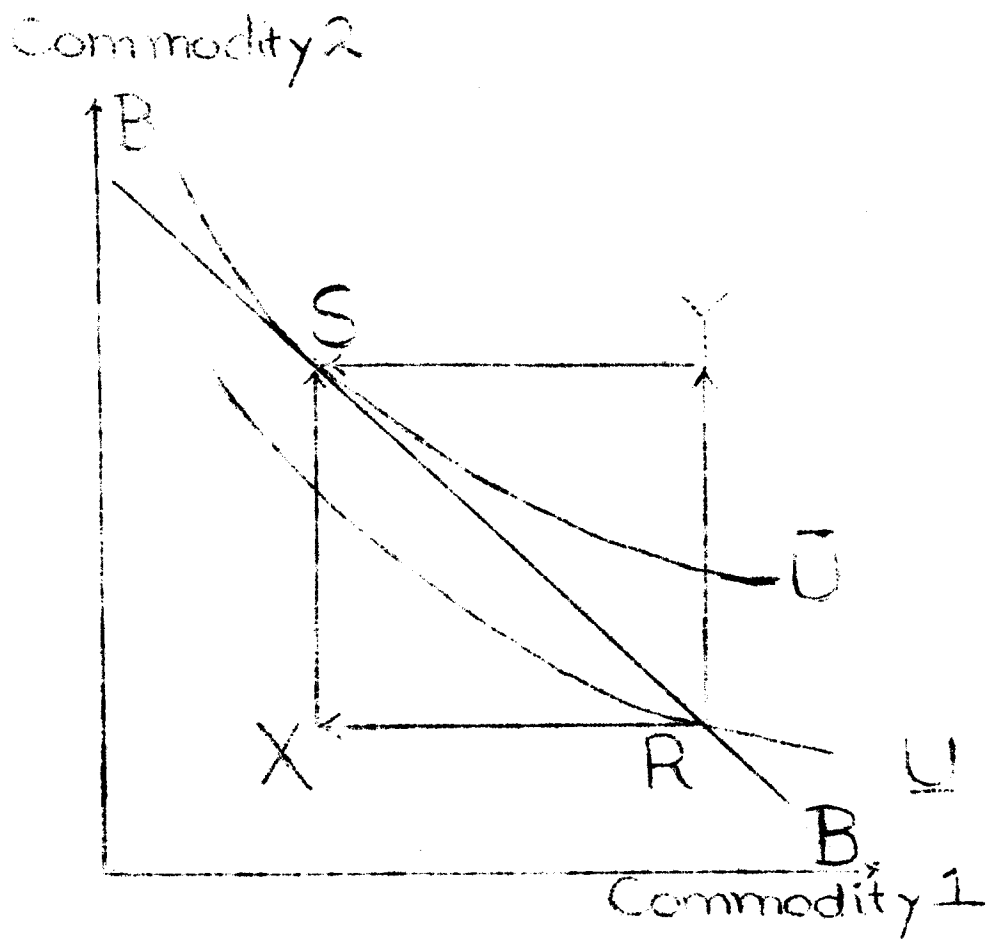The above three features of EDD trades are illustrated in the following
diagram.

FIGURE 1

The straight line, BB, is the budget line. It goes through the initial endowment, point R, and the CE allocation, point S. EDD trades begin and end on the budget line but in the interim they depart from it. The path R→X→S indicates a trading sequence in which sale preceded purchase while the path R→Y→S indicates the reverse. There is no guarantee that the intermediate positions, indicated here by the points X and Y, will lie within the region whose lower bound is $\underline{U}$, the indifference curve passing through the initial endowment, and whose upper bound is $\overline{U}$, the indifference curve passing through the CE allocation.

IV. Equilibrium

Recall that in defining the trading rule, $\rho = \{\rho_i^t\}$, it was assumed that individuals had no choice. Each revealed what he knew to the best of his knowledge. However, the trading rule does not require this accuracy. All that is required and all I shall assume is that no one says he has more of a commodity than he knows he has (he may say he has less).

Let $h_i^t(I_i^t) = \hat{I}_i^t$ be the information i conveys to the broker. It is his decision whether or not to misrepresent what he knows. A strategy for i is a system $h_i = \{h_i^t(I_i^t)\}$, detailing what he will say he knows given what he actually knows at each trading opportunity. Therefore, $\{h_i^t\}$ rather than $\{I_i^t\}$ will be the informational inputs determining the course of trade.

Suppose a trading rule $\rho = \{\rho_i^t\}$ and strategies h_i, $i = 1,\ldots,n$ have been selected. The outcome is uniquely determined by the initial state. If $S^1 = (A, W^1; \pi)$, this determines $\{I_i^1(S^1)\}$ and therefore $\{h_i^1(I_i^1)\}$, and then $\{\rho_i^1(h_1^1,\ldots,h_n^1)\}$ which determines W^2 and therefore $S^2 = (A, W^1, W^2; \pi),\ldots$, etc. We may summarize this recursive relation by saying that if the initial state is $(A, W; \pi)$ and the trading rule is given by $\rho = \{\rho_i^t\}$ and individual strategies

are given by $h_1, \ldots, h_n$, the end result is $g_i[\rho, h_1, \ldots, h_n | (A,W;\pi)] = w_i^{\tau+1}$, $i=1, \ldots, r$

Now, we may say that the CE allocation is an equilibrium if for all $(A,W) \varepsilon \, \mathcal{U}$ and $\pi \varepsilon \, \Pi_\tau$ there exists a trading rule $\rho^* = \{\rho_i^{*t}\}$ and strategies $h_1^*, \ldots, h_n^*$ such that for all $i = 1, \ldots, n$,

$$(14) \qquad g_i[\rho^*, h_1^*, \ldots, h_n^* \mid (A,W;\pi)] = a_i$$

and

$$(15) \qquad u_i(a_i) = \max_{h_i} u_i(g_i[\rho^*, h_1^*, \ldots, h_i, \ldots, h_n^* \mid (A,W;\pi)]).$$

The CE allocation is an equilibrium if the CE allocation is informationally feasible (14) and if it is in no individual's interest to depart from his strategy given the trading rule and strategies of the others (15).[19]

Underlying the question of the equilibrium of the CE allocation is the necessary condition of budget balance (BUB),

$$(16) \qquad p \cdot (w_i^{\tau+1} - w_i^1) = 0, \quad i = 1, \ldots, n.$$

Over the course of trade, if not at each trade, (16) says the purchases and sales balance.

Supposing individuals to have no choice so that they are compelled to satisfy BUB, it follows immediately from the definition of the CE allocation (see (1)) that

PROPOSITION 8: If the CE allocation is informationally feasible and if BUB is imposed, the CE allocation is an equilibrium.

If BUB were not imposed from outside, would individuals voluntarily choose strategies leading to it -- i.e., would they choose to reveal what they know?

If not, the CE allocation is not an equilibrium.

The only means of imposing BUB from within is through BB. Suppose that all $i \neq j$ give instructions to their brokers that they will refuse to partake in any trade for which $p \cdot (w_i^{t+1} - w_i^t) > 0$; then j cannot do other than satisfy BUB over the course of trade. It has already been determined (Proposition 6) that such a restriction precludes the informational feasibility of the CE allocation in the direct exchange model. But to give up BB is to give up the only means of imposing BUB and, therefore, to give up the incentive to reveal what one knows. The summary conclusion is

PROPOSITION 9: <u>The CE <u>allocation is not an equilibrium for the direct exchange</u> model.</u>[20]

Of course, if the individuals are willing and able to allow more time for trade, the opposite conclusion may be drawn. Consider the following "turn-taking" routine: Individual 1 goes to each of the others in turn asking for commodities to fulfill his (positive) excess demands and paying for them with his excess supplies so that BB is maintained. After $(n-1)$ periods he will reach his CE allocation. Next, 2 takes his turn with $3,\ldots,n$ in the same manner,$\ldots$, etc., so that after $n(n-1)/2$ periods the CE allocation is achieved.[21]

V. Money as a Record-Keeping Device

How to enforce BUB without imposing BB? Rather than ask how this enforcement is actually effected, I shall focus on the conditions which must precede enforcement. Again, it is a matter of information.

Consider the direct exchange model and assume that if an individual who has "over-balanced" his budget is found out, he will be made to give more than will put him back in balance; but if he is not found out, he may keep what he has. This represents a shortcut to the conclusions derived from a more extensive version (Ostroy (1971)) which supposed that individuals were in an economy

consisting of a large number of repetitions of the above direct exchange model.

At the completion of trade, we may ask i whether j balanced his budget.
Recall from the definition of I_i^{T+1}, what he knows at the completion of trade,
that i cannot say. Individual i knows only his own trading history and this
will not suffice to infer what j has done. However, he does know one fact
about j's trading history. At t, when $\pi^t(i) = j$, $p \cdot (w_j^{t+1} - w_j^t) = -p \cdot (w_i^{t+1} - w_i^t)$.
If the information possessed by all $i \neq j$ were added together, we could compute
$\sum_{t=1}^{t=T} p \cdot (w_j^{t+1} - w_j^t) = p \cdot (w_j^{T+1} - w_j^1)$. As it stands, however, this information is
scattered among the individuals with no one other than j, himself, able to
determine whether he has balanced his budget.

As monetary version of the model of a trading economy, introduce a
central receiving station called a monetary authority. Its function is to collect
and collate the bits of information individuals have about each others' trading
histories. Each will require his trading partner to write a signed statement,
a check, indicating the amount by which the partner's purchases exceed his sales.
This record is forwarded to the monetary authority who revises individual
accounts on the basis of this new information. <u>Sellers, by requiring payment in
money, are guaranteeing a steady flow of information such that the monetary
authority, and it alone, is able to monitor trading behavior.</u> Of course, there
is every incentive to require and deposit this information with the monetary
authority; otherwise, one would not receive credit for sales and so have to
cut back on purchases. Therefore, in the monetary version of the direct exchange
model of a trading economy, if individuals evaluate commodities by their CE
prices the CE allocation is an equilibrium.

There is a small slip. We have seen that for any $(A,W) \in \mathcal{U}$, any vector
$p = (r,r,\dots,r)$, $r > 0$, is a CE price vector. If the monetary authority is
to be able to make trades between different individuals commeasurable, they
must all agree to the same value of r — i.e., we require a common unit of account.

While this convention is essential to the operation of the record-keeping system, it is not identical to it. Money is not simply a unit of account.[22]

VI. Conclusion: Integrating Monetary and Value Theory

In the Walras-Hicks-Patinkin tradition, the goal of monetary theorists has been to present a picture of a money economy which would be a logical extension of the standard theory of value. Walras brought the equation of the offer and demand for money into line with the rest of his system by making a distinction between the stock of money, assumed to be without any utility of its own, and the "services of availability" of the stock which does contribute to one's well-being. Just as no inquiry is made into the sources of satisfaction from other goods, the services of availability are similarly unquestioned.

Recently, several theorists have suggested that it might be useful to dig a little more deeply.[23] While the traditional approach could be readily applied to determine why individuals might hold more or less money given that they valued it in certain ways in the first place, perhaps, just perhaps, it might clarify some contemporary monetary muddles if we asked why money is held at all.

This poses a dilemma. How to make money appear without making the standard theory disappear? Normal research strategy says that for a theory to be complete and consistent it must be derivable from the standard theory. But the standard theory has been cultivated to its present high level as a model of exchange in which money does not appear. Unlike the Walras-Hicks-Patinkin approach which left the standard theory intact and relied on conceptual appendages to introduce money, the recent approach forces us to look for modifications within the body of the theory. The following are some suppositions as to where to look and on which I have not relied. I shall argue that they are superseded by the conclusions obtained from the model of a trading economy.

1. <u>Money enlarges the set of feasible transactions.</u>[24] In the standard theory,
any redistribution of commodities which preserves their totals is feasible.
Into this model we can introduce the problems of exchange as a kind of transport
cost of getting from one bundle of goods to another. We may then reason that
monetary exchange represents a least cost network, so that without the money
commodity the set of feasible transactions must shrink. While this may be
adequate metaphorically, it misses the point. It is fairly well-established
that the term "feasible" denotes what <u>could</u> happen, ignoring individual behavior,
not what <u>would</u> happen. Monetary exchange does not enlarge the set of feasible
transactions; it merely enables trades, which must be feasible in the first
place, to be realized.

2. <u>Money is held because we do not know what prices will be.</u>[25] Price
uncertainty is neither necessary nor sufficient to explain the presence of a
medium of exchange. The model of a trading economy assumed that exchange
rates were known; yet it required a record-keeping device. Suppose, however,
that exchange rates were unknown but that individuals voluntarily agreed to
keep accurate records of their transactions (in terms of a common unit of account)
in order to balance their budgets. There would be no need for a medium of exchange.

3. <u>The advantages of money have their origin in the properties of the money</u>
<u>commodity.</u>[27] In the model of a trading economy, all commodities are perfectly
portable, durable, divisible, and recognizable, yet there is a need for money.
The origin of this need is the decentralized trading arrangement. I chose to
introduce a monetary authority and bookkeeping entries as a kind of ideal
monetary arrangement because the record-keeping function of money is conceptually
distinct from the properties of the commodities traded. Of course, to understand
a particular monetary arrangement, it becomes a matter of recognizing a minimum
cost method of imposing budget balance and in a society unfamiliar with double
entry bookkeeping, the monetary version of the model of a trading economy would

not be ideal. Then, bilateral balance might be the only means of insuring that individuals keep accurate records and balance their accounts and we would have to look for a minimum cost method of imposing bilateral balance. In such a situation, the principle would not change but the practice might well be to choose as a method of enforcing budget balance a commodity which is most portable, durable, divisible, and recognizable.

To the standard theory of value, the phenomenon of monetary exchange is surprising and distressing; surprising because the phenomenon is inexplicable and distressing because the phenomenon would seem to be one of the most elemental conclusions to be derived from any theory of exchange. Once we give up the standard theory framework which allows the execution of exchange to be the province of a centralized agency and concentrate on the logistics of more disaggregated trading arrangements, monetary exchange becomes explicable as a matter of course. It follows that these logistical considerations are worthy of attention by general equilibrium theorists.[28]

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FOOTNOTES

*   Assistant Professor of Economics, University of California, Los Angeles. I have benefited from discussions with Ronald Britto, Jack Hirschleifer, Ben Klein, Axel Leijonhufvud, Louis Makowski, Richard Sweeney, Earl Thompson, and Joseph Wharton. Bryan Ellickson, Peter Howitt, and Ross Starr made many improvements in the formulation and presentation. My introduction to the topic came through the research of Robert Clower whose continued advice and encouragement were invaluable.

1   By this phrase I mean the model of general equilibrium first proposed by Walras and recast into its definite mathematical form by such contributors as Arrow and Debreu.

2   Recently, formal notice has been taken of the fact that the valuable exchange services rendered by the auctioneer are costly to provide. Much of the motivation for these studies has been a desire to fit monetary exchange into the standard theory. We may learn from some of them that costly exchange can be introduced without giving up the assumption that exchange is coordinated by a central agency -- an auctioneer who charges for his services. See Hahn (1971), Kurz, Niehans (1971), Sontheimer and Wallace. According to the present treatment, however, it is only when the exchange process is decentralized that the role of money can be understood. See Clower (1971).

We may learn from Niehans (1969) that even when exchange is restricted to pairs it need not be completely decentralized. The selection of a least cost bilateral trading network can be made by a central planner who solves a complicated programming problem. A similar difficulty occurs in Starr (1970) where individuals choose optimal _sets_ of bilateral transactions but require a central agency to hook them together.

Radner and Brunner and Meltzer have approached monetary exchange as a reflection of imperfect information. Radner has suggested that money might arise

from the unpredictability of future spot prices and Brunner and Meltzer
have indicated that money arises because of the need for a commonly recognizable
asset. I shall discuss these points in Section VI.

The present treatment is related to the work of Hicks, Starr (1972), and
Veendorp.

3   It is this simultaneity - while one pair is exchanging other pairs are
not standing still - which contributes to the informational demands on trade.

4   It would have been possible to formulate a model in which the costs of
exchange varied with the amounts of commodities and the number of individuals
with whom one traded per unit time. Ignoring the resulting complexities, the
outcome must concede that not everyone exchanges everything at once if it is
to gain insight into monetary exchange.

5    $p \cdot w_i \equiv \Sigma_c p_c w_{ic}$ is the dot product of p and w_i.

6   For example, individual j could be a central distributor for (A,W) if
$w_{jc} > \delta$, $c = 1,\ldots,m$, and if $\max|a_{ic} - w_{ic}|$ over all c and $i \neq j$ were less than δ/n.

7   For example, commodity d could be used as a medium of exchange for (A,W)
if $w_{id} > \delta$, $i = 1,\ldots,n$, and if $\max|a_{ic} - w_{ic}|$ over all i and $c \neq d$ were less
than δ/m.

8   These qualifications as well as their consequences are more fully discussed
in Ostroy and Starr.

9   Proof: Suppose the contrary; then it would require that for some subset,
T, consisting of fewer than n individuals, $\Sigma_{i \in T}(a_i - w_i) = 0$. This defines a
less than full-dimensional class of economies in $\mathcal{U}$.

10   The following was obtained in correspondence with L. Shapley: To complete
the solution to minimizing the number of time periods $\tau = (\log_2 n) + 1$ if
n is even and not a power of 2 and $\tau = (\log_2 n) + 2$ if n is odd.

11   Proposition 3 holds for all $n \geq 4$, the smallest number for which the

advantages of indirect trade appear. See the Remark following Proposition 5 for a demonstration in the case n = 4.

12   Proofs of Proposition 4 are in Ostroy and Starr and Bradley. I conjecture that the direct exchange sequence with $\tau = n - 1$ may be the fewest number of periods for which the CE allocation is technically feasible under BB.

13   Getting individuals to reveal what they know, e.g., their tastes, has been recognized as the principal difficulty in allocating collective goods (Samuelson). We shall see that there are similar strategic issues in a barter economy.

14   This is the game-theoretic method for describing imperfect information. Harsanyi has shown how this may be applied to the case of players in a game who do not know each others' payoffs. The treatment above was developed independently in Ostroy (1970).

15   The less one knows the smaller is his set of possible strategies. See Radner for a restriction similar to (11).

16   The indirect exchange sequence, by explicitly denying the informational sufficiency of prices, provides the kind of environment hospitable to the activities of specialists in exchange. See Ostroy (1970). All of this is of a piece with money. Nevertheless, when our interest is just monetary exchange, we may use the direct exchange model to isolate the essential aspects of the problem in the context of a neater solution.

17   Proofs of Proposition 6 follow along the lines of the demonstration of Proposition 5. Clearly, for n = 1 and 2, it is false. It is also false for n = 3. The reason is that once any two information sets $\{I_i^t\}$, i = 1,2,3, are known the other may be inferred so there is effectively perfect information. For n = 4, it holds only if individuals do not know the number of the period in which they are trading (see Ostroy and Starr). Bradley has shown that for n=5,

Proposition 6 holds with the pattern of information assumed here.

18 Applying (13) when $W = I$, we have that for any i and $\pi^t(i) = j$,

$$\rho_{ic}^t = \begin{cases} w_{ic}^t + a_{ic}, & \text{if } j = c \\ w_{ic}^t - a_{jc}, & \text{if } i = c \\ o, & \text{otherwise.} \end{cases}$$

Therefore, when $p = (r, r, \ldots, r)$, $r > 0$, $p \cdot (w_i^{t+1} - w_i^t) = 0$ if and only if $a_{ij} = a_{ji}$.

19 Condition (15) is the definition of a non-cooperative equilibrium proposed by Nash.

20 This result does not hold when $n \leq 4$ (see fn. 17, above).

21 This is an upper bound and not necessarily a minimum estimate of the number of periods required to make the CE allocation informationally feasible with BB. This result demonstrates that to achieve the same end additional time can be substituted for lack of information. The trade-off may be pushed further. Feldman has shown that when individuals know nothing but their own tastes and endowments but have an unlimited number of bilateral trading opportunities, if they accept only utility-increasing trades they will, under certain assumptions, eventually reach a Pareto-optimal allocation.

22 I have avoided use of the term "transactions costs" because its meaning varies from writer to writer. However, the present treatment seems consistent with the usage of my colleague, Earl Thompson, who defines transactions costs as those losses arising from differences in information. With his definition, we may say: Money reduces transactions costs.

23 Cf. fn. 2. That we recognize there is a problem at all is due largely to the important papers of Patinkin, Marschak, Hahn (1965), and Clower (1967).

24 See Hahn (1971), Kurz, Niehans (1971) and Sontheimer.

25 See Radner.

26   When individuals are groping for equilibrium exchange rates in a world where such voluntary restraint is lacking, monetary exchange would be essential. Imagine how much more difficult would be the approach to equilibrium if payment for one commodity were made in an arbitrary collection of other commodities. On this issue of price dynamics, I have benefited from reading Peter Howitt's study of stability in a decentralized regime of monetary exchange.

27   See Brunner and Meltzer

28   Any reader familiar with the work of Stigler and the contributors to Phelps et. al. will recognize that these logistical considerations have already received some attention as determinants of search unemployment in labor markets.