

TAXATION AND NATIONAL DEFENSE

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In the traditional theory of taxation, the government provides collective consumer or producer goods. Because governmental provision of such goods does not alter any of the familiar necessary conditions for the efficient utilization of the resources remaining for private use, efficient taxation when the private-goods-sector is perfectly competitive is achievable only with lump-sum taxes (such as head taxes, land taxes, or equal, ad valorem taxes on all consumer benefits). Any tax other than a lump-sum tax yields violations of the familiar necessary conditions for achieving Pareto optimality given the resources remaining for private use.

In the real world, we observe that lump-sum taxes in the United States have been consistently rejected in favor of non-lump-sum taxes (which are relatively costly to administer and cannot be plausibly rationalized as user or sumptuary taxes) despite the obiter dictum of traditional economic theory.

This paper is a development of a theory of taxation which is based upon a view of the world in which the government provides collective defense of the property of its citizens. The traditional model is apparently attempting to describe the same world. But it is an error to describe collective defense as a collective consumer or producer good. A tank is neither a consumer nor a producer good. Efficient taxation in an appropriate model of the nature of collective defense implies non-lump-sum taxes.

In section I, a model of government expenditures and efficient taxation is developed in which the private accumulation of certain kinds of capital creates an extra defense burden for the country protecting that capital. In such a model, efficient taxes discriminate against the accumulation of capital that is coveted by potential foreign aggressors.

Section II develops a model in which the optimal capital tax is achieved with a tax solely on produced outputs by the use of a simple income tax complemented by: (1) realistic depreciation allowances, (2) tax write-offs

for charity and abnormal, non-cosmetic medical expenses, (3) no write-offs on income taxes of education, relocation, or "tools-of-trade" expenses, and (4) a theoretically specified, positive: (a) percentage depletion allowance to natural resource owners, (b) degree of progression in the income tax rates, (c) minimum income exemption for each individual and his dependents on his income tax, (d) corporate profits tax, and (e) excise tax on each consumer durable.

In contrast, in the traditional economic theories of government expenditures and taxation, all of these special features of the income tax -- and the income tax itself -- are inconsistent with a Pareto Optimum.¹

Section II also presents some rough and ready quantitative estimates of the theoretically derived, optimal rates in 4 (a-e) above. The results indicate a striking degree of overall efficiency of the U.S. Federal Tax structure. Our recommendations for policy changes have relatively minor effects on economic incentives.

The results raise the question of what in the U.S. political system has permitted the evolution of such an efficient tax structure in spite of the fact that the only existing tax theories deprecated every major feature of the structure.

I. THE BASIC MODEL

A. The Environment

We shall employ a capital model with discrete time, an infinite horizon, no joint production, and perfect knowledge about the environment.²

We assume that each of the N^α individuals in country α has a utility function defined over all feasible sequences of his consumption benefits, $U^i (B_1^i, B_2^i, \dots)$, $i = 1, 2, \dots, N^\alpha$, where $U^i()$ is a monotone increasing, differentiable, strictly quasi-concave function.³ Aggregate consumption

benefit over these individuals during the t^{th} period is given by the differentiable, quasi-concave production function,

$$(1) \quad \sum_{i=1}^{N^{\alpha}} B_t^i = C^t(K_{1Ct}, K_{2Ct}, \dots, K_{MCt}), \quad t = 1, 2, \dots,$$

where K_{kCt} represents capital of the k^{th} kind ($k = 1, \dots, M$) devoted to the production of consumption benefits at time t . Aggregate capital in country α in each future period is the result of devoting capital in the preceding period to its production or acquisition so that

$$(2) \quad K_{kt+1} = I^{kt}(K_{1kt}, \dots, K_{Mkt}), \quad t = 1, 2, \dots,$$

where $I^{kt}(\)$ is differentiable and quasi-concave. Foreign aggression is treated as a form of investment, the capital obtained from a current act of aggression being unavailable for consumption until the following period.

National defense effort at time t is

$$(3) \quad D_t = G^t(K_{1Gt}, \dots, K_{MGt}),$$

where $G^t(\)$ is also differentiable and quasi-concave. The aggregate capital stock of kind k in any period is the sum of the amounts of capital used in the above activities plus the amount taken by foreign aggressors, K_{kAt} . That is,

$$(4) \quad K_{kt} = K_{kCt} + \sum_{y=1}^M K_{kyt} + K_{kGt} + K_{kAt}.$$

Relations (1) - (4) insure the absence of joint production and collective goods in that they state that no particular unit of a capital good serves several functions simultaneously, such as producing consumption goods and producing itself in the following period.

Similar relations hold for each country.

We assume that the distribution of capital between countries is an equilibrium distribution, meaning that each of the countries has rationally decided (rational in the Paretian sense) which property to claim and defend in each period and that the decisions are mutually consistent given the world's aggregate stocks of capital. The equilibrium initial capital stock of country α is given by

$$(5) \quad K_{k1} = K_{k1}^*, \quad k = 1, 2, \dots, M.$$

We assume that each country knows the rational strategies of the others so that no actual aggression occurs in determining this equilibrium distribution of capital between countries. In equilibrium, the country possessing a unit of capital in a given period is the country that has made a prior commitment to impose damages on any country attempting to acquire the capital which are at least as great as the value of the capital to the aggressor.⁴ Since the only way to subvert the prior commitment of another country is to take control over the entire country, all foreign aggression is all-or-nothing. So if the net return from aggression is ever positive, it is greatest for $K_{kAt} = K_{kt}$. Hence, we can write the profit to aggression against country α for a particular aggressor as

$$(\pi) \quad \pi_{At} = A(K_{1t}, K_{2t}, \dots, K_{Mt}) - C(D_t),$$

where $A(\)$ is the aggressor's evaluation function of the assets he acquires and $C(D_t)$ is his corresponding cost of the aggression, which we assume to be a monotonic increasing function. The rational foreign aggressor acquires no K_{kt} if the profit to the aggression is nonpositive.

Equilibrium also implies that each country always retains its rationally produced capital. This is because if a subsequent unit of capital were not successfully defended, it would have been better for the country to consume

the capital which was devoted to its production. This follows from the monotonicity of utility functions and the consumability of capital expressed in relations (2) - (5). In equilibrium, then, α makes D_t just high enough in each period that for each potential aggressor, $\pi_{At} \leq 0$ for all $K_{kAt} > 0$. I.e., it makes D_t just high enough that the solution value of K_{kAt} is equal to zero for all potential aggressors. From (π) , this level of D_t obviously depends upon K_{1t}, \dots, K_{Mt} , which determines the return to foreign aggression. Hence, we set D_t equal to $D^t(K_{1t}, \dots, K_{Mt})$, the defense requirement of the country at time t , the level of defense required to dissuade all potential aggressors. Also, since there are no foreign aggression activities in equilibrium, the only output of the "government" is D_t . Hence, equations 3 and 4 are written:

$$(3') \quad D^t(K_{1t}, \dots, K_{Mt}) = G^t(K_{1Gt}, \dots, K_{MGt}) \text{ and}$$

$$(4') \quad K_{kt} = K_{kCt} + \sum_y K_{kyt} + K_{kGt}.$$

B. Conditions for Pareto Optimality

Maximizing $U^1(B_1^1, B_2^1, \dots)$ subject to $U^j(B_1^j, B_2^j, \dots) = U^{j*}$, $j \geq 2$, and equations (1), (2), (3'), (4') and (5), we find that necessary for a Pareto Optimum in our environment is that the allocation of resources (i.e., K_{kCt} , K_{kyt} , K_{kGt} , and B_t^i for all k, y, t and i) satisfies, in addition to the constraint equations above, the following marginal equalities:

$$(6) \quad \frac{C_k^t}{C_y^t} = \frac{I_k^{zt}}{I_y^{zt}} = \frac{G_k^t}{G_y^t} \text{ for all } t, k, y, \text{ and } z, \text{ and}$$

$$(7) \quad \frac{\partial U^i / \partial B_t^i}{\partial U^i / \partial B_{t+1}^i} = \frac{C_y^{t+1} I_k^{yt}}{C_k^t} \cdot \left(1 - \frac{D^{t+1}}{G_y^{t+1}} \right) \text{ for all } t, k, y \text{ and } i,$$

where subscripts on function symbols indicate partial derivatives of the

function with respect to capital of the type specified by the subscript.

Equation (6) states the familiar condition that in an optimum, different kinds of capital are allocated between sectors so as that their relative marginal productivities are equal whatever they produce. Equation (7) states that in an optimum, the marginal rate of time preference of B_t over B_{t+1} is less than the familiar marginal rate of time productivity by a percentage equal to the increase in defense requirement caused by the capital which is produced to create the extra B_{t+1} relative to the defense productivity of this capital.

C. Competitive Equilibrium

We now give each individual in country α an initial endowment of capital $(K_{11}^{i*}, \dots, K_{M1}^{i*})$ such that

$$(8) \quad \sum_i K_{kl}^{i*} = K_{kl}^* \text{ for all } k.$$

We also give each of these individuals a set of quasi-concave production functions for each period which read:

$$(9) \quad \begin{aligned} C^{ti} &= C^{ti}(K_{1Ct}^i, \dots, K_{MCt}^i) \text{ and} \\ I^{kti} &= I^{kti}(K_{1kt}^i, \dots, K_{Mkt}^i) \text{ for every } k. \end{aligned}$$

The aggregate functions described in (1) and (2) must then be derived by maximizing aggregate output for given aggregates of inputs devoted to the production of the output. That is,

$$(10) \quad \begin{aligned} C^t(K_{1Ct}, \dots, K_{MCt}) &= \max_i \sum C^{ti}(K_{1Ct}^i, \dots, K_{MCt}^i) \text{ subj. to } \sum_i K_{yCt}^i = K_{yCt}, \text{ and} \\ I^{kt}(K_{1kt}, \dots, K_{Mkt}) &= \max_i \sum I^{kti}(K_{1kt}^i, \dots, K_{Mkt}^i) \text{ subj. to } \sum_i K_{ykt}^i = K_{ykt}, \end{aligned}$$

all k, y , and t .

We assume that each individual may economically participate somewhat in the production of each output (which will imply non-increasing returns to scale in the individual functions), so that the above maximizations obviously occur when and only when

$$(11) \quad \begin{aligned} C_k^{ti} &= C_k^{tj} (= C_k^t) \\ I_k^{yti} &= I_k^{ytj} (= I_k^{yt}) \text{ for all } k, i, j, \text{ and } t. \end{aligned}$$

Hence, if the equilibrium in the economy satisfies (11), it generates the aggregate production functions in (1) and (2) given the constraints in (10).

We now introduce prices. Our prices are all initial period, unit-of-account prices; that is, they describe the amount of wealth one must currently surrender in order to obtain delivery of a good at a specified date. To obtain such prices from prices that would rule in actual transactions in future periods, a suitable discount must be applied to the price in the future to reflect the value of early payment in the form of initial wealth. The price of capital of type k delivered in period t is written P_{kt} , and the price of consumption goods delivered in period t is written P_t . An individual is also taxed an amount whose present cost is given by T^i .

Each individual is assumed to freely choose B_t^i , K_{kCt}^i , and K_{kyt} so to maximize $U^i (B_1^i, B_2^i, \dots)$ subject to production functions in (9) and his budget,

$$(12) \quad T^i + \sum_t P_t B_t^i = \sum_k P_{kl} K_{kl}^{i*} + \sum_t P_t C^{ti} + \sum_{ty} \sum_{yt+1} P_{yt+1} I^{yti} - \sum_{tyk} \sum_{kt} P_{kt} (K_{kCt}^i + K_{kyt}^i).$$

The solutions represent a competitive equilibrium when prices are set⁵ so that, for all t and k ,

$$(13) \quad \sum_t B_t^i = \sum_i C^{ti}, \quad \sum_i (K_{kCl}^i + \sum_y K_{kyl}^i) = K_{kl}^*, \quad \text{and} \quad \sum_i (K_{kCt+1}^i + \sum_y K_{kyt+1}^i) = \sum_i I^{kti}.$$

D. The Case of Lump-Sum Taxation

When taxes are lump-sum so that they do not vary with the individual's behavior, the individual utility maximizing choices are seen to satisfy the following marginal equalities:

$$\frac{C_k^{ti}}{C_y^{ti}} = \frac{I_k^{zt}}{I_y^{zt}} = \frac{P_{kt}}{P_{yt}} \text{ and}$$

$$\frac{\partial U^i / \partial B_t}{\partial U^i / \partial B_{t+1}} = \frac{C_y^{t+1, yt} I_k^t}{C_k^t} = \frac{P_t}{P_{t+1}}, \text{ all } k, y, z, t, \text{ and } i.$$

These conditions are inconsistent with the condition for Pareto optimality in (7) except when $D_y^{t+1} = 0$ for every y and t , which is the implausible special case in which the returns to aggression by the marginal foreign aggressors are never affected by the size of the victim's capital stock.

E. The Pareto Optimality of a Competitive Equilibrium with Certain Capital Taxes

We now assume that

$$(14) \quad T^i = \sum_{tk} a_{kt} P_{kt} K_{kt}^i + T^{oi},$$

where a_{kt} is a constant present tax rate on capital of type k at date t and the T^{oi} is a lump sum tax or subsidy to the individual set so that $\sum_i T^i$ satisfies the government's budget condition,

$$(15) \quad \sum_i T^i = \sum_{tk} P_{kt} (1 + a_{kt}) K_{kt}^i.$$

Equation (15) reflects the fact that the capital tax is levied on sellers of capital rather than buyers and that prices are the net prices to sellers. We assume that the government minimizes costs using fixed factor prices so that

$$(16) \quad \frac{P_{kt}(1 + a_{kt})}{P_{yt}(1 + a_{yt})} = \frac{G_k^t}{G_y^t}.$$

Maximizing $U^i(\)$ subject to (9), (12) and (14) yields the following marginal conditions:

$$(17) \quad C_k^{ti} = P_{kt}(1 + a_{kt})/P_t \quad \text{and} \quad I_k^{yti} = P_{kt}(1 + a_{kt})/P_{yt+1},$$

$$(18) \quad \frac{C_k^{ti}}{C_y^{ti}} = \frac{I_k^{yti}}{I_y^{yti}} = \frac{G_k^t}{G_y^t} = \frac{P_{kt}(1 + a_{kt})}{P_{yt}(1 + a_{yt})} \quad \text{and}$$

$$(19) \quad \frac{\partial U^i/B_t}{\partial U^i/B_{t+1}} = \frac{C_y^{t+1} I_k^{yt}}{C_k^t} \cdot \frac{1}{1 + a_{yt+1}} = \frac{P_t}{P_{t+1}}, \quad \text{all } k, t, z, t \text{ and } i.$$

(17) satisfies the conditions in (11) so that (10) holds. We now need only set

$$(20) \quad a_{yt+1} = a_{yt+1}^o = \frac{D_y^{t+1}}{G_y^{t+1} - D_y^{t+1}}$$

in order for (18) and (19), together with (8), (10), (13), (3'), and (4') to represent the same equation set as (6) and (7) together with (1), (2), (3'), (4') and (5) -- in order for any competitive equilibrium with such capital taxes to be a Pareto Optimum.⁶

Note that no particular tax rate on capital in the initial period is implied by optimal capital taxes. This is reasonable because such capital has already been produced so that taxing it is equivalent to applying a lump-sum tax. Nevertheless, we often times below apply the harmless procedure of applying the optimal tax rate on future capital to capital in the initial period. Note also that the optimal capital tax is equivalently a tax on the value of the capital output, I^{kt} .

It is easy to show, using (17), allowing the optimal capital tax rate to apply in period 1, and assuming linear homogeneity of $D^t(\)$ and $G^t(\)$ with

respect to their respective arguments, that $\pi^{0i} = 0$ so that optimal capital taxes alone are just sufficient to finance government expenditures. While we do not maintain these homogeneity assumptions in the paper, the result indicates, to the extent that the homogeneities are roughly plausible, a relatively minor role for lump-sum taxation or subsidization in a world employing an optimal capital tax.

F. Specification of the Marginal Aggressors' Marginal Profit Functions

Since the marginal aggressors' profits are kept at zero by the potential victim's defense effort, we have, differentiating (π),

$$(24) \quad \frac{\partial A}{\partial K_{xt}} = A_{xt} = \frac{dC(D^t)}{dD^t} D_{xt}^t.$$

Hence,

$$(25) \quad \frac{A_{xt}}{A_{yt}} = \frac{D_{xt}^t}{D_{yt}^t} \text{ if } A_{yt} > 0.$$

We assume that for some subset of $(k) = (1, 2, \dots, M)$, written $a(k) = (1, 2, \dots, M_a)$, $A_{kt} > 0$. We call any kind of capital in this subset a part of the country's "coveted capital". For the rest of the capital stock, $A_{kt} = 0$.

Equations (20) and (24) tell us that if $A_{xt+1} = 0$, $a_{xt+1}^0 = 0$. That is, if a particular kind of capital is not part of the country's coveted capital, the optimal tax on the capital is zero. Now we assume that for all x and y in $a(k)$,

$$(26) \quad \frac{A_{xt}}{A_{yt}} = \frac{P_{xt}}{P_{yt}}.$$

That is, the relative marginal values which foreign aggressors place on different kinds of the country's coveted capital are equal to the corresponding

relative values to the defending country. There are several reasons that this is not a strictly justifiable assumption. It does, however, serve to maintain reasonable orders of magnitude. A Jet Plane is a lot more valuable than a light bulb, to the aggressor as well as the defender. From (25) and (26),

$$(27) \quad \frac{P_{xt}}{P_{yt}} = \frac{D_x^t}{D_y^t} \text{ for all } x, y \in a(k).$$

Hence, from (27), (18) and (20), optimal capital taxes are nondiscriminatory, i.e.,

$$(28) \quad a_{xt}^0 = a_{yt}^0 = a_t^0 \text{ for all } x, y \in a(k).$$

G. Problems in Implementation

We have as yet produced no model specific enough to indicate which types of capital comprise a country's coveted capital stock. Also, since it is practically very costly to tax the value of capital in every period when there are not transactions in the capital during every period, a problem arises as to how one can create, if possible, a tax system which levies only on transactions but which is still equivalent in effects to the idealized system of optimal capital taxation described above. These problems of implementation are the subject of Section II.

H. A Possible Generalization

Admitting Marshallian joint production of consumption and investment goods would open up the possibility that some units of produced capital would be optimally surrendered in the future to a potential foreign aggressor. To have determinate units of such capital, we would, in effect, have to introduce different defense costs for different units of equally valuable capital, thus violating (27) and giving rise to discriminatory taxes. While these cases of "surrenderable capital" are excluded from our formal analysis below, an informal discussion is provided in section IIIH.

II. ACHIEVING AN OPTIMAL CAPITAL TAX

A. The Transaction Structure and Income Taxes

We now allow our economy to have an explicit transaction structure-- a particular set of trades between individuals which achieves the optimal competitive equilibrium described above. Suppose each producer sells his entire output, purchasing all of his inputs all over again for his production in the following period. Then there would be no difficulty in implementing the optimal capital tax. A simple income tax, a tax on all producer sales, with tax exemptions granted for sales of the outputs of noncoveted capital, would obviously be sufficient to produce an equivalent to the optimal capital tax. However, this supposition is far from realistic; in our model, producers may retain some of their capital output for their own future use. To acquire an equivalence between an income tax and the optimal capital tax then requires amendments to the simple income tax besides exemptions for sales of outputs from noncoveted capital. The income tax on the k^{th} kind of capital of individual i is given by

$$(29) \quad T_b^{ki} = \sum_t b_{kt} (P_t C_{kt}^{tim} - X_{kt}^i),$$

where b_{kt} is the income tax rate, X_{kt}^i represents deductions from the tax base, and K_{kt}^{im} represents the capital that i uses to produce goods for the market in period t . By definition,

$$(30) \quad K_{kt}^i = K_{kt}^{im} + K_{kt}^{ii},$$

where K_{kt}^{ii} is the capital that individual i uses to produce goods which are not sold in the market. The optimal income tax exists when, for each k and i , b_{kt} and X_{kt}^i take on values that make (29) equivalent to (14) and (20), or,

$$(31) \quad \sum_t b_{kt} (P_t C_{kt}^{im} - X_{kt}^i) = \sum_t a_{kt}^o P_{kt} K_{kt}^i + T^{oi} \text{ for all } K_{kt}^i, \text{ or}$$

$$(32) \quad dT_b^{ki}/dK_{kt}^i = a_{kt}^o P_{kt} .$$

Solutions to (31) and (32) are written (b_{kt}^o, X_{kt}^{io}) .

Note that income taxes have an independence property expressed as

$$(33) \quad \sum_k T_b^{ki} = T_b^i .$$

B. Noncoveted Capital

Capital may generate benefits which apply to only certain, specified individuals within a country -- such capital is "people-specific". One's psychic capital - e.g., his acquired ability to appreciate nature and various activities, his stock of pleasant memories, his acquired ability to entertain himself - is part of the country's people-specific capital stock. The rest of a country's "people-specific capital" is "friendship capital" -- where a specified individual (e.g., a husband) can command the services (e.g., good cooking) of another (e.g., a wife) because the other either feels a sense of gratitude for past favors by the specified individual or has confidence that the specified individual will reciprocate in the future without contracts or monetary exchange, the cost of which would preclude such favors from having a positive net value to the recipient. People-specific capital is of no value to individuals outside of the country and is not part of a country's coveted capital stock. Since the returns on people-specific capital do not generate market transactions so that $K_{kt}^{im} = 0$ for people-specific capital, an income tax operates in an optimal fashion by not taxing people-specific capital.

While returns from friendship capital frequently come in the form of a monetary payment in the cases of gift and charity income, such income is, appropriately, substantially disregarded for tax purposes in the U.S.⁷

When an aggressor takes over a country, he does not benefit from human capital to the extent that such capital is necessary for the "subsistence" of the individual. An individual is below "subsistence" if he would sooner die in rebellion than pay the taxes of the successful foreign aggressor. One's subsistence includes support for his family as well as any non-cosmetic family medical expenses. Hence, one's human capital below that required for his family's subsistence is not part of his country's coveted capital stock. As a result, an optimal income tax has an exemption, X_{ht} , on incomes from human capital required for normal family subsistence of market goods and for abnormal, non-cosmetic medical expenses. The observed U. S. income tax exemption of about \$1,000 per dependent person appears to approximate the subsistence fairly well, and the observed write-off for non-cosmetic, abnormal medical expenses also corresponds exactly to its treatment under an optimal income tax (the level of the subsistence exemption can be adjusted to include normal medical expenses).

It is likely that a modern foreign aggressor would not let his victims who produce less than their own subsistence die. Assuming this to be the case, any increase in the assistance requirement of below-subsistence individuals has the same external social product as any other decrease in the coveted capital stock of equal value. The decrease in the capital stock in this case appears as an increase in expenditures on charity. Hence a receipt of charity is not only a return on non-coveted capital; it typically serves as a reward to individuals who produce less than their subsistence for a reduction in their non-coveted human capital. So expenditures on charity should be treated for

income tax purposes as any other expenditure which reduces the capital stock of the country. In fact, charitable expenditures in the U.S. are treated substantially the same as business expenditures for income tax purposes.^{8/}

Certain consumer durables such as furniture, portraits, trophies and certain antiques are also part of the country's noncoveted capital stock. While there are no taxes on these goods as $K_{ct}^{im} = 0$, consumer durables which are part of the coveted capital stock are discussed in Section E below.

Cash, i.e., paper currency, may exist in the economy with explicit transactions as an intermediate asset which allows for the achievement of the no-transactions-cost economy described in Section I. However, with costless currency creation, a successful foreign aggressor can also costlessly create any feasible level of real cash balances by altering the rate of growth of the currency supply. Hence, the level of real cash balances used by the defending country is irrelevant to the aggressors and not part of a country's coveted capital stock.

The definitions of capital which Smith, Marshall, and Knight inferred from discussions of men of affairs excluded people-specific capital, subsistence-producing capital, certain personal consumer durables, and paper money. Using our theory, we can rationalize the exclusion of these forms capital from the concept of capital used by men of affairs by arguing that these men are only discussing coveted capital because it is the only capital which should be taxed. The only remaining types of private-good-capital in the Smith-Marshall-Knight taxonomies, the types comprising the capital stock as seen by men of affairs, are natural resources, produced producer durables, human capital above subsistence requirements, and the remaining consumer durables. These forms will be assumed to comprise the country's coveted capital stock, and the achievement of an optimal tax on each of these forms of capital is examined separately below.

C. Producer Durables which are Originally Produced for Sale

No tax on the gross income from producer durables which are originally purchased but not resold in every period (labelled e for equipment) is equivalent to the optimal tax. The derivative of taxes with respect to K_{et}^i is positive in (32), and, in view of (30), is zero in (29) for fixed X_{et}^{im} if $X_{et}^i \equiv 0$. So, for optimal taxation, $X_{et}^i \neq 0$.

To achieve the effects of an optimal capital tax, we consider a tax on e 's capital income reduced by an estimate of the depreciation of the original, observed value of the durable good. We are assuming, until section G below, that capital that is not used to produce current marketed output is used only to produce itself in the following period. That is, $K_{et}^{ii} = K_{eet}^{ii}$ and $K_{fet}^{ii} = 0$ for $f \neq e$. Hence, letting X_{et} represent realistic depreciation of the original purchased capital in terms of current consumer goods, (29) becomes

$$(34) \quad T_b^{ei} = \sum_t b_{et} [P_t C_e^t (K_{et}^i - K_{eet}^{ii}) - P_{et+1} (K_{et}^i - I_e^{et} \cdot K_{eet}^{ii})].$$

Using (17) this simplifies to

$$(35) \quad T_b^{ei} = \sum_t b_{et} [P_t C_e^t - P_{et+1}] K_{et}^i.$$

Hence, to obtain an optimal net income tax on e in time t , in view of (31) and (35), we need only pick a b_{et} which satisfies

$$(36) \quad b_{et} = \frac{a_t^o P_{et}}{P_t C_e^t - P_{et+1}}.$$

Using (17) this optimal income tax rate becomes

$$(37) \quad b_{et}^o = \frac{a_t^o}{1 + a_t^o} \cdot \frac{I_e^{et}}{I_e^{et} - 1}.$$

From (19) and (17) we can see that when there is a stationary solution such that $a_t^o = a_{t+1}^o$ and $C_k^t = C_k^{t+1}$,

$$(38) \quad b_{et}^o = \frac{a_t^o(1 + \rho_t)}{(1 + \rho_t)(1 + a_t^o) - 1},$$

where $1 + \rho_t = \frac{\partial U / \partial B_t}{\partial U / \partial B_{t+1}}$, the marginal rate of time preference.

Thus, the use of realistic, physical depreciation allowances converts the inefficient income tax into an efficient one under an appropriate income tax rate.⁹

The only other existing rationalization of the realistic depreciation write-off this author has seen is by Samuelson (1964). Samuelson shows that a write-off on income taxes of all forms of interest income will create a tax structure with no effect on prices only if the tax on income includes a write-down for realistic depreciation. However, no tax system in existence subsidizes interest returns and taxes net receipts so as to have offsetting revenue and incentive effects. Such a system would be a lot of trouble for no purpose. The tax system Samuelson claims to represent is a U.S.-like system, a system containing a tax on interest income as well as a corresponding write-off of interest expenses. But in such a system there is no net tax or subsidy on borrowing or lending -- nor should there be in our optimality model because borrowing represents a mere redistribution of purchasing power rather than the creation of any real asset. Samuelson erred by failing to allow the after-tax contractual rate of interest to rise to reflect the equal shifts up in the interest-demand-price for loans and the interest-supply-price of loans resulting, respectively, from the tax write-off of interest expenses and the tax on interest income. Once gross market rates of interest are raised to reflect the tax on interest income, the reduction of the gross market rate by applying the tax write-off on interest expenses (or income) to obtain the after-tax borrowing (or lending) rate relevant for discounting only serves to pull the discount rate back down to the original real interest rate for the original allocation of real resources.

And with no reduction in the discount rate in Samuelson's model, no positive income tax rate satisfies Samuelson's price invariance condition, whether or not there are depreciation allowances.

D. Natural Resources

Natural Resources (i.e., minerals and oil and gas) pose a different problem in the lack of transactions to correspond to each act of production. Like producer goods which are purchased upon their original creation, natural resources are accumulated by the owner without any corresponding transaction. But unlike such producer goods, there can be no transaction corresponding to original creation for a natural resource. Because tax collectors cannot be assumed to know the value of the natural resource at any given date in the past, they cannot be assumed to know the change in its value over time. However, once a natural resource is utilized by converting it into some other good by an act of withdrawal from nature, there is, we assume, always a sale of the withdrawn resource. Hence, the obvious method of achieving the effects of a tax on the accumulation of natural resources is to apply taxes at withdrawal in a way which subsidizes early withdrawal. The ordinary income tax does not do what it may appear to - tax early withdrawal - because the profit from withdrawal increases with the rate of interest so that delaying withdrawal merely increases future taxes by the rate of interest and has no tax-saving or tax-increasing effect in a world with a constant income tax rate. Obversely, a subsidy to net withdrawal income would not encourage early withdrawal. We can specify a tax or subsidy on transactions which will encourage early withdrawal only after specifying some special, technological features of the natural resource industry.

In particular, (a) in producing a natural resource for next year, the only input one uses is the same resource in the current year, and (b) the amount of the natural resource produced is identical to the amount of the resource devoted to its production. I.e.,

$$(39) \quad K_{nt+1}^i \equiv I^{nti} \equiv K_{nnt}^{ii}$$

where n is the natural resource. From this and (17), in an optimum,

$$(40) \quad P_{nt}(1 + a_{nt}) = P_{nt+1} = P_t C_n^t.$$

Similarly, the same physical units of natural resources become consumable once they are withdrawn so that, letting Q_t^i be i 's withdrawal of natural resources in time t , $P_{Qt} K_{nQt}^i$ represents the consumption value of the withdrawn resource. It follows that $P_{Qt} - P_{nt}(1 + a_{nt}) = P_{Qt} - P_{nt+1}$ is the direct withdrawal cost per unit of the resource assuming linear homogeneous withdrawal functions.¹⁰

Thus, $P_t C_n^t = P_{Qt} \frac{\partial K_{nQt}^i}{\partial K_{nQt}^i} - (P_{Qt} - P_{nt+1}) = P_{nt+1}$ as in (40) above. Finally, there is the observation that the spot price of withdrawn natural resources has not substantially changed over time (See Barnett and Morse.) Thus we can write $P_{Qt} = P_t$ and $Q_t^i = C_t^i = K_{nQt}^i$ for a withdrawer of a natural resource.

With a constant spot price of the withdrawn natural resource, an obvious encouragement to natural resource exploitation (when there is a positive marginal rate of time preference) is a subsidy which is a fixed percentage of the revenue from the sales of the withdrawn natural resource. For the producer would rather have a given subsidy this year than next. Thus we set

$$(41) \quad X_{nt} = \lambda P_t K_{nt}^{im}.$$

λ is the "percentage depletion rate".¹¹ Hence,

$$(42) \quad T_b^{ni} = \sum_t b_{nt} [P_t C_n^t (K_{nt}^i - K_{nnt}^{ii}) - \lambda P_t (K_{nt}^i - K_{nnt}^{ii})].$$

Using (39) and (40),

$$(43) \quad T_b^{ni} = \sum_t b_{nt} (P_{nt+1} - \lambda P_t) (K_{nt}^i - K_{nt+1}^i).$$

Assuming $b_{nt} = b_n$ and using (40) and (19),

$$(44) \quad T_b^{ni} = b_n (P_{n2} - \lambda P_1) K_{n1}^{i*} + \sum_t b_n [P_{nt+1} (1 + a_{nt+1}) - \lambda P_t \left(\frac{1}{1 + \rho_t} \right)] K_{nt+1}^i - \sum_t b_n (P_{nt+1} - \lambda P_t) K_{nt+1}^i, \text{ or}$$

$$(45) \quad T_b^{ni} = b_n (P_{n2} - \lambda P_1) K_{n1}^{i*} + b_n \sum_t (a_{nt+1} P_{nt+1}) \left(\frac{\lambda \rho_t P_t}{1 + \rho_t} \right) K_{nt+1}^i.$$

Hence, for an optimal tax, letting the first lump-sum term in (45) be absorbed in T^{oi} and using (31),

$$(46) \quad a_{t+1}^o = b_n^o \left(a_{t+1}^o + \frac{\lambda^o \rho_t}{(1+\rho_t)} \frac{P_t}{P_{nt+1}} \right) .$$

Assuming that the income tax rate is that optimal rate applied to equipment, and again assuming that $a_{t+1}^o = a^o$, we find, substituting (38) into (46), that

$$(47) \quad \lambda^o = \frac{P_{nt+1}}{P_t}$$

Hence, the optimal percentage depletion allowance is the ratio of the market value of the conserved natural resource to the value of the product obtained by currently exploiting the resource.

The most reliable data on natural resources we have covers the oil industry. Here, the ratio of mineral right value to output value has been relatively constant at about 23 percent. (Source: Joint Associations Survey. This mineral right value is obtained by adding amortized oil lease payments to royalties.) The current U.S. percentage depletion allowance is 22 percent. There is thus a close correspondence in this industry between the optimal and actual percentage depletion allowances. While precise data are not available, there is also a correspondence between the low, 5 percent depletion allowances given to producers of gravel, peat, pumice, shale and stone and the obviously low value of mineral rights to these natural resources relative to the prices of the withdrawn resources to consumers. And in the extreme case in which mineral rights are essentially free such as for soil, dirt, moss, minerals from sea water, and air, there is a zero percentage depletion allowance.

We now estimate the optimal depletion allowance for the minerals industry in the aggregate. Note first that because of the constancy over time of the spot price of withdrawn natural resources, spot withdrawal costs must fall over time so as to make the spot price of a natural resource rise at the productive

rate of interest. In particular the percentage reduction in spot withdrawal costs over time times withdrawal costs relative to the spot price of a particular natural resource at withdrawal must equal the rate of interest. Therefore, the price of a natural resource at withdrawal relative to the withdrawn resource equals the capitalized rate of cost reduction divided by one plus this capitalized rate. Since the rate of decrease of withdrawal costs in the minerals industry is about two percent per annum (Barnett and Morse), and we are using a productive interest rate of 10 percent, the price of natural resources at withdrawal relative to withdrawn resource price over all valuable minerals is estimated to be $16\frac{2}{3}$ percent. In fact, the bulk of the statutory depletion allowances fall between 14 and 22 percent (with effective rates slightly lower because of a limitation of the allowance to 50 percent of the net income of the taxpayer).

The observed spot price of natural resources relative to withdrawn resources may be constant over time despite its increase for a given natural resource because the quality of the resources exploited may decrease over time. For example, the ratio of oil royalties to the value of the withdrawn oil, a well-known empirical constant, could never have remained constant had not the more easily withdrawable oil deposits been tapped at the earlier dates.

E. Consumer Durables

Consumer durables, like producer durables, are sold when they are originally created but, unlike producer durables, do not create future benefits for others. There is therefore no "income" from consumer durables to tax. Hence, an excise tax on the production of consumer durables goods is in order. Using again an interest rate of 10 percent, equation (37), and an optimal income tax rate of approximately 25 percent, we have an optimal capital tax rate of 2.5 percent. Therefore, consumer goods lasting 5 years and depreciating

in a sum-of-years-digits fashion, should be taxed at an initial excise tax rate of 2.5 percent $\times (1 + \frac{10}{15(1.1)} + \frac{6}{15(1.1)^2} + \frac{3}{15(1.1)^3} + \frac{1}{15(1.1)^4}) = 5.35$ percent, and consumer goods lasting 15 years should be taxed at 10.29 percent using these assumptions. **Until very recently**, U. S. Federal excise taxes on consumer durables ranged between 5 percent and 10 percent with the lower rates generally applying to the relatively short-lived goods.¹²

An important consumer durable that is not federally taxed in the U.S. is an individual's home. And apartment building depreciation write-offs are so generous that, in view of the ease of transferring these buildings, there is also no substantial Federal taxes on these consumer durables. But local property taxes seem to compensate for these apparent inefficiencies as effective property tax rates typically are about 2 percent per year, which is close to the Federal rate on the other durables treated above. My guess is that the cumbersome local property tax and the provision of free education to minors is somehow required by the federal government before a locality can exercise local police power or float tax-exempt bonds. In this view, the locality is merely an adjunct of the federal government. The chief reason for this suspicion is that it is not plausible that freely competing localities would offer free education to a partially mobile, heterogeneous populace or would use the property tax as a means for financing it. Yet the education of minors is an activity whose federal subsidization is easy to support -- not as a collective good -- as a good falling within our model which would be privately underproduced within governmental subsidy because the private decision makers (parents) do not substantially gain from the increase in future productivity which their decisions (education for their minor children) create. We shall employ still another implication of the lack of appropriate parental

rewards in the following section. While it is fairly obvious that the inefficient parental reward structure serves to rationalize special laws against polygamy, prostitution, divorce, and child labor (as well as the minimum wage for teenagers), the traditional, inappropriate theory of public finance has buried its important effects on the efficiency of the tax system in a sea of imagined inefficiencies.

F. Human Capital

Human capital as used below is coveted human capital, or "skill", that part of one's human capital stock which he uses to produce goods for the market (or to produce future skill) in value exceeding his subsistence. Newly created skills are reproduced in each future period through the worker's taking care of himself and making any necessary expenditures to retain his skill, the latter being treated as the former for tax purposes by granting it a write-off as a current expense.

Since skill carries its own maintenance out of what would otherwise not have been coveted capital, its value to the aggressor in each period is the present value of its entire future product. Thus, optimal capital taxes on i 's skill amount to

$$(47) \quad T_a^{sio} = \sum_t a_t^o K_{st}^i \sum_{\tau=t} P_{\tau} C_{\tau}^T \cdot \frac{1}{1 + a_{\tau}^o} \cdot \frac{13}{}$$

On the other hand, income taxes are

$$(48) \quad T_b^{si} = \sum_t b_{st} P_t C_s^t (K_{st}^i - K_{sst}^{ii}).$$

We assume a stationary optimum so that $a_{\tau}^o = a^o$, $C_s^T = C_s$, and $\rho_t = \rho$. We consider an accumulation of a durable skill from the time v onward equal to ΔK_{sv}^i , where $v > 1$.

First consider the case in which the accumulation in v is not the result of foregoing income from skill so that $\Delta K_{sst}^{11} = 0$ for all t . Then,

$$(49) \quad \Delta T_a^{s10} = \frac{a^0 P_{v-1} C_s \Delta K_{sv}^1 (1 + \rho)}{(1 + a^0) \rho^2} \quad \text{and}$$

$$(50) \quad \Delta T_b^{s1} = \frac{b_s P_{v-1} C_s \Delta K_{sv}^1}{\rho} \quad \text{where } b_{st} = b_s.$$

Hence, using (32),

$$(51) \quad b_s^0 = \frac{a^0 (1 + \rho)}{(1 + a^0) \rho}.$$

Tax write-offs for expenditures on education, job search, and worker-owned equipment would substantially subvert this efficient income tax. The optimal income tax in (51) is only slightly different than the optimal income tax on purchased producer durables.

Now consider the case in which the initial accumulation of a durable skill is accomplished by foregoing income from current skill and keeping leisure time constant. Here, (49) remains the same, but

$$(52) \quad \Delta T_b^{s1} = -b_{sv-1} P_{v-1} C_s (\Delta K_{sv-1}^1) + \sum_{t=v} b_{st} P_t C_s \Delta K_{sv}^1$$

$$= -b_{sv-1} \frac{P_{v-1} C_s \Delta K_{sv}^1}{1 + \rho + a} + \sum_{t=v} b_{st} P_t C_s \Delta K_{sv}^1.$$

Assuming $b_{st} = b_{sv}$ for $t > v$,

$$(53) \quad T_b^{s1} = -b_{sv-1} \frac{P_{v-1} C_s \Delta K_{sv}^1}{1 + \rho + a} + b_{sv} \frac{P_{v-1} C_s \Delta K_{sv}^1}{\rho}.$$

Using (49), for optimal taxation, we have,

$$(54) \quad \frac{a^0 (1 + \rho)}{(1 + a^0) \rho} = b_{sv}^0 - \frac{b_{sv-1}^0}{1 + \rho + a}.$$

Equation (54) states that an optimal income tax is progressive, as the marginal tax rate applying to increases in future wages, b_{sv}^0 , must exceed the average tax rate, b_{sv-1} , which one avoids by training rather than working in the $v-1^{st}$ period. If we let $\frac{a^0(1+\rho)}{\rho(1+a^0)}$ approximate the average income tax rate in the investment period, we find that

$$(55) \quad b_{sv} \approx b_{sv-1} \left(1 + \frac{\rho}{1 + \rho + a} \right).$$

Using 10 percent time preference rates and 2.5 percent optimal capital tax rates, we can fit (55) to actual data on U. S. tax rates.¹⁴

The fit is again very close. The actual U.S. Marginal income tax rates are always within two percentage points of these theoretically optimal rates for all reasonable levels of human capital investment (i.e., for all levels of foregone annual incomes of \$25,000 or less). It is not implausible that the truly optimal tax rate on leisure-produced skill rises with income rather than remaining at the constant expressed in (51) because the excess of the foregone earnings of an individual over average foregone earnings is a plausible measure of the extent of his parental overtraining to surrender leisure (as the reward structure given parents induces them to instill greater lifetime estimates of the value of work relative to leisure in their young children than they would if they could collect as much of their child's leisure benefits as they can the child's work benefits) while the deficit of an individual's foregone earnings below average foregone earnings is a plausible measure of an individual's parental undertraining concerning the value of investing in human capital.

The above analysis applied to accumulating durable skill so that depreciation could be ignored. This appropriately describes most education and training, as most education and training is undertaken by young people who are going to use their training for very many years. However, some

training is undertaken by older people and some training of youth is for short-term or risky careers. In these cases, the above tax on human capital accumulation is too high. Some sort of subsidy for retraining or for training in occupations with short durations is called for. A depreciation allowance on human capital investments would be insufficient. For since the wage increases required to justify a short-term human capital investment are greater than those required to justify a durable investment, the progressive income tax effects a greater tax rate on short-term human capital investments than on long-term investments. We do observe a tax break for risky and short-term human capital investments in the form of an "income averaging" opportunity, but we have made no attempt to quantify the effects of this tax provision.

G. Producer Durables which are Not Sold when Produced

Individuals may produce their own intermediate capital goods so that tax authorities cannot practically employ the slow expense-write-off implied by the depreciation allowances in subsection B above. If expenditures for such accumulation exist at all, individuals can immediately write them off on their income tax. For example, tax authorities explicitly permit immediate tax write-offs for "maintenance" expenditures. Capital so produced generates streams of accounting costs and returns identical to the streams resulting from one's production of skill with leisure or by foregoing income so our analysis here is no different than that above as concerns a single individual when his capital is continuously maintained. However, in this case, we have no rationale for higher optimal tax rates on higher individual incomes. To overcome this problem, we allow an individual to "incorporate" his non-human capital and avoid the progressive personal income tax, taking his return from the corporation in a form in which he pays only about a ten percent tax rate.¹⁵ Our problem is then to specify an efficient tax on

corporations in light of their ability to produce their own capital. The simplest policy is to tax the value of the company (or the value of its stock, still taxing the interest income of the creditors) at the efficient, 2-3 percent capital tax rate and drop the tax on dividends and capital gains. Such a policy would directly tax any capital in the company -- regardless of how it is accumulated. The costly implementation of realistic depreciation and efficient depletion allowances would be avoided, as would the taxation of dividends, capital gains, and corporate profits. But we assume that such a policy is not available, or, if it is, that corporations remain whose stock does not trade at observed prices. The problem then is to specify an efficient corporation income tax in view of the corporation's ability to produce some of its own capital. Now purchased capital is recorded on the books of a company as an asset, but internally produced capital which the company expenses is not included on the books as an asset. However, this latter kind of capital is included in the market evaluation of the company so that the depreciated stock of externally purchased assets in a corporation relative to its total capital stock can be represented by the ratio of the company's book value to its market value. We assume this is constant within each corporation.

Letting B_{ft}/K_{ft} represent the ratio of book to market value of the assets of the firm, replacing K_{et}^1 in (35) with B_{ft}^1 , using (31) and (37), and adjusting for a 10% effective tax on dividends and capital gains, the optimal corporation profits tax is:

$$(57) \quad b_f^o = \left(\frac{K_f}{B_f} \right) \frac{b_e^o}{1.10} .$$

The average K_f/B_f has been estimated to be 1.6.¹⁶ Thus, given our other estimates, the average optimal tax rate on corporate income is about 37 percent. While this is somewhat lower than the statutory rate of 48 percent for a large company, the presence of the 7 percent investment tax credit in

recessions has served to substantially lower the effective tax on produced producer durables. Assuming that half of the years are "recession" years, the average about 3 1/2 percent. Assuming, in addition, that the typical age of produced producer durables is 10 years, the optimal effective excise tax (as computed above for consumer durables) on the production of producer durables is about 7% x 1.6. This means that the effective 3 1/2 percent investment credit subsidizes capital by about 30 percent of the optimal capital tax. But since the corporate profits tax rate is greater than the optimal rate by about 30 percent of the optimal rate, the combined 48 percent corporate profits tax and 7 percent investment credit in recessions effects very close to an optimal tax on producer durables in corporations.

H. Surrenderable Capital

Apparently, the only empirical cases in which the costs of defending certain units of capital are significantly higher than the costs of defending equally valuable coveted capital are cases in which the capital is located on foreign soil. In accord with our general results, the U.S. government neither substantially commits itself to defend such capital nor significantly taxes the income from such capital. Rather, the U.S. sells insurance against foreign confiscation to the various companies with foreign holdings, in effect charging a price for providing only those protection services that it would rationally supply as an insurer in order to avoid claims by the insured.

III. SUMMARY

Our results indicate that the U.S. tax structure does not produce the deluge of malincentives and economic inefficiencies that one finds when using the traditional theory of public finance. Rather, using a more accurate theory of the nature of national defense, our tax structure produces

roughly optimal incentives. Because no model such as ours was available to the advisors or decision makers during the development of the U.S. tax structure, our results strongly suggest that our political system, using the self interested calculations of its citizens, has somehow been able to systematically produce a substantially more efficient tax structure than our economists, using the traditional theory of taxation, have been able to recommend.

While incentives under the U.S. tax system appear to be remarkably efficient, we are not advocating the status quo. In particular, we have argued that the current U.S. tax system could be improved by reinstating excise taxes on those consumer durables which are not used for business purposes and replacing corporate income taxes on companies with publicly traded shares with an annual tax on about $2\frac{1}{2}\%$ on the market value of their common and preferred stock, dropping the personal income tax on dividends and capital gains on such stock.

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¹Samuelson (1964) has attempted to provide some kind of rationalization of realistic depreciation allowances in an apparently traditional environment, but we shall find a simple economic error in his exercise. Also, some authors have assumed that an income tax does not generate a "double taxation of savings" and an inefficiently discriminating tax on future (relative to present) consumption in the traditional model. Since the double tax is only on the interest from savings in the traditional model and there may be a zero interest rate, these authors can be defended, but only in a special case.

²The central results in this paper have also been established for a mathematically less familiar model containing continuous time and Marshallian joint production, although dropping this assumption requires a substitute assumption, as noted in Section IH below. In the case of a finite horizon, several of our tax-equivalence theorems hold in only an approximate sense. The perfect information assumption can be substantially relaxed and is made largely to simplify the discussion.

³The "strict quasi-concavity" of a function here means that if $f(x) = f(x')$, $x \neq x'$, then $f(ax + (1-a)x') - f(x) > \delta \sup_j (x_j - x'_j) f_j$ for some $\delta > 0$ where $1 > a > 0$ and $x = x_1, x_2, \dots$. This may also be termed "asymptotically strict quasi-concavity". It assures the absence of infinite quantities in maximizing f over all x subject only to linear equalities in x with positive coefficients.

This type of quasi-concavity, as well as the standard quasi-concavity assumptions stated below [$f(x)$ is quasi-concave if $f(x) = f(x')$, $x \neq x'$ implies

that $f(ax + (1-a)a') - f(x) \geq 0$]; also are used to insure the existence of a competitive equilibrium.

The differentiability assumptions in this paper, together with the absence of traditional non-negativity assumptions, are made to facilitate the mathematical argument, its being obvious that technical non-substitutabilities do not disturb the optimality of incentives in our efficient tax system.

⁴It may be of interest to note that such commitments are generally impossible to make in a pure democracy, as the voters can always vote against a war by voting down war appropriations. Such a government is sure to lose, bit by bit if not all at once, all of its transferable capital to a non-democratic foreign aggressor that can make commitments to fight wars over property at war costs to both parties which are greater than the value of the property at stake. (The reason such commitments are rational is that once they are made, the democratic country rationally surrenders its capital so that fighting the war is unnecessary.) However, in a constitutional democracy, where certain government policies are not subject (except at great cost) to future voter disapproval, the constitution, by giving proper incentives to the government leaders and by allowing them to command war resources without voter approval, may effect the necessary war commitment. Constitutionally granted wartime finance policies such as the draft, debt financing and government currency creation, and price controls are thus a necessary part of our wartime financial structure. The cost of having such a government is that the leaders may use the same means of financing for peacetime goals or to fight wars other than those to defend or acquire current property.

An important implication of this necessary, confiscatory, wartime financing is that there is then an insufficient accumulation of war-relevant capital during peacetime. This is the economic bases of the classical "national

defense argument" for peacetime subsidization of certain domestic industries (see Thompson for an elaboration and empirical application). In the formal model above, such subsidies appear as government purchases of capital used to produce national defense.

⁵We assume that $\liminf_{t \rightarrow \infty} [\sum_{ty} p_{yt+1}^y I^{yt} - \sum_{tyk} p_{kt} K_{kyt} - T^i]$ exists to assure finite solutions. Our prices are therefore "Malinvaud Prices".

⁶Since we have not precluded decreasing marginal costs of defense with respect to the protected capital, there may be several allocations satisfying (6) and (7) for given utility levels of $N^\alpha - 1$ individuals. And some of these allocations may be Pareto non-optimal. In such cases, however, we assume that the government picks tax rates (a_{kt} and T^{oi}) which correspond to their shadow values in a global Pareto optimum. That such taxes succeed in inducing global Pareto optima despite the possible economies of scale in protecting capital is easily proved: First note that our quasi-concavity, non-increasing returns, and bounded wealth assumptions insure that all solutions to (18) and (19), given (8), (10), (13), (3') and (4'), are privately optimal. It follows that we need only pick the levels of a_{kt} and T^{oi} that correspond to their shadow equivalents in any specified Pareto optimum in order to have the privately selected allocations of the equilibrium coincide with the Pareto optimum. For if this were not the case, then one allocation satisfying (18) and (19), given (8), (10), (13), (3') and (4'), would make someone worse off than would another such allocation.

⁷It should be pointed out that our assertion that people-specific capital is non-coveted rests on an assumption that a successful foreign aggressor cannot substantially switch this kind of capital into the production of benefits which are not people-specific. Thus, it is assumed

that the human capital that an individual devotes to producing benefits for himself (i.e., producing "leisure"), cannot be substantially converted to the production of goods for the aggressor. Our foreign aggressors therefore do not make slaves out of their victims; they merely tax them to subsistence. This implication appears to be fairly realistic. The behavioral basis of it appears to be that an individual acquires certain work-leisure habits, which cannot be substantially broken at any reasonable cost by the foreign aggressors.

⁸Note, however, that a standard type competitive model such as that developed above applies only to cases of private-good charity -- cases in which one enjoys his giving of charity rather than enjoying the beneficiary's receipt of the charity (i.e., in which there are no external economies in the giving of charity) and in which an individual can costlessly avoid the knowledge of another's suffering (i.e., in which there are no external diseconomies in the creation of charity-inducing attributes). Including both of these excluded realistic cases would probably have little total effect on the tax-treatment of charity because their separate effects are substantially cancelling.

⁹In the real world, depreciation allowances are typically granted on a fixed schedule for a particular kind of capital good regardless of how the good actually wears out. But, also in the real world, depreciation typically takes the form of Marshallian joint production of marketed output and future capital. Under such depreciation, the optimal tax on a new investment under stationary conditions is a single tax on the present value of the capital in each future period resulting from the investment while an income tax without a depreciation allowance is a tax only on the initial capital value. It is easy to see by an argument similar to that used above that a realistic depreciation allowance converts the income tax into a tax on the present value of the future capital values implied by an

investment and thus can lead to an optimal choice of investments in a world in which actual depreciation takes the form of Marshallian joint production of marketed output and future capital for the firm.

There are real-world cases of physical depreciation of producer durables which have been produced for sale in which the depreciation does not take the form of Marshallian joint production. This appears for originally produced timber, wine, and various agricultural products. Here, we observe in the U.S., the variable depreciation allowances of the kind we have specified; these are frequently called "cost depletion allowances".

¹⁰ Thus an implicit production function for j , a withdrawer of natural resources, can be written:

$$Q^{tj} = \min (K_{nQt}^j, g^t(K_{1Qt}^j, \dots, K_{n-1Qt}^j, K_{n+1Qt}^j, \dots, K_{MQt}^j)),$$

where Q is the withdrawn resource. Selecting "derivatives" of this function to be such that (6) and (7) are satisfied, as we are free to do, this specification is not inconsistent with our general model. The cost of g^t is the "withdrawal cost" described above while $P_{Qt} Q^{tj} = P_{Qt} K_{nQt}^j = P_t C_t^{tj}$.

¹¹ The Traditional, Harberger analysis of such percentage depletion allowances assumes that manufacturing and oil "investments" should be taxed equally if they generate the same streams of cash income. It fails to recognize that if the oil "investment" is not undertaken, there is still an accumulation of oil reserves, which is a true social investment. Therefore, it is necessary to net the disinvestment of oil reserves out of Harberger's oil "investment" before taxes should be equated on his equal investments. The Harberger study should also be corrected for the fact that a depletion allowance is capitalized in the value of the land, thus serving to increase the costs as well as the returns to current oil "investments". Making these adjustments in Harberger's analysis and making intertemporal investment possibilities and taxes explicit leads to our own analysis.

¹²For the various excise tax rates, see Commerce Clearing House. For relative depreciation rates, see Prentice-Hall. As excise tax rates have recently been volatile, there is no simple method of evaluating the post-1964 excise tax structure. Some federal excise taxes fall substantially on producer durables, the most notable of which are "business" machines (such as typewriters and computers) and cars and trucks. The business machine case is fairly easy to understand once it is recognized that the sellers of the more expensive machines normally avoid capital taxation by renting their outputs. The less expensive machines are frequently used by consumers, so that an excise tax is in order. The same applies to cars and trucks. However, businessmen should be allowed to expense their purchases of this already-taxed equipment.

¹³In a case in which produced skill is maintained out of that skill itself, equation (47) would, of course, take the standard form of equation (14) above. It is easy to verify that our results below also hold to a close approximation for that case. (One need only note below that in such a case $K_{sst}^{ii} (1 + \rho_t + a_t) = K_{st}^i$ for $t \geq v$ so that (50) and (53) are correspondingly reduced to approximately match the reduction in (49).)

¹⁴Future tax rates were computed assuming the individuals will be married while current rates, b_{v-1} , were computed assuming the investor is single. (Source, Lasser.)

¹⁵Partial dividend exclusions and taxes on realized capital gains at about half the tax rate applicable to ordinary income seem to achieve about this effective rate for the typical investor. The number may seem a little low, but it reflects the significant advantages to delaying realized gains, and giving charity and bequests in the form of appreciated stock.

¹⁶This was done by multiplying an estimate of the rate of return to book value of equity for U.S. manufacturing corporations in 1966 (Source: Pechman, p. 307) by an estimate of the price-earnings ratio for U.S. industrials for the same year (Source: Moody's Investors' Service). This was then adjusted to represent the ratio of the book value of companies relative to their market value by adding on the ratio of debt to net worth and dividing by one plus this ratio (Source: Pechman, p. 307).