#32

SPECULATION AND INFORMATION IN SECURITIES MARKETS

by

J. HIRSHLEIFER
Economics Dept., UCLA

MARK E. RUBINSTEIN
School of Business Administration, UCB

UCLA Economics Dept.
Discussion Paper Number 32

Revised Jan. 1973

SPECULATION AND INFORMATION IN
SECURITIES MARKETS

J. Hirshleifer
Mark E. Rubinstein

ABSTRACT

This paper studies the allocation of consumption income over states of the world, where it is anticipated that information will emerge so as to modify prior probability beliefs. Two regimes of markets are considered: (1) STATE-COMPLETE MARKETS, with S tradable contingent claims (securities) corresponding to the S states, and with two rounds of trading--one prior to, one posterior to the emergence of information. (2) INFORMATION-STATE-COMPLETE MARKETS, with a single round of trading but SE securities spanning the S states and E possible information-events. Solutions under the two market regimes are equivalent.

"Speculation" corresponds, under STATE-COMPLETE MARKETS, to planning for portfolio revision upon emergence of information. However, this portfolio-revision definition of speculation is obviously inapplicable for the single trading round under INFORMATION-STATE-COMPLETE MARKETS. The corresponding concept, constituting a generalized definition of speculation, is that a nonspeculator is one who uses available markets to achieve identical contingent consumptions over information-events (but not in general over states).

The necessary and sufficient condition for nonspeculative behavior, following from this definition, is independent of endowments and preferences and depends only on the relationship between the individual's beliefs and the market prices of securities. Moreover, complete agreement of beliefs is not required among individuals who do not speculate; the extent of agreement necessary is specified in the paper.

J. Hirshleifer
Mark E. Rubinstein

January 1973

SPECULATION AND INFORMATION IN SECURITIES MARKETS

I. INTRODUCTION

The nature and consequences of speculation in securities markets constitute problems for both positive and normative analysis. On the normative level, debates as to the desirability of "speculation" as contrasted with "investment" in securities remain largely unresolved. While in part motivated by such policy issues, the specific aim of this paper is limited to building a foundation of better understanding at the positive level of analysis.

It will turn out in what follows that speculative behavior is closely associated with: (1) beliefs about the likelihoods of alternative states of the world, but only in association with beliefs as to the content of emergent information bearing upon those likelihoods; and (2) the nature and extent of markets provided by the economy for adjusting portfolios in the light of beliefs.

The present paper is complementary with a piece by one of the authors[1] that was concerned to explain the nature of speculative versus "hedging" decisions in the face of uncertainty about relative price changes between more and less risky commodities. In this paper we deal instead with only a single commodity representing consumption income; claims to that commodity may however be contingent upon a given state of the world or a given information-event (or both). Each such type of claim will be called a <u>security</u>. The model here is also timeless, except for a distinction between decisions <u>before</u> and <u>after</u> an essentially instantaneous emergence or injection of information--such information tending to modify probability beliefs and, consequently, prices.

II. STATE-COMPLETE MARKETS

Consider a world of pure exchange. Each individual is concerned to balance his holdings of securities in the form of elementary state-claims yielding a unit of consumption income if and only if the associated state of the world obtains. There are S states and thus, under the assumption here of State-Complete Markets, S securities tradable against each other. The individual's decision problem is to exchange his endowment vector into a preferred combination of securities that will yield him consumption incomes over the various states of the world.

In a <u>non-informative</u> situation the individual does not anticipate that new information will emerge so as to change prices before the finalizing of his plans for consumption. Thus he would take account of only a single round of trading, in which he moves directly to his optimal position by maximizing utility subject to his wealth constraint:

$$(1) \quad \underset{\{Z_s\}}{\text{Max}} \; \Sigma_s \Pi_s u(Z_s) - \lambda[\Sigma_s P_s Z_s - \Sigma_s P_s Z_s^+]$$

Here Z_s is his holding of the state-s security, where s=1,...,S; $u(Z_s)$ is the "cardinal" preference-scaling function defined in the usual way; Π_s is the subjective probability attached to state-s; P_s is the price of the state-s security; and Z_s^+ is the endowed quantity thereof.

We are not concerned here with this familiar[2/] problem, however. Rather, we seek to analyze the effect of <u>the anticipation of emergent information</u> upon trading and consumption decisions. More specifically, we will assume that any individual may exchange first to an interim or trading position $\{Z_s^o\}$ -- in what will be called the "prior round" of trading that takes place before the anticipated emergence of information. This prior trading determines and takes place at prior prices P_s^o. While individuals are at their trading positions there will then occur some one of the information-events e=1,...,E having the effect of changing individuals' beliefs as to the likelihoods of the different states s=1,...S.[3/] Markets will then reopen for a "posterior round" of trading in which individuals will move to their final consumptive positions $\{Z_{s.e}\}$ in the light of their revised beliefs $\Pi_{s.e}$. In the posterior round the equilibrium prices consequent upon information-event e will be denoted $P_{s.e}$.

The individual's decision problem may be expressed as:

$$(2)\quad \underset{\{Z_s^o\}}{\text{Max}}\ \Sigma_e \Pi_e \ \{ \underset{\{Z_{s.e}\}}{\text{Max}}\ \Sigma_s \Pi_{s.e}\, u(Z_{s.e}) - \lambda_e [\Sigma_s P_{s.e} Z_{s.e} - \Sigma_s P_{s.e} Z_s^o] \} - \lambda_o [\Sigma_s P_s^o Z_s^o - \Sigma_s P_s^o Z_s^+]$$

In order for the individual to choose the optimal Z_s^o in the prior round of trading he must have first solved the conditional posterior optimization problems within the large braces in (2). But this requires that he already know, in the prior round, the posterior equilibrium prices $P_{s.e}$ that will not be determined until the posterior round! We will specify below a set of circumstances under which these posterior prices can indeed be computed from data already available in the prior round of trading. For the moment, let us assume that these circumstances apply. Then the first-order optimality conditions fall into groups as indicated below:

$$(3a)\quad \Pi_{s.e}\, u'(Z_{s.e}) = \lambda_e P_{s.e} \qquad \text{(SE eqs)}$$

$$(3b)\quad \Sigma_s P_{s.e} Z_{s.e} = \Sigma_s P_{s.e} Z_s^o \qquad \text{(E eqs)}$$

(3c) $\Sigma_s P^o_s Z_s = \Sigma_s P^o_s Z^+_s$ (1 eq)

(3d) $\Sigma_e \Pi_e \lambda_e P_{s.e} = \lambda_o P^o_s$ (S eqs)

It is evident from the structure of the equations that the individual's decisions will depend only upon the conditional posterior state-claim price <u>ratios</u> $P_{s'.e}/P_{s.e}$ -- where s' and s signify two different states of the world -- together with the prior state-claim price <u>ratios</u> $P^o_{s'}/P^o_s$. For example, from (3a) we obtain:

(4) $$\frac{\Pi_{s'.e}\ u'(Z_{s'.e})}{\Pi_{s.e}\ u'(Z_{s.e})} = \frac{P_{s'.e}}{P_{s.e}}$$

And conversely, the interaction of all individuals' decisions will determine equilibrium values in the prior and posterior markets only of the respective price <u>ratios</u>, the absolute prices retaining a degree of freedom in each case.

Let us tentatively define an <u>investor</u> (i.e., a <u>nonspeculator</u>) as someone who, in the light of the information currently and prospectively available to him, proposes to move directly in the prior round to his final risky consumptive optimum. That is, he regards it as optimal to select a portfolio in initial trading that is so balanced as not to require him to take advantage of (and so does not expose him to any risk involving) the price changes that will ensue upon emergence of the new information. This may be called the "no portfolio revision" concept of nonspeculation.

For whom is it optimal to behave as an investor rather than speculator? We can determine this from the condition (implied by the definition above) that $Z_{s.e} = Z^o_s$ regardless of e, for each s.

Using this property in (3a), we can multiply each side by Π_e and then sum both sides over e to obtain for the nonspeculator:

(5) $$u'(Z_s^o)\,\Sigma_e \Pi_e \Pi_{s.e} = \Sigma_e \Pi_e \lambda_e P_{s.e}$$

The probability summation on the LHS is simply the unconditional probability Π_s, while the RHS is seen in (3d) to equal $\lambda_o P_s^o$. Thus:

(6) $$\Pi_s\, u'(Z_s^o) = \lambda_o P_s^o$$

Then for any two states s and s':

(7) $$\frac{\Pi_{s'}\, u'(Z_{s'}^o)}{\Pi_s\, u'(Z_s^o)} = \frac{P_{s'}^o}{P_s^o}$$

Equation (7) together with (4) -- which holds for both speculators and investors -- imply as a necessary condition for nonspeculation:

(8) $$\frac{\Pi_{s'.e}/\Pi_{s'}}{\Pi_{s.e}/\Pi_s} = \frac{P_{s'.e}/P_s^o}{P_{s.e}/P_s^o}$$

If the u function is assumed concave (risk aversion), this condition is also sufficient. In words: An individual will be a nonspeculator if and only if, for any information-event, the ratio of posterior to prior state probabilities is proportional over states to the ratio of posterior to prior state-claim prices. Note the absence of parameters associated with endowments or preference functions in the no-revision condition (8). The decision to speculate depends <u>only</u> on the relationship between beliefs and prices.[4/]

We now can face the question by-passed earlier: Under what conditions can the <u>posterior</u> price ratio be computed using only the public data available in the <u>prior</u> round of trading? In general, of course, such computation could not be made by any individual without knowing very detailed private data about other individuals.[5/] However, (8) indicates that using only knowledge of the beliefs of individuals who hold no-revision portfolios, it is possible to infer posterior prices.

In the extreme case of a world of representative individuals, identical in all respects, portfolio revision would be impossible and so (8) would necessarily apply for all. But we need not impose so severe a condition. For, if despite varying endowments and preferences it remains true that probability beliefs are identical ("concordant"), then individuals will still hold no-revision portfolios and (8) will continue to apply for all[6/]. Moreover, there may even be some patterns of discordant beliefs in which disparities in prior and posterior probability estimates compensate for each other so that (8) can continue to hold.[7/]

Suppose there were an "almost-concordant" world in which there exist one or more persons who are belief-deviant -- but of negligible social weight in determining prices. The deviant individuals would not find (8) holding for their own decisions, and indeed would be engaging in portfolio revision (speculation). But they could still use (8) to forecast prospective price changes (assuming they were aware of the concordant probabilities), thus leading to optimum solutions as in (3) or (4) above.

As a practical matter, we would want to interpret <u>average</u> beliefs in the market as if they were the <u>concordant</u> beliefs required by our theoretical development. This is not strictly legitimate, evidently. Yet condition (8) would surely hold to a good degree of approximation if interpreted in terms of average beliefs. One difference is that essentially everyone might then become "belief-deviant" and therefore a speculator, i.e., portfolio revision would then be indicated by conditions (3). Again, as a practical matter, transaction costs would set some check on portfolio revision wherever deviation from average beliefs remains relatively small.

III. INFORMATION-STATE-COMPLETE MARKETS

Up to this point the "no portfolio revision" concept of speculation has been employed. But this is not ultimately satisfactory, since whether or not revision is needed to achieve desired final portfolios is a function of the nature and extent of markets provided. In particular, it has been shown elsewhere that if there are fewer markets than states of the world, portfolio revision will in general be required even for nonspeculative behavior.[8/] Here we will take the opposite tack, showing that under certain market regimes providing more markets than states of the world there can be speculative behavior even without portfolio revision. This development leads to a more general and fundamental interpretation of the nature of speculation.

In the regime of markets to be specifically considered here, an elementary security is a claim to a unit of income conditional upon the advent of both state of the world s and information-event e. All the securities are separately tradable at prices P_{se}.

The individual's decision problem takes the form:

$$(9) \qquad \underset{\{Z_{se}\}}{\text{Max}}\ \Sigma_e \Pi_e [\Sigma_s \Pi_{s.e} u(Z_{se})] - \lambda[\Sigma_{s,e} P_{se} Z_{se} - \Sigma_{s,e} P_{se} Z^+_{se}]$$

The first-order optimality conditions are:

$$(10a) \qquad \Pi_{se}\ u'(Z_{se}) = \lambda P_{se} \qquad \text{(SE eqs)}$$

$$(10b) \qquad \Sigma_{s,e} P_{se} Z_{se} = \Sigma_{s,e} P_{se} Z^+_{se} \qquad \text{(1 eq)}$$

The probability in (10a) is of course the joint probability $\Pi_{se} = \Pi_e \Pi_{s.e}$ that both state s and event e obtain.

Again, for whom is it optimal to behave as an investor rather than speculator? Let us consider an individual whose optimal portfolio does not vary over information-events. Then $Z_{se} = Z_s$ regardless of e. It follows from (10a) and the law of corresponding addition that

$$(11) \qquad \frac{\Pi_{se}}{P_{se}} = \frac{\Pi_{se'}}{P_{se'}} = \ldots = \frac{\Sigma_e \Pi_{se}}{\Sigma_e P_{se}} .$$

From probability theory we have $\Pi_s = \Sigma_e \Pi_{se}$, while economic logic tells us that the price P_s representing the price of a unit claim to income conditional only on s can be regarded as the sum over e of prices of unit claims conditional upon both e and s. (If not, profitable arbitrage would be possible.) Consequently:

$$(12) \qquad \frac{\Pi_{se}}{\Pi_s} = \frac{P_{se}}{P_s}$$

for all s and e.

To determine a definition and condition for speculation which are independent of the market regime, for purposes of comparison with State-complete markets, assume that individuals confront consistent security prices under the two regimes. Since arbitrage would force the price of a unit of consumption conditional upon both state s and information-event e (P_{se}) to equal the product of the price of a unit of consumption conditional only upon information-event e (P_s) and the price of a unit of consumption conditional upon state s given the occurrence of information-event e ($P_{s.e}$), then $P_{se} = P_e P_{s.e}$. Using this and the corresponding property, $\Pi_{se} = \Pi_s \Pi_{s.e}$, (8) can be reformulated as :

$$(13)\qquad \frac{\Pi_{se}/\Pi_s}{\Pi_{s'e}/\Pi_{s'}} = \frac{P_{se}/P_s}{P_{s'e}/P_{s'}}$$

for any two states s and s'. Since $\Pi_s = \Sigma_e \Pi_{se}$, $\Pi_{s'} = \Sigma_e \Pi_{s'e}$, $P_s = \Sigma_e P_{se}$ and $P_{s'} = \Sigma_e P_{s'e}$, it is not difficult to prove that the numerators and denominators are separately equal. Therefore, conditions (8) and (12) are identical.

IV. CONCLUSION

We have compared two alternative market regimes: (a) STATE-COMPLETE MARKETS, in an informative situation (i.e., with two rounds of trading permitting revision of portfolios after emergence of information); (b) INFORMATION-STATE-COMPLETE MARKETS, with only a single round of trading, but where trading in claims contingent upon both state of the world and information-event can take place. Under State-complete markets, only individuals whose beliefs satisfy equation (8) will not speculate, in the sense that they will choose a prior-round trading position that does not require them to revise their portfolios upon the emergence of the information. Individuals with other beliefs will adopt a speculative position, requiring portfolio revision.

But under Information-state-complete markets no-one ever revises portfolios.[9] What then is the concept corresponding to the no-revision condition applicable under State-complete markets? The answer, the generalized definition of speculation sought for in this paper, can be put as follows: An investor (i.e., nonspeculator) uses the available markets to achieve identical holdings (representing contingent consumption possibilities) over information events, i.e., for him $Z_{s.e} = Z_{s.e'}$ for any two information-events e and e', for all states s. We emphasize that his planned consumption may and generally will vary over states s. But for any given state he does not plan to be better off under any one information outcome as against any other -- which is what we ultimately mean in saying that he is not speculating upon the information-event.

As for the circumstances leading to such behavior, the degree of risk-aversion and the scale or composition of endowments might plausibly be assumed to be involved. However, interpreting the LHS of (12) as a conditional posterior belief, $\Pi_{e.s}$, and the RHS similarly as a conditional posterior price,[10/] our results show: An individual will be an investor (i.e., nonspeculator) if and only if his conditional posterior beliefs are equal to the corresponding conditional posterior prices. This leaves open the possibility that two individuals may both be nonspeculators even though they have different beliefs; only their conditional posterior beliefs must be identical.

FOOTNOTES

1. Hirshleifer [3].

2. See Arrow [1] and Drèze [2].

3. Note that "states of the world" are defined here in terms of the configuration of events other than information-events. This usage may diverge from that of other authors, for example Radner [6], who define states in terms of a complete description of the world environment.

4. The same conclusion was arrived at in the models studied in Hirshleifer [3].

5. Drèze [2, pp. 144-45] makes a similar objection to Arrow's [1, pp. 92-94] securities model. See also Myers [5, pp. 25-29] and scattered comments in Radner [6]. Keynes [4, p. 156] held the extreme view that successful investing (i.e., speculating) derives not from the ability to forecast states and information-events but from the ability to forecast the decisions of other participants in the market.

6. This equilibrium will be unique if u is strictly concave and $S \geq E$. If $E > S$, in the prior round of trading the individual will be indifferent among a number of optimal portfolios -- one of which will be a no-revision portfolio.

7. It may be reasonable to assume, as an implication of the emergence of information, that there will be a higher degree of posterior than of prior unanimity. With absolute posterior unanimity, (8) could only hold if there were also prior concordance.

8. Hirshleifer [3, pp. 32-46].

9. Indeed, even if a posterior trading round were available it would be redundant. No posterior trading would occur.

10. This is defined of course, as a price <u>ratio</u>.

REFERENCES

1. Arrow, K. J. "The Role of Securities in the Optimal Allocation of Risk-Bearing," Review of Economic Studies, April, 1964.

2. Drèze, J. H. "Market Allocation under Uncertainty," European Economic Review, Winter, 1970-1971.

3. Hirshleifer, J. "Foundations of the Theory of Speculation: Information, Risk, and Markets" Working Paper #189, Western Management Science Institute, UCLA, June, 1972.

4. Keynes, J. M. The General Theory of Employment, Interest and Money, Harcourt, Brace and Company: New York, 1936.

5. Myers, S. C. "A Time-State-Preference Model of Security Valuation," Journal of Financial and Quantitative Analysis, March, 1968.

6. Radner, R. "Competitive Equilibrium under Uncertainty," Econometrica, January, 1968.