

INTERNATIONAL TRADE
WITH
FLUCTUATING PRICES

by

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There is a long history of concern with the implications of international price fluctuations for the welfare and policy of trading nations. However an important issue which has not been given sufficient formal attention is the necessity of making input decisions before world demand for the finished goods is known.

To focus on this problem a simple model of an open economy is developed in Section I. The essential feature is that the domestic country must make all production decisions knowing only that world prices follow some unchanging probability distribution.¹ The problem may be simply to pick the outputs for next period, or to install permanent capacity (i.e., with prohibitive costs of adjustment) for an indefinite number of periods: the analysis is the same. Initially, the further assumption is made that storage costs of both goods and foreign exchange are prohibitive.

It is shown that the introduction of mean-preserving price uncertainty systematically affects the optimal level of output of all the traded goods. Factors critical in the determination of the direction of these changes are

¹Our model contains only final goods. A somewhat more elaborate case is Linder's (Staffen Linder, Trade and Trade Policy for Development, New York: Frederick A. Praeger, Inc., 1967), where material ("operation") imports are needed to support final goods production. With resource allocation frozen, fluctuating prices can result in the balance of payments constraint forcing unemployment of primary resources. The problem is not essentially different from that of the text, since the Linder case simply has an efficient long run transformation surface of more dimensions and allows some short run substitution (inferior of course to movement on the long run surface). The basic problem is that resources must be allocated ex ante, and the simpler production assumption allows a more convenient analysis.

Linder and others of course have also been concerned with domestic sources of fluctuation (e.g., varying crop yields due to weather and pestilence). This will be the subject of a later paper.

then isolated.

Section I next turns to the related issue of decentralization. First the use of a stock market, rather than a complete set of contingent markets, is discussed. Second, all capital markets are ruled out and it is shown that, under the assumption of expected profit maximization, decentralization can be achieved with a production tax-subsidy scheme.

Given the imperfections in stock markets in developing countries, and the large foreign interest in many branches of industry, the latter scenario is perhaps a justifiable approximation. If so, it follows that attempts by these countries to partially insulate themselves from the effects of international price fluctuations, do have a formal basis. Trade protection, the usual tool employed, is however, a second best approach, involving unnecessary distortions between marginal rates of substitution and marginal rates of foreign transformation.

A further question of much interest is whether welfare is improved by price fluctuation (assuming optimal allocation). Interestingly, in Section II, it is shown that whether society gains or loses depends critically on the degree of specialization. If production conditions are such that the optimal allocation of resources requires concentration in an export whose price fluctuates, welfare will very likely diminish as compared with the fixed price case. This has of course been the long voiced complaint of many developing countries. Since the result differs from that obtained by Hueth and Schmitz in their recent paper in the Q.J.E.² an explanation is sought in Section III.

²Darrell Hueth and Andrew Schmitz, "International Trade in Intermediate and Final Goods: Some Welfare Implications of Destabilized Prices", Q.J.E., LXXXVI, No. 3, (August, 1972), 351-65.

I. Optimal Resource Allocation

The basic model used in most of this paper is that of a small country facing fluctuating international prices. All goods are final and are produced subject to a long run efficient transformation surface. In the short run production is fixed. Payments must balance at each point in time and storage is impossible. The economy's social planners wish to maximize a weighted sum of expected utilities

$$W = \sum_{i=1}^N \lambda^i EU^i(c^i)$$

by choosing a single production point from the feasible set to most efficiently exploit the set of uncertain trade opportunities.

While the solution can be readily obtained for the general case, we will, from the outset, assume that every individual has the same probabilistic beliefs. Then social welfare can be written as:

$$\begin{aligned} W &= E\left\{\sum_{i=1}^N \lambda^i U^i(c^i)\right\} \\ &= EU(c) \quad \text{with } \sum_i c^i = c \end{aligned}$$

where U is a Bergson-Samuelson social welfare function of aggregate consumption possibilities. It will be assumed that each individual's utility function is very well-behaved, and hence, that U is a twice differentiable, strictly quasi-concave function.

The decision process of the economy is in two stages. The second stage involves maximizing U , subject to a fixed production point and the trade opportunities at fixed prices.

$$\begin{aligned} \text{i.e. Max } & U(c) \\ & c, \hat{x} \\ \text{st } & p \cdot c \leq p \cdot \hat{x} \\ & \hat{x} \leq x \end{aligned}$$

where $\hat{x} \equiv$ production vector

$x \equiv$ vector of production capacities

and $p \equiv$ price vector, random ex ante with known probability distribution $F(p)$.

At each price there is an optimal trade vector

$$t = c - x$$

which is a function of both x and p . Hence, we can write down the solution of the second stage in terms of the indirect utility function.³

$$V = U(x + t(x,p)) = V(p \cdot x, \bar{v}) \quad (1)$$

where $V_i = -t_i V_0 \quad i=1...H$

and V_0 is the marginal utility of income.

The first stage of the decision process is to maximize expected (indirect) utility subject to the constraint on production possibilities

$$\text{i.e. } \text{Max}_x E_p V(p \cdot x, p) \quad \text{st } g(x) \geq 0$$

where $g(x)$ is a concave twice-differentiable transformation surface.

By applying standard Lagrange techniques the following first order conditions are readily obtained

$$\frac{E_i}{E_h} = \frac{E_p (P_i V_0)}{E_p (P_h V_0)} \quad i = 1...H$$

Suppose that the prices of the first $H-1$ goods are denominated in terms of the H -th good 'gold', whose price is certain and set at unity. Then we can re-write these necessary conditions as:

³The indirect utility function is obtained by substituting the optimal values of consumption, $c = c(p \cdot x, p)$ from the usual utility maximization problem into the utility function.

$$\frac{\xi_i}{\xi_H} = \frac{E(p_i V_o)}{E_p(V_o)} \quad i = 1 \dots H \quad (2)$$

The elements of expression (2) are simply the elements of the normal to the transformation surface at the optimal production point. To determine the impact of introducing price uncertainty, we can compare this normal vector with the normal when the price vector is $\bar{p} = E(p)$ with certainty. In the latter case the marginal rates of transformation will be proportional to foreign prices.

$$\text{i.e. } \frac{\xi_i}{\xi_H} = \bar{p}_i \quad i = 1 \dots H \quad (3)$$

Then introducing the vector τ as the divergence between the two normals we have,

$$\tau = \frac{E(pV_o)}{E(V_o)} - \bar{p} = \frac{1}{E(V_o)} \{E(pV_o) - E(p)E(V_o)\} \quad (4)$$

The bracketed term is simply a vector of covariances between $V_o(p \cdot x, p)$ and p . While it is not generally possible to derive simple conclusions about the signs of the components of the covariance vector, three special cases are of interest: (1) the two good (or one variable price) case, (2) independent distribution of $p_1 \dots p_{H-1}$ and (3) small variations in the price vector. In each case the conclusion depends critically on the derivatives of V_o , the marginal utility of income.

In the two good case the sign of τ depends only on $\frac{\partial V_o}{\partial p_1}$. If the derivative is positive over the range of feasible values of p_1 then τ is also positive. It follows that for such cases the optimal output of x_1 is higher when its price is uncertain. Similarly if $\frac{\partial V_o}{\partial p_1}$ is always negative so is τ , and hence optimal output of x_1 is lower with uncertainty.

Intuitively, the sign of the covariance of a monotonic function with its

independent variable must depend only on the first derivative of the function, as may be seen by graphing the function. Formally we have the following simple lemma.

Lemma: If $y(p)$ is strictly monotonic and differentiable then $y'(p) \{E(py) - E(p)E(y)\} > 0$

Proof: First note that

$$\int_a^z (p-\bar{p})dF < \int_a^b (p-\bar{p})dF = 0 \quad a < z < b$$

where $F_p(p) > 0$ on $[a,b]$ only

Then $E(py) - E(p)E(y) = \int_a^b (p-\bar{p})y(p)dF$ and integrating by parts

$$= - \int_a^b y'(z) \int_a^z (p-\bar{p}) dF dz$$

$> (<) 0$ if $y'(z) > (<) 0$

Q.E.D.

If the prices are distributed independently, the $H-1$ non-zero covariances of the vector τ can be treated separately and hence just as in the two good case.⁴ Each element of τ then depends for its sign only on $\frac{\partial V}{\partial p_i}$, as long as this is one-signed for the interval over which p_i varies.

The third case allows any probability distribution for the price vector restricted to the neighborhood of the mean. Expanding $V_0(p)$ in a Taylor's series to the first order about \bar{p} we have,

⁴ The H good case of the above lemma is, without loss of generality,

$$\begin{aligned} E(p_1 Y(p)) - E(p_1)E(Y(p)) &= \int_{a_1}^b \dots \int_{a_H}^b (p_1 - \bar{p}_1) Y(p_1 \dots p_H) dF(p_1) \dots dF(p_H) \\ &= \int_{a_1}^b (p_1 - \bar{p}_1) \left[\int_{a_2}^b \dots \int_{a_H}^b Y(p_1 \dots p_H) dF(p_2) \dots dF(p_H) \right] dF(p_1) \end{aligned}$$

The lemma then goes through as before. Note of course that $a_H = b_H$ and $F(p_H) = 1$, since there is no price uncertainty for good H .

$$\begin{aligned}
 E(pV_0) - E(p)E(V_0) &= E(p - \bar{p})V_0(\bar{p}) + E((p-\bar{p})(p-\bar{p})' \nabla_p V_0(\bar{p})) \\
 &= \Omega_{pp} \nabla_p V_0(\bar{p}) \tag{5}^5
 \end{aligned}$$

where Ω_{pp} is the variance covariance matrix of the price vector. Unlike previous cases the i th element of τ is not signed solely by $\frac{\partial V_0}{\partial p_i}$, since the covariance of the price must be exploited. We have

$$\tau_i = \frac{1}{E(V_0)} \left[\text{Var}(p_i) \frac{\partial V_0}{\partial p_i} + \sum_{j \neq i} \text{cov}(p_i, p_j) \frac{\partial V_0}{\partial p_j} \right] \quad i=1, \dots, H-1$$

Note that the above expression may provide a good indication as to the sign of τ_i , and hence changes in the optimal production point, even well away from the local region of its strict validity. Whether this is indeed the case will depend upon the curvature of V_0 and on the probability distribution. Better approximations, of course, require examination of additional terms.

Since all cases depend upon $\nabla_p V_0$, we now turn to its evaluation. First, note that $V_0(p \cdot x, p)$ is subject to the Slutsky decomposition into income and substitution terms. We have

$$\frac{\partial V_0}{\partial p_i} = \frac{\partial V_0}{\partial p_i} \Big|_{\text{comp}} - t_i \frac{\partial V_0}{\partial m} \quad i = 1, \dots, H \tag{6}$$

where $m \equiv p \cdot (c - x) = p \cdot x = 0$

Moreover as part of the symmetry of the Slutsky matrix we have

⁵ $E[(V_0 - E(V_0))(p - E(p))] = E[(V_0 - V_0(\bar{p})) (p - \bar{p})]$, since $E[(E(V_0) - V_0(\bar{p})) (p - \bar{p})] = 0$.

$$\left. \frac{\partial v_0}{\partial p_i} \right|_{\text{comp}} = -v_0 \frac{\partial c_i}{\partial m} \quad i = 1, \dots, H$$

Therefore we can rewrite (6) as

$$\frac{\partial v_0}{\partial p_i} = -v_0 \frac{\partial c_i}{\partial m} - t_i v_{00} \quad i = 1, \dots, H \quad (7)$$

The marginal utility of income is positive (we ignore problems of satiation), so the first term is positive or negative as the i th good is inferior or normal. The sign of the second term depends upon two terms which in general may take on either sign. But if society is assumed averse or at most neutral towards fair gambles over lump sum income m , utility must be concave and the latter implies that the marginal utility of income must diminish ($v_{00} \leq 0$). In the limiting case of risk neutrality the sign of $\frac{\partial v_0}{\partial p_i}$ depends only on whether the i th good is normal or inferior. For the risk averse case, assuming normality, the second term reinforces the first if the good is exported and offsets if the good is imported.

Consider resource allocation when only the price of the i th good is uncertain. With risk neutrality and normality the optimal divergence between the normals, τ_i , is negative, and it follows that output of the i th good is lower with the introduction of mean-preserving price uncertainty. For a risk averse society the same results must hold if the good is exported at all feasible prices. However if the i th good is imported it is quite possible that the aversion term outweighs the first term, and hence that optimal output rises with the introduction of price uncertainty. Indeed as risk aversion becomes large it becomes optimal always to move towards the no-trade equilibrium point; that is, the point of tangency between an indifference curve and the production possibility frontier.

Intuitively, this is because letting risk aversion become large means moving towards the objective of maximizing the minimum utility, and production at the no trade point assures always doing at least as well as the no trade utility level.

A simple example clarifies this insight. Suppose utility is given by

$$U = \frac{1}{1-\sigma} (c_1^\alpha c_2^{1-\alpha})^{1-\sigma} \quad \sigma \geq 0, 0 < \alpha < 1;$$

This has a corresponding indirect utility function

$$V = V(I, p_1, p_2) = \frac{1}{1-\sigma} [(p \cdot x) p_1^{-\alpha} p_2^{-(1-\alpha)}]^{1-\sigma}.$$

Then when the optimal production vector x^* has been selected,

$$\frac{\partial V_0}{\partial p_1} = B(p, x^*) \left[\alpha \left(1 - \frac{1}{\sigma}\right) - \frac{p_1 x_1^*}{p \cdot x^*} \right] \text{ where } B > 0.$$

Note that $\sigma = - \frac{(p \cdot x) V_{00}}{V_0}$, that is, the coefficient of relative aversion to fair gambles about income. Therefore we can fairly describe increases in σ as increases in risk aversion. If $\sigma \leq 1$ the expression for $\frac{\partial V_0}{\partial p_1}$ is negative whatever values the vectors p and x^* may take. However for larger σ this is no longer true. In particular, for sufficiently small values of p_1 or x_1^* , $\frac{\partial V_0}{\partial p_1}$ must be positive. This result can be strengthened by considering the expression,

$$\left[\alpha \left(1 - \frac{1}{\sigma}\right) - \frac{p_1 x_1(p)}{p \cdot x(p)} \right]$$

where $x(p)$ is the production point when p is the certain price. It is easy to show that the bracket is a decreasing function of p_1 . Moreover, if $\sigma > 1$ there is some price \tilde{p}_1 such that the bracket is positive for $p_1 < \tilde{p}_1$ and negative for $p_1 > \tilde{p}_1$.

Now suppose the price of the first good is uncertain but belongs always to the range $R = \{p_1 | p_1 < \tilde{p}_1\}$. While we cannot solve explicitly for the optimal value of x , without further specification of the probability distribution and the production frontier, it is clear that $x^* \in \{x(p) | p \in R\}$. But then $x_1^* < x_1(\tilde{p})$ and it follows that $\frac{\partial V_0}{\partial p_1}$ is definitely positive. Similarly if p_1 is uncertain, but always greater than \tilde{p}_1 , the derivative of V_0 must be negative. This is illustrated in Figure 1. The ray $OL(\sigma)$ intersects the production frontier at the point with slope $-\tilde{p}_1$. If the price is always such that $x_1(p)$ is to the left of this line, output is higher with uncertainty. If $x_1(p)$ is to the right for all p , the optimal output is lower with uncertainty. For $\sigma \leq 1$ the ray is the vertical axis. Then as σ increases still further the ray swings clockwise, approaching in the limit ON .

We now note again that quite broad results have been obtained based on the monotonicity of the marginal utility of income. Previous work in uncertainty has utilized convexity properties of V and V_{x_i} to compare the optimal uncertainty point with the certainty point, so we have obtained simpler, more usable conditions. As yet unanswered however is the additional interesting question as to whether a mean-preserving spread in the distribution of p_i (an increase in riskiness) will cause the optimal x_i to move further in the same direction. Rothschild and Stiglitz⁶ have obtained strong sufficient conditions to predict the movement of a single control variable with a mean preserving spread in a the distribution of a single random variable. However, even with $\frac{\partial V_0}{\partial p_1}$ one signed in the interval

⁶ Michael Rothschild and Joseph Stiglitz, "Increasing Risk: I, A Definition", Journal of Economic Theory, II No. 3 (Sept., 1970), 225-243 and "Increasing Risk: II, It's Economic Consequences," Journal of Economic Theory, III No. 1 (March, 1971), 66-84.

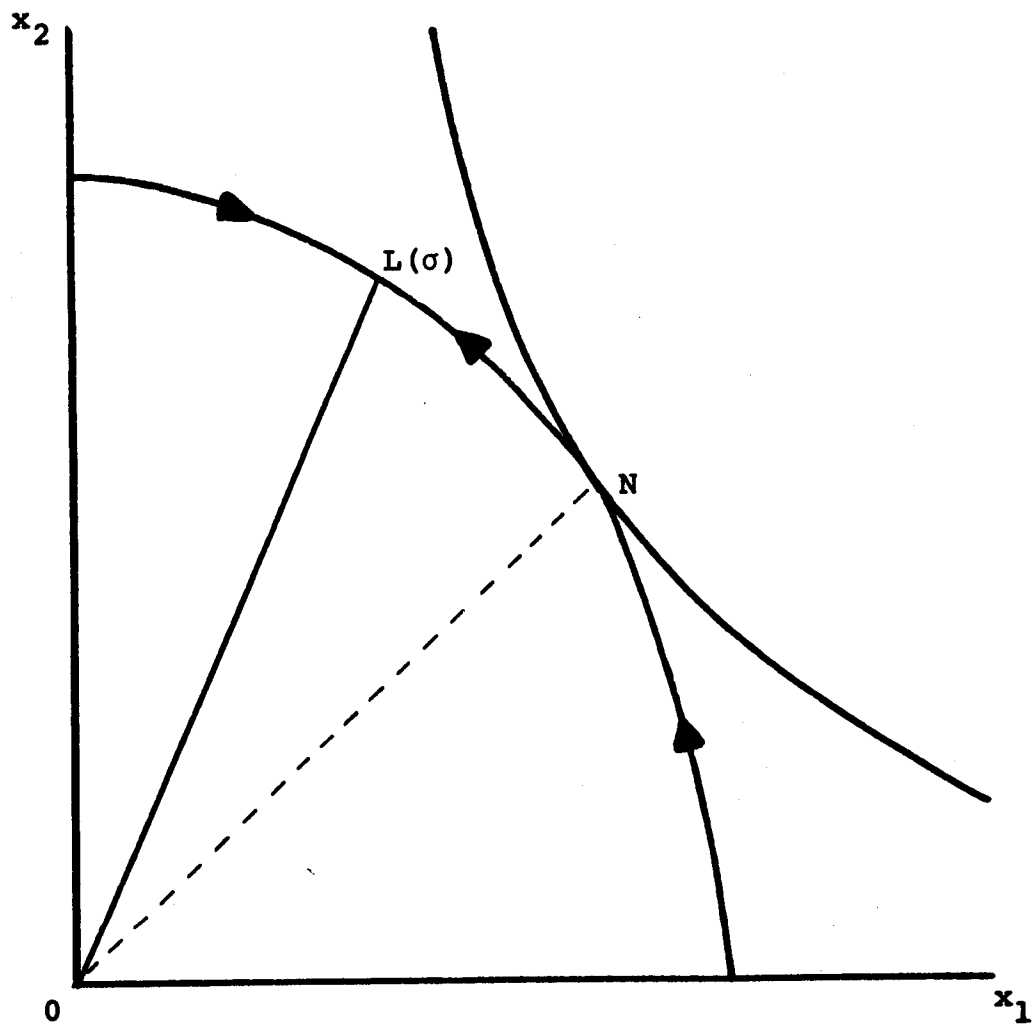


Figure 1

of variation, our model need not meet their conditions. Indeed counterexamples can be constructed to show that a mean preserving spread need not lead to a further movement in x_1 away from the laissez faire output level. Conditions eliminating all such counterexamples are derived in an appendix. It is shown that by restricting only moderately the allowable class of mean preserving spreads the monotonicity of the marginal utility of income will again suffice to sign the impact of such a spread.

Having discussed, in some detail, the allocational implications of price uncertainty, it is natural to turn next to decentralization issues. If individuals' beliefs about the future differ, it is reasonable that they should change over time, as the actual price distribution is revealed. In this case there will be a desire to reopen markets in each period, or at least until all beliefs merge. However given our assumption of identical, correct beliefs a complete set of contingent futures markets can achieve the optimum.

Also implicit in our formulation is the independence of welfare in different future periods. This additional simplification makes the contingent markets unnecessary. Instead the optimum can be achieved with a stock and bond market. Individuals maximize expected utility by selling or buying either bonds or shares in the different firms. Each of the latter is assumed to produce only one final good. Owning a share entitles an individual to that share in the profits of the firm regardless of the uncertain outcome.

From such behaviour, the following simple rule can be derived:

$$\text{market value of a firm} = \left(\frac{\text{value of output}}{\text{output level}} \right) \cdot \text{output level} - \text{value of inputs}$$

It follows that each firm can very simply calculate the value of output per unit from the market value of its shares. Making the usual price taking assumption implies that this unit output value will be perceived as being independent of

actual output. Then it can be shown that if each firm behaves so as to maximize its market value, the full optimum is achieved.⁷

However, as noted in the introduction, the assumption of the existence of such a stock market is also a strong one - especially for a developing country. In such cases the preferences of the owners of a firm should be explicitly considered in analyzing the firm's decision. For simplicity we consider here the extreme case in which ownership is by individuals or foreign firms, wealthy enough that risk aversion can be ignored. In such a world firms will operate to maximize expected profits.

For the j th firm profits are

$$p_j x_j(L_j) - w \cdot L_j$$

where $x_j(\cdot)$ is the production function, L_j is a vector of inputs and w the factor price vector. Maximization of expected profits requires

$$E(p_j) \frac{\partial x_j}{\partial L_{jk}} = E(w_k)$$

With all firms behaving in this fashion, the economy will produce where

$$\frac{E(p_j)}{E(p_H)} = \bar{p}_j = \frac{\varepsilon_j}{\varepsilon_H} \quad j = 1, \dots, H \quad (8)$$

But these are exactly the production conditions in the case where the prices are \bar{p}_j with certainty. Thus under laissez-faire the optimal production vector x is unaffected by the introduction of price uncertainty.

Such suboptimal behaviour is the result of firms not taking account of the

⁷ For a full discussion of the stock-market model, see the paper by P.A. Diamond, "A Stock Market in a General Equilibrium Model". A.E.R. LVII No. 4 (Sept. 1967), 759-776. The details of the adaptation of his one-good model with technological uncertainty are available on request from the authors.

impact of their decisions on the marginal utility of income. This inefficiency can, however, be overcome by changing the expected price for firms from \bar{p} to $E(pV_0)/E(V_0)$. All that is necessary is the introduction of a subsidy on production equal to the difference. But this is just the vector τ so its components can now be reinterpreted as production subsidies ($\tau_i > 0$) or taxes ($\tau_i < 0$).

For the special case in which only the i th good has an uncertain price, and it is exported, the previous analysis has indicated that the optimal τ_i is negative. Then given profit maximizing behaviour by producers it follows that it is optimal for the government to tax production of this good.

All this suggests that there is a formal basis for government intervention owing to price uncertainty. However the use of tariffs in such circumstances is clearly second best, since this has the undesirable effect of driving a wedge between domestic rates of substitution and international price ratios.

To conclude this section we now relax the assumptions of period by period trade balance, and of prohibitive storage costs. Each is defensible as a first approximation to reality, since storage of either goods or foreign exchange is costly and seldom undertaken on the potentially enormous scale necessary to offset price fluctuations. Nevertheless, it may be useful to examine the polar cases of costless storage and of only a long run (average) foreign exchange constraint.

In the latter problem, we assume a Santa Claus is willing to lend unlimited exchange at no interest subject only to being paid back eventually. We simplify the discussion considerably by reinterpreting the probability distribution as a frequency distribution, derived from an unchanging cycle of international prices.

Then

$$E(p) = \sum_k p(\theta_k) h(\theta_k) = \frac{1}{T} \sum_{s=1}^T p^s$$

where T is the length of the cycle.

Assuming no discounting of the future, society's problem is the following.

$$\begin{aligned} \text{Max } E U(c) &= \frac{1}{T} \sum_{s=1}^T U(c^s) \\ \text{s.t. } \sum_{s=1}^T p^s (c^s - x) &\leq 0 \text{ and } g(x) \geq 0 \end{aligned}$$

Solving for the first order conditions we obtain

$$\frac{E(U_j)}{E(U_H)} = \frac{E(p_j)}{P_H} = \frac{\varepsilon_j}{\varepsilon_H} \quad j = 1, \dots, H$$

Comparing the last expression with equation (8) it follows immediately that expected profit maximization is no longer sub-optimal. The reason is straightforward. Maximizing expected profits is equivalent here to maximizing total profit, and hence national income, over the cycle. Then given the ability to freely borrow and lend, and therefore to redistribute income over time, such a policy is bound to be optimal.

Turning now to the implications of the alternative assumption of costless storage of goods, the problem must be further modified. We continue with the frequency distribution interpretation, in order to avoid a dynamic programming problem. In such circumstances the small country assumption is no longer tenable, since infinite purchases at below average prices, and infinite sales at above average prices, would result. If instead we assume that the domestic country faces a fluctuating offer curve, the solution will involve complete stabilization through intertemporal arbitrage.⁸

Of course storage and foreign exchange credits are costly so the real story, lying between those polar cases, involves some borrowing at increasing cost and

⁸See the discussion of the Hueth and Schmitz paper in Section III.

some storage of durable goods. Production taxes will still be necessary in the absence of a perfect stock market. Moreover neither future price fluctuations nor probability distributions are known with certainty so that any position taken now about the future involves additional uncertainty. However any attempt at such refinements would take us beyond the scope of this paper.

We conclude the discussion of resource allocation by noting that although the analysis is highly theoretical, it may have some empirical applicability. Suppose truncation of the Taylor's series expansion of V_0 is justified. Previously we had $\tau = \frac{1}{E(V_0)} C(V_0, p)$ and from equation (5), $C(V_0, p) = \Omega_{pp} \nabla V_0(\bar{p})$. From equation (7), $\nabla V_0(\bar{p}) = -V_0 \frac{\partial c}{\partial m} - tV_{00} = V_0(\bar{p}) \left[-\frac{\partial c}{\partial m} - t \frac{V_{00}}{V_0} \right]$ and we note that $\frac{-V_{00}}{V_0} = r(\bar{p})$, the coefficient of absolute risk aversion evaluated at \bar{p} . Then:

$$\begin{aligned} \tau &= \frac{V_0(\bar{p})}{E(V_0)} \Omega_{pp} \left[-\frac{\partial c}{\partial m} + tr \right] \\ &\approx -\Omega_{pp} \left[\frac{\partial c}{\partial m} - tr \right], \text{ given the truncation assumption.} \end{aligned}$$

All the terms in this last expression are in principle measurable except for r . Then with agreement on r by social planners τ may also be calculated.

II. Welfare and Price Fluctuations

We now turn to the issue of whether international price fluctuations benefit or hurt a small trading country. Not surprisingly Jensen's inequality is useful. As generalized by Hartman,⁹ it can be expressed as follows:

The expected value of a real valued convex (concave) function increases (decreases) or remains unchanged when the joint distribution of the arguments undergoes a mean preserving spread.

⁹ R. Hartman, "The Effects of Price and Cost Uncertainty on Investment," Journal of Economic Theory, V, No. 2 (October 1972), 258-266.

Then if we can find ranges of price variation R over which the indirect utility function $V(p \cdot x^*, p)$, $x^* \in \{x(p) | p \in R\}$ is either convex or concave in prices, we can infer immediately the welfare implications of increasing price variation over such ranges. This perhaps deserves elaboration. Let s and q be two random price vectors, $s, q \in R$, and s is riskier than q . With convexity holding for all p and x^* , we note that $E[V(s \cdot x_q^*, s)] \geq E[V(q \cdot x_q^*, q)]$ where x_q^* is the optimal value of x given the random variable q . Hence a fortiori, $E[V(s \cdot x_s^*, s)] \geq E[V(q \cdot x_q^*, q)]$, where x_s^* is the optimal value of x given the random variable s . With concavity holding, $E[V(s \cdot x_s^*, s)] \leq E[V(q \cdot x_q^*, q)]$, hence a fortiori, $E[V(s \cdot x_s^*, s)] \leq E[V(q \cdot x_q^*, q)]$.

As we shall see, it is not true, even for very well behaved welfare functions, that a country always gains from externally generated price uncertainty. However quite broad conclusions can be drawn, especially for the case of uncertainty in the price of the i th good alone. We therefore examine the second derivative of V with respect to this price.

$$\frac{\partial V}{\partial p_i} = x_i V_0 + V_i$$

But from the derivation of the indirect utility function

$$V_i = -c_i V_0$$

therefore

$$\frac{\partial V}{\partial p_i} = -t_i V_0 \quad (9)$$

and

$$\frac{\partial^2 V}{\partial p_i^2} = -V_0 \frac{\partial t_i}{\partial p_i} - t_i \frac{\partial V_0}{\partial p_i}$$

With x_i chosen ex ante, the first term simplifies and we have

$$\frac{\partial^2 V}{\partial p_i^2} = -V_0 \frac{\partial c_i}{\partial p_i} \Big|_{\text{comp}} + V_0 t_i \frac{\partial c_i}{\partial m} - t_i \frac{\partial V_0}{\partial p_i}$$

Then substituting for $\frac{\partial V_0}{\partial p_i}$ from equation (7) the second derivative can be expressed as follows.

$$\frac{\partial^2 V}{\partial p_i^2} = t_i^2 V_{00} + 2t_i V_0 \frac{\partial c_i}{\partial m} - V_0 \frac{\partial c_i}{\partial p_i} \Big|_{\text{comp}} \quad (10)$$

Assuming that the i th good is normal, the second and third terms in this expression are both positive, if at all feasible prices the i th good is imported ($t_i > 0$). Therefore if society is risk neutral ($V_{00} = 0$) and the probability distribution $F(p)$ is such that good i is never exported, we can conclude that society will be better off under a mean preserving increase in price uncertainty.

However if the i th good is exported, or society is risk averse and a large trader in the i th good, it is quite possible that V is concave over some range of p_i . It should be noted also that in the vicinity of the no trade point, expression (10) takes on the sign of the third term implying that V is convex. Therefore if price uncertainty is restricted to this range society must gain from a mean preserving spread. Intuitively, if p is always near p^N , the no trade equilibrium price, x^* will be near the corresponding no trade point x^N . But at x^N any price is at least as good as p^N so society gains from the possibility of prices other than the latter.

These results are generalized to allow for fluctuations in more than one price in Appendix 2. Here we return to the special Cobb-Douglas case and show that V is always concave in the price of a good if the country is sufficiently specialized in the production of that good. Moreover, it is demonstrated that if the degree of risk aversion is sufficiently large, V is also concave when the country is a heavy importer of the good.

From Section I

$$V = \frac{1}{1-\sigma} [(p_1 x_1 + p_2 x_2) p_1^{-\alpha} p_2^{-(1-\alpha)}]^{1-\sigma}$$

Differentiating with respect to p_1 and noting that for the Cobb-Douglas case

$$\frac{p_1 c_1}{p_2 c_2} = \frac{\alpha}{1-\alpha}$$

we have
$$\frac{\partial^2 V}{\partial p_1^2} = A(p, x^*) \left[\frac{\alpha}{\alpha + \sigma(1-\alpha)} - (1-\alpha)^2 \left\{ \frac{p_1 x_1^*}{p_2 x_2^*} - \frac{p_1 c_1}{p_2 c_2} \right\}^2 \right] \quad (11)$$

where $A(p, x^*) > 0$

To determine the sign of the square bracket we first examine the related expression

$$\begin{aligned} S &= \left[\frac{\alpha}{\alpha + \sigma(1-\alpha)} - (1-\alpha)^2 \left\{ \frac{p_1 x_1(p)}{p_2 x_2(p)} - \frac{p_1 c_1}{p_2 c_2} \right\}^2 \right] \\ &= a_0(\sigma) - (1-\alpha)^2 \left\{ \frac{p_1 x_1(p)}{p_2 x_2(p)} - \frac{\alpha}{1-\alpha} \right\}^2 \end{aligned}$$

At the no trade equilibrium price the bracket $\{ \}$ is zero. Therefore $S(p_1)$ takes on a maximum at p_1^N , and, since $a_0(\sigma) > 0$, the maximum value is positive. As p_1 increases further, the second term decreases without bound hence there must be a price $p_1'(\sigma)$ at which the sign of S switches from positive to negative. Similarly as p_1 declines from p_1^N the second term decreases to a lower bound $-\alpha^2$. For small σ this does not offset $a_0(\sigma)$. However if $\sigma > 1 + 1/\alpha$ the lower bound does more than offset $a_0(\sigma)$, hence there is another switch point $p_1''(\sigma)$. Between the switch points S is positive and beyond them it is negative.

Now suppose p_1 is uncertain but lies in one of these three regions (denoted R_j , $j = 1, 3$). Specifically suppose p_1 lies in R_2 i.e. $p_1''(\sigma) < p_1 < p_1'(\sigma)$. Then x^* must lie on the production possibility surface between $x(p'')$ and $x(p')$,

$$\text{i.e.} \quad \frac{x_1(p'')}{x_2(p'')} < \frac{x_1^*}{x_2^*} < \frac{x_1(p')}{x_2(p')}$$

and since p_1 belongs to R_2 we must also have,

$$\frac{p_1''}{p_2} \frac{x_1(p'')}{x_2(p'')} < \frac{p_1'' x_1^*}{p_2 x_2^*} < \frac{p_1 x_1^*}{p_2 x_2^*} < \frac{p_1' x_1^*}{p_2 x_2^*} < \frac{p_1' x_1(p')}{p_2 x_2(p')}$$

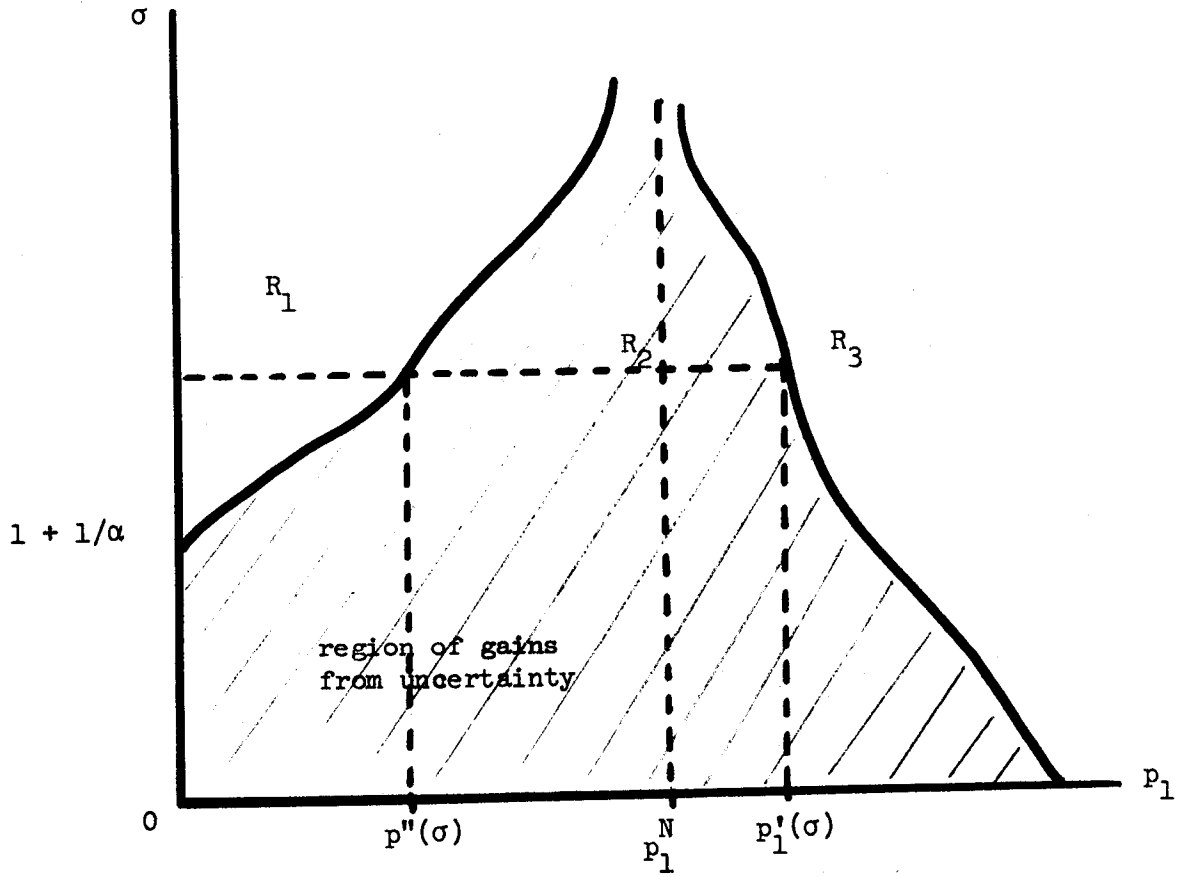
It follows that the square bracket in (11) is definitely positive and hence that $\frac{\partial^2 V}{\partial p_1^2}$ is positive. By an almost identical argument it can be shown that the opposite must be true if p_1 lies always in either R_1 or R_3 . This is depicted in figure 2. Note that as σ increases the two switch points approach one another, and in the limit both tend towards the no trade equilibrium price.

This last result is explained by noting again that the limiting case ($\sigma = \infty$) is equivalent to maximizing the minimum utility. If x^N is the optimal output vector under price certainty, then the minimum utility is at least as great as $U(x^N)$ under a mean preserving introduction of price uncertainty. However if any other output vector is optimal under certainty, the introduction of uncertainty introduces the possibility of being worse off.

Finally we note that while similar results should be obtainable for a much wider class of utility functions there is no reason to expect to be able to always obtain this 3 region conclusion. This is obvious from an examination of equation (10). For example as soon as the assumption of constant relative aversion is dropped there is no simple relation between V_0 and V_{00} . However there is always a region of convexity of V around the no trade price, and for heavy enough trade and high risk aversion, V will always be concave.

Figure 2

The Welfare Implications of Price Uncertainty



III. The Social Surplus Approach

The result of Section II that gains or losses from price fluctuations are not certain a priori, is in sharp contrast to the results obtained by Hueth and Schmitz. The latter, using a social surplus partial equilibrium approach, studied a case where production was freely variable and storage was costless. The trading country was not 'small' and the fluctuations took the form of a parallel shift in either the domestic or the foreign supply schedule, with the two positions alternating from one period to the next. Their conclusion was that the domestic country was always better off without stabilization if the source of the disturbance were foreign and would always prefer stabilization if the source were domestic.

There are two misleading aspects to their work besides difficulties associated with using partial equilibrium. We shall put their analysis in general equilibrium terms and show that their treatment of stabilization was at least incomplete, and that their measure of gain was not the one appropriate to analysing welfare under conditions of instability. Countries always gain from stabilization where costless storage is available under perfect foresight no matter what the source of disturbance. If storage is impossible, they may gain or lose from either type of disturbance and again the source does not matter.

Hueth and Schmitz use consumer's and producer's surplus measures to analyze a single market keeping all other prices constant. It is more general (and more revealing) to rid the analysis of partial equilibrium supply and consider the aggregate marginal cost curve. For the domestic country it is optimal to equate the price of a good not only with the marginal cost of domestic production but also with the marginal cost of importing. Therefore the relevant marginal cost

curve is obtained by summing horizontally the domestic supply curve and the marginal cost curve for imports. This is shown in Figure 3. Each 'supply' curve is in fact the marginal cost, for a large trading country, of moving around the Baldwin envelope.¹⁰ The area behind it then has a simple interpretation as the change in income from production and trade.¹¹ The demand curve depicted in the figure is the compensated demand for the i th good at the utility level associated with the stable price p_i^s . Then the area behind D_i also has an income interpretation. It is the additional income needed to return utility to the level at p_i^s .

Clearly intertemporal arbitrage, with costless storage by the home country, will stabilize the price at p_i^s between p_i^0 and p_i^1 , where the quantity displacements about the intersection of D_i with p_i^s are equalized. Moreover the arbitrage brings with it a social gain depicted by the shaded triangles. Hueth and Schmitz arrive at their conclusions because they (implicitly) assume that while storage is costless for domestic disturbances, it is prohibitively expensive for foreign disturbances. If the source of disturbance is foreign the question implicitly is whether the domestic economy would prefer fluctuations of prices to the stabilized price if the foreigner did the stabilizing. Their answer is yes, using social surplus, and is simply another example of the Waugh and Oi Theorems.¹² Reinterpreting D_i as compensated excess demand and MC_i as marginal

¹⁰Note that this is not the usual supply function which adds, for each price, the domestic supply and the foreign offer. The vertical distance between the two curves gives the optimal tariff.

¹¹James E. Anderson, "A Note on Welfare Surpluses and Gains from Trade in General Equilibrium," American Economic Review, forthcoming.

¹²These are summarized in P.A. Samuelson in his note on "Feasible Price Stability", Quart. J. Econ., LXXXVI, No. 3 (August 1972) 476-493. With the economy in question not closed it is possible to apply the theorems, comparing the fixed price regime with the high-price low-price regime, each with prices set by the foreigner. The latter always dominates in areal measure, but intertemporal arbitrage (if possible) (cont. on bottom of page 22.)

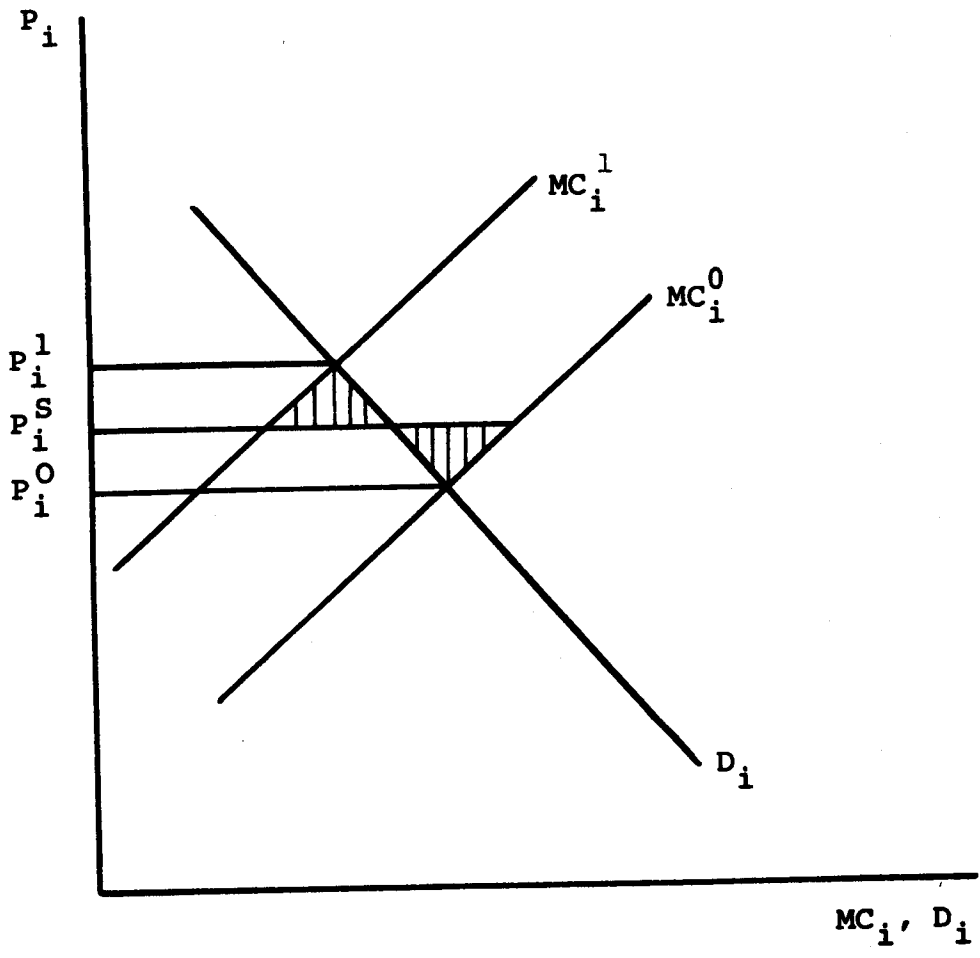


Figure 3

cost of imports, foreign stabilization results in some MC_i^S (not drawn) with an equilibrium price of p_i^S . The high-price, low-price regime then dominates in an areal sense, since the area behind D_i lost with the high price is more than offset by the area behind D_i gained from the low price.

In essence Hueth and Schmitz integrated equation (9) ignoring changes in V_0 to obtain

$$W = \int_{p_i}^{p_i'} V_0 t_i(p) dp_i \approx V_0 \int_{p_i}^{p_i'} t_i(p) dp_i$$

Since $t_i(p)$, the demand for imports, is a decreasing function of price they concluded that the gains from the low price exceeded the losses from the high price.

While the diagram is unimpeachable, the analysis is not. It is impossible for V_0 to be constant with respect to both income and prices, and the analysis of section II shows that "approximate constancy" is a dangerous assumption in the context of trading nations facing fluctuating prices. It is in fact perfectly possible for a risk averse nation to lose from import price fluctuations. The social surplus approach, with its equal weighting of each dollar of income, is often adequate in determining the direction of welfare changes but may

¹²(cont. from page 23) does still better. From the world point of view, arbitrage dominates, as Hueth and Schmitz note, at least in the sense of the compensation principle. Our diagram attributes all the arbitrage gain to the home country consistent with the Cournot-type assumptions of the optimal tariff treatment, while Hueth and Schmitz attribute it to the country whose supply shifts. In general, both countries will engage in arbitrage, the gain will be split, and without compensation one could lose. Properly interpreted, their contribution appears to lie in making this point.

be very misleading if used to determine magnitudes. Figure 3 provides a simple example. It is correct to conclude that arbitrage is beneficial since the gain is the sum of the two shaded areas. However it would be quite incorrect to conclude that half the gain should be imputed to each period just because (in the case drawn) the triangles have equal area.

As Section I demonstrated we can say a lot about the variation in the marginal utility of income owing to price changes. Explicit treatment of the marginal utility of income in welfare economics is usually eschewed because of its cardinal nature. However, unless one rejects the hypothesis of expected utility maximization in the analysis of uncertainty, cardinality is automatically introduced. Therefore for this class of problem perhaps the social surplus approach has outlived its usefulness.

APPENDIX 1. The Impact of a Mean Preserving Increase in Price Uncertainty

In Section I it was shown that the monotonicity of V_0 with respect to price was sufficient to determine the sign of the optimal change in production of a good resulting from the introduction of price uncertainty. Unanswered was the question of whether a further movement of x^* could be so determined by a further mean preserving spread in the distribution of p_i . Rothschild and Stiglitz have presented strong sufficient conditions for such a result involving a third derivative of V . However for our problem these turn out to be very messy so instead we look to extend the results of Section I by restricting the class of allowable mean preserving spreads.

The notation we use follows that of Diamond and Stiglitz.¹³ We denote the initial distribution by $F(p_i, r)$ where increases in the shift parameter r represent increases in risk. Then the following conditions must be satisfied.

$$T(z, r) = \int_a^z F_r(p_i, r) dp_i \geq 0; \quad a \leq z \leq b$$

with $F_r(a, r) = F_r(b, r) = T(a, r) = T(b, r) = 0$

We now examine the effect of an increase in r on the optimal value of τ_i . Since the results are best interpreted in the case of decentralization with firms maximizing expected profits we will use this case, and hence write of a tax τ_i on the i th good.

From equation (4) in Section I this is given by:

$$\tau_i = \frac{E(V_0(p)p_i)}{E(V_0(p))} - \bar{p}_i = \frac{\int p_i V_0(p) F_{p_i}(p, r) dp_i}{\int V_0(p) F_{p_i}(p, r) dp_i} - \bar{p}_i$$

¹³P.A. Diamond and J.E. Stiglitz, "Increases in Risk and in Risk Aversion," M.I.T. Working Paper No. 97, January, 1973.

For the remainder of the discussion we will for notational convenience drop the subscript i . It should be remembered however that we are dealing only with uncertainty in the price of a single good. Differentiating totally with respect to r we obtain:

$$\frac{\partial r}{\partial r} = \frac{\int p V_0'(p) F_{pr} dp \int V_0(p) F_p dp - \int V_0(p) F_{pr} dp \int p V_0'(p) F_p dp}{[\int V_0(p) F_p dp]^2}$$

We can evaluate the numerator, Q , with integration by parts.

$$\begin{aligned} Q &= - \int (V_0 + p V_0') F_r dp \int V_0 F_p dp + \int V_0' F_p dp \int p V_0 F_p dp \\ &= - \int V_0 F_r dp \int V_0 F_p dp - \int V_0' F_r [\int (p-z) V_0(z) F_z(z,r) dz] dp \end{aligned} \quad (12)$$

The first term can be integrated again and written as

$$\int V_0'(p) T(p,r) dp \int V_0 F_p dp$$

Since T is positive the term must be negative if V_0 is increasing everywhere in p and positive if decreasing everywhere. Therefore if the second term does not offset it completely, our result in Section I does generalize. That is, the monotonicity of $V_0(p)$, not only determines the sign of the optimal tax, but also the impact of mean preserving increases in risk.

However it is not always true that the second term is dominated by the first so further conditions are necessary.

Writing p^* as the optimal price before the increase in risk we have

$$p^* = \bar{p} + \tau = \frac{E(V_0(p)p)}{E(V_0(p))}$$

Then it is immediate that for $p < p^*$ the square bracket in expression (12) is negative and for $p > p^*$ the bracket is positive.

In general $F_r(p,r)$ may change sign any odd number of times. However in its most intuitive form a mean preserving spread involves a simple shift in weight to the tails of the distribution and hence a single 'crossing' at some price \tilde{p} . This is shown in Figure 4. Restricting ourselves to this class of 'simple mean preserving spreads' we have $F_r(p,r)$ either positive or zero for smaller p and negative or zero for larger p . It follows that

$$H(p) = F_r \left[\int (p-z)V_0(z)F_z(z,r)dz \right] \geq 0 \quad \min(\tilde{p}, p^*) < p < \max(\tilde{p}, p^*) \\ \leq 0 \quad \text{all other } p.$$

Then introducing the notation $p' = \min(\tilde{p}, p^*)$, $p'' = \max(\tilde{p}, p^*)$ we can rewrite the numerator as,

$$Q = -\int_0^\infty V_0'(p)T(p,r)dp \int V_0 F_p dp - \int_0^{p'} V_0'(p)H(p)dp - \int_{p''}^\infty V_0'(p)H(p)dp - \int_{p'}^{p''} V_0''(p)H(p)dp$$

From the above discussion the first three terms all have the sign of $-V_0'(p)$, and the fourth term has the opposite sign. Therefore as long as $[p'' - p']$ is sufficiently small the last term is definitely dominated by the other three. But $p'' = p'$ if the increase in risk does not alter the probability that p is less than p^* , the expected price for producers. (This case is depicted in Figure 4).

Therefore if a simple mean preserving increase in risk does not alter too

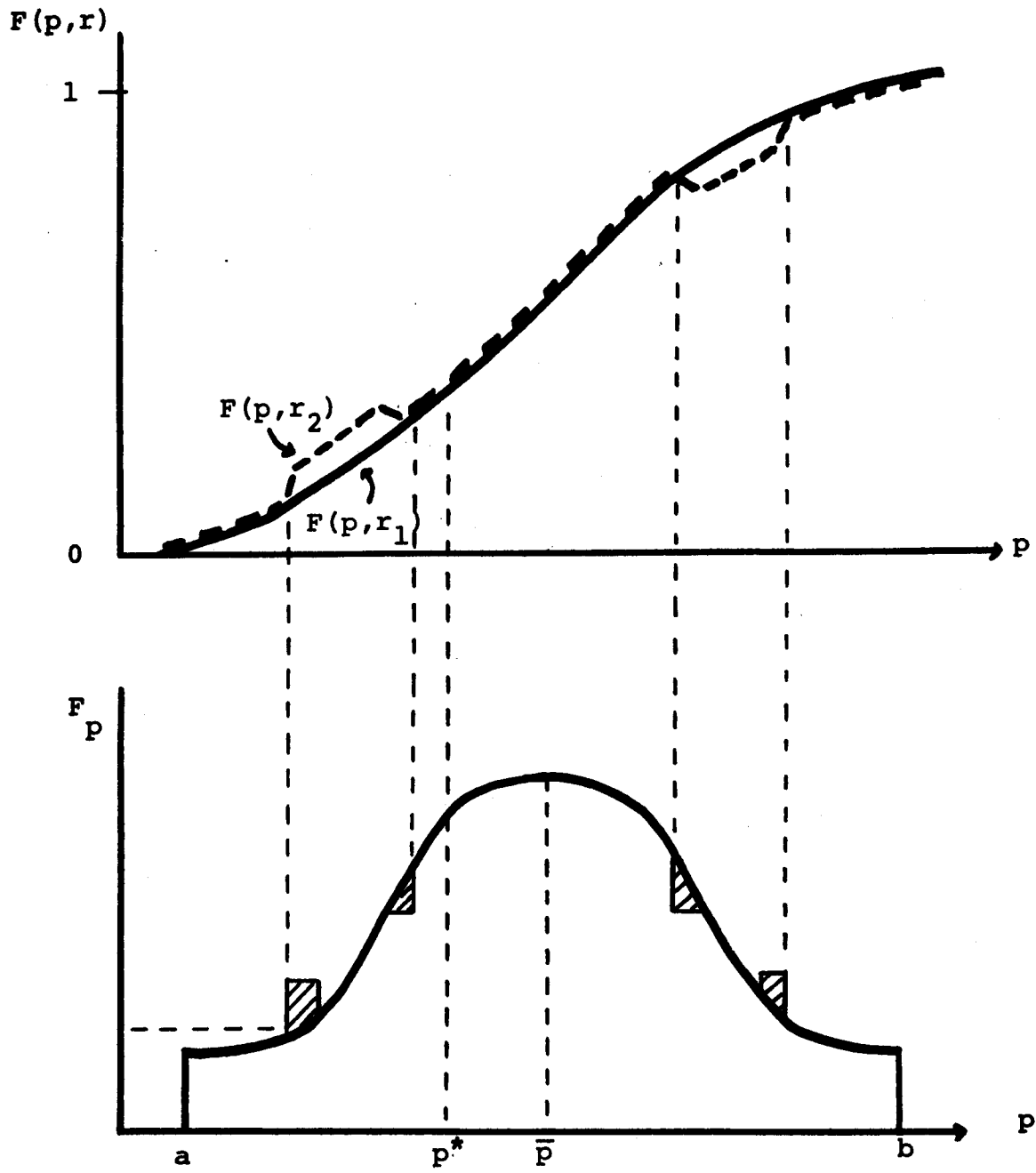


Figure 4

Appendix 1
page 4

much, the probability that p_i is less than p_i^* , we can conclude that the size of the optimal tax (subsidy) will increase, as long as the marginal utility of income is a monotonic function of p_i .

APPENDIX 2. Welfare Implications of Multi-price Fluctuations

At the beginning of Section II it was demonstrated that in the absence of risk aversion (to fair gambles over lump sum income) a country would benefit from uni-price uncertainty with normality (a) if the range of the price distribution lay sufficiently close to the no trade equilibrium price, and (b) if the good with the uncertain price was always imported. We now extend these results to allow for multi-price uncertainty.

From equation (8) in Section II;

$$\frac{\partial V}{\partial p_i} = -t_i V_0 \quad i = 1, \dots, H$$

therefore

$$\frac{\partial^2 V}{\partial p_i \partial p_j} = \frac{-\partial c_i}{\partial p_j} V_0 + t_i \frac{\partial V_0}{\partial p_j} \quad i, j = 1, \dots, H$$

Introducing the Slutsky decomposition and substituting from equation (7) this can be rewritten as:

$$\frac{\partial^2 V}{\partial p_i \partial p_j} = t_i t_j V_{00} - V_0 \left\{ \left. \frac{\partial c_i}{\partial p_j} \right|_{\text{comp}} - t_i \frac{\partial c_j}{\partial m} - t_j \frac{\partial c_i}{\partial m} \right\}$$

At the no trade point and in the absence of risk aversion this reduces simply to

$$\left[\frac{\partial^2 V}{\partial p_i \partial p_j} \right] = -V_0 \left[\left. \frac{\partial c_i}{\partial p_j} \right|_{\text{comp}} \right]$$

where each of the square brackets is an HxH matrix. But the matrix on the right hand side is just the Slutsky matrix and is therefore negative semi-definite. Then, since V_0 , the marginal utility of income is positive the left hand matrix

is positive semi-definite. It follows that in some neighborhood of the no trade point, the indirect utility function is convex in the price vector p . Therefore applying Jensen's Inequality the domestic country gains from a mean preserving increase in international price uncertainty if the price vector is always sufficiently close to the no trade equilibrium price vector p^N .

We next assume that price uncertainty exists only for a subset of the H commodities. Without loss of generality suppose these are the first I commodities. Then retaining the assumption of risk neutrality we have

$$\left[\frac{\partial^2 V}{\partial p_i \partial p_j} \right] = -V_0 \left[\frac{\partial c_i}{\partial p_j} \Big|_{\text{comp}} \right] - \left[t_i \frac{\partial c_i}{\partial m} + t_j \frac{\partial c_j}{\partial m} \right]$$

where the square brackets are $I \times I$ square matrices. The second matrix on the right hand side is clearly symmetric. If in addition the first I goods are normal goods ($\frac{\partial c_i}{\partial m} > 0$) and are imports ($t_i \geq 0$) this matrix is positive. But subtracting a positive symmetric matrix from a negative definite matrix yields a negative definite matrix. Therefore the left hand matrix is positive definite implying that V is convex in the first I prices. The desired result then follows immediately from Jensen's Inequality.

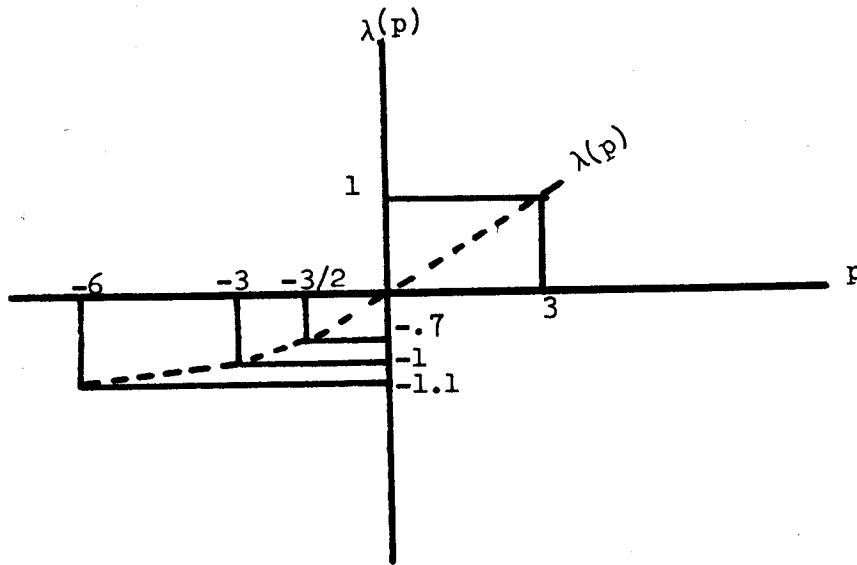
Additional Note

Demonstration that the covariance of a random monotonic function with its argument need not increase as the variance of the argument increases.

We utilize two simple distributions, with an unspecified monotonic function, and work in deviations about the mean (λ and p have the mean subtracted).

Distribution 1: $p = (3, -3)$; $pr(1/2, 1/2)$; $\lambda(3) = 1$; $\lambda(-3) = -1$; $E(p) = 0$;
 $V(p) = 9$; $c(p, \lambda(p)) = 3$

Distribution 2: $p = (3, -3/2, -6)$; $pr(1/2, 1/3, 1/6)$; $\lambda(3) = 1$; $\lambda(-3/2) = -7$;
 $\lambda(-6) = -1.1$; $E(p) = 0$; $V(p) = 11 \frac{1}{4}$; $c(p, \lambda(p)) = 2.95$



List of Symbols and Characters
(in order of appearance)

U	utility	N	no trade equilibrium point
x	production vector	R	range of feasible values of p
t	trade vector	C	consumption vectors
p	price vector	A(p,x)	some positive function
F(p)	probability distribution function		
V	indirect utility		
E	expectation operator		
g(x)	transformation surface		
H	number of final goods		
l_{ji}	factors of production		
W_j	factor rentals		
τ	'divergence vector'		
$\left. \begin{matrix} Y \\ z \end{matrix} \right\}$	functions used in the lemma		
Ω_{pp}	covariance matrix		
Var(P_i)	variance		
C(P_i, P_j)	covariance		
∇	gradient		
α	coefficient of Cobb-Douglas utility		
σ	degree of risk aversion		
B(p,x)	some positive function		
L(σ)	switch point related to Figure 1		