

Competitive Signalling

by

John G. Riley*

Discussion Paper No. 50

June 1974

Preliminary Report on Research in Progress
Not to be quoted without permission of the author.

*I am indebted to M. Spence and many of my colleagues at UCLA, especially F. Welch, for helpful discussions.

Abstract

With imperfect information about product quality there are incentives for buyers to make use of proxy variables as 'signals'. Receive theory suggests that under certain circumstances there exists an infinite number of 'signalling equilibria' all of which successfully distinguish quality differences.

In this paper it is shown that from the family of 'equilibria' only one, the Pareto supreme member, survives plausible experimentation by firms. With moderately more sophisticated experimentation the market is shown to be unstable.

I. Introduction

In several recent papers, Spence has examined the implications of using an imperfect proxy to predict quality prior to market transactions. The critical elements of his analysis are the alterability of the proxy and a negative correlation between the cost of upgrading it, and actual product quality. As a result, it pays sellers of higher quality products to upgrade the proxy further, thereby differentiating these products. In Spence's terminology, the level of the proxy becomes a 'signal' to potential buyers.

While the author suggests a wide variety of markets in which signalling might occur, his primary application is the labor market. The actual productivity of job applicants is unknown to potential hirers (firms) and possibly to the applicants themselves. However, assuming the cost of education is negatively correlated with underlying ability, individuals find it profitable to invest in education as a signal. This takes place whether or not the information carried by the signal is productive.

The main inferences are threefold. First of all, there is a whole class of equilibrium wage profiles, each of which is self confirming in the sense that ex-ante expectations about the relationship between productivity and education are fulfilled. Second, the private return to the signal exceeds its direct contribution to productivity as embodied human capital, in each of the possible equilibria. Third, this overinvestment can be eliminated, and hence total output can be maximized, by government control over the wage profile, or equivalently, by a tax on the signal.

In the following sections it will be demonstrated that the first two

conclusions must be significantly modified, with only modest relaxation of Spence's basic assumptions. Initially the notion of a self-confirming wage profile is explored. Implicitly Spence assumes that firms passively extrapolate the relationship between education and job skill into the region of no acceptance. However, it seems very reasonable to suppose instead, that firms experiment with higher offers, for lower educational requirements, until they do find some takers. Then only if such offers are unprofitable will we describe an equilibrium as being fully-confirmed.

It is shown below that of the family of equilibrium wage profiles only one, the Pareto-supreme member, is a fully-confirmed equilibrium. The properties of this equilibrium are then examined. All individuals are seen to be investing in education to a level where marginal costs exceeds marginal social product, except the least able, who invest as they would in a world of free information.¹

None of these conclusions require firms to be aware of the signalling phenomenon or of its implications. We therefore turn in Section III to the possibility that this information might become 'unscrambled'. It is shown that under such circumstances a competitive (Nash) equilibrium is unstable.

This problem of instability has also been analyzed in the context of competitive insurance markets by Rothschild and Stiglitz [1]. These authors asked whether different risk classes might not be distinguished by the degree of coverage which they purchased. Using a two class model they concluded that, for a wide range of plausible assumptions, there was no stable Nash equilibrium.² They also conjectured that for the case of a continuum of risk classes, a stable equilibrium was most unlikely. Since our proof of instability is readily adaptable to the Rothschild-Stiglitz problem, this

conjecture is indeed confirmed.³

II. Fully-Confirmed Signalling

The starting point for our analysis is a model discussed by Spence in [4]. Each person seeking a job has an underlying ability 'n', within the domain $[n^0, n^1]$ and the fraction of the population with ability less than n is assumed to be a differentiable function $F(n)$. Given general assumptions, outlined below, about the factors determining job skill and the cost of education, it is shown that there exist a family of equilibrium wage profiles

$$w = w(y, k) \quad \text{with } w_y, w_k > 0$$

where y is the level of educational attainment and k is an underdetermined parameter. Since a higher value of k implies a higher wage at all education levels, the multiple equilibria are ranked by the Pareto criterion.

In this section it is first demonstrated that there exists a Pareto-supreme signalling equilibrium, and the characteristics of this equilibrium are explored. Next it is argued that plausible experimentation by firms, even if they do not recognize that signalling is occurring, will lead to the breakdown of any Pareto-inferior equilibrium. Only the Pareto-supreme member of the class of Spencian equilibrium profiles survives such experimentation.

Following Spence's notation,⁴ individual job productivity 's' is assumed to be an increasing function of underlying ability and education.

$$\text{i.e. } s = s(n, y) \quad \text{with } s_n > 0, s_y \geq 0.$$

Costs of education are assumed representable by a function $c(n, y)$ with

$c_{ny} < 0$. That is, marginal cost of an additional unit of education is lower for a person with greater underlying ability. For analytical convenience both functions are assumed twice differentiable. Since firms cannot distinguish underlying ability, wage offers are based on the proxy, education ($w = w(y)$). Each individual chooses an education $y(n)$ in order to maximize wage income less education costs.

$$\text{i.e. } \max_y i(n,y) = w(y) - c(n,y) \quad (1)$$

A signalling equilibrium is said to exist if there is a profile of wage offers $w(y)$ which is (partially) self-confirming. That is, net income maximizing individuals choose an education $y = y(n)$ such that actual productivity s , which is determined only at the end of the period, is equal to the wage offered.

For individual maximization of net income (1), at positive education levels, the following first and second order conditions must be satisfied.

$$w_y - c_y = 0 \quad (2)$$

$$w_{yy} - c_{yy} \leq 0 \quad (3)$$

Then since s_n is, by assumption positive, one can in principle invert the ex-post condition $s(n,y) = w(y)$, substitute for n in the first order condition and so obtain a first order differential equation for $w(y)$.

This is best clarified by way of an example. Suppose job productivity, $s(n,y) = (ny)^{1/2}$ and the cost of investing in an education $c(n,y) = (y/n)^{1/2}$. An individual of type n maximizes net income

$$i(n,y) = w(y) - (y/n)^{1/2}$$

by choosing y such that

$$i_y = w_y - \frac{1}{2}(ny)^{-1/2} = 0$$

In equilibrium, wages paid out, $w(y)$, must equal the actual job skill which is determined ex post.

$$\text{i.e. } w(y) = (ny)^{1/2}$$

Combining this with the first order condition yields

$$2ww_y = 1$$

and integrating we have

$$w(y) = (y + k)^{1/2}$$

where k is an arbitrary constant of integration.

Generally the solution of the differential equation can be expressed as

$$w = w(y, k) \quad \text{with } w_y, w_k > 0 \quad (4)$$

Each value of k yields a potential signalling equilibrium. However we shall now show that there is always a maximum value of k , denoted k^P , for which $w(y, k)$ represents a complete signalling equilibrium. Since all lower k values yield lower wages at all education levels, the wage profile $w = w(y, k^P)$ is the Pareto-supreme profile.

First we note that $s_n > 0$, hence the ex-post equilibrium condition $s(n, y) = w$ can be inverted and written as

$$n = n^*(y, w)$$

Substituting for w from (4) we have

$$n = n^*(y, w(y, k)) = n(y, k) \quad (5)$$

But differentiating $s(n, y)$ totally with respect to y we can eliminate $w'(y)$ and obtain the differential equation,

$$\frac{dn}{dy} = \frac{c_y - s_y}{s_n} \quad (6)$$

Since (5) is a one parameter family of functions which satisfy this differential equation it is indeed the complete solution. However only those members of the family which are defined over $[n^0, n^1]$ and which satisfy the second order condition (3) are actually full signalling equilibria. Totally differentiating the first order condition (2) we have, for all $y > 0$,

$$w_{yy} - c_{yy} = c_{ny} \frac{dn}{dy}. \quad (7)$$

The cross-partial deviation c_{ny} is by assumption negative, thus for a signalling equilibrium we require

$$\frac{dn}{dy} \geq 0 \quad \text{over} \quad [n^0, n^1]. \quad (8)$$

Then from (6), the entire equilibrium schedule must lie on or to the right of the curve $c_y - s_y = 0$.

While the slope of this boundary curve,

$$\left(\frac{c_{yy} - s_{yy}}{s_{ny} - c_{ny}} \right)$$

is not critical to the analysis, several weak additional assumptions are sufficient to sign it. For example if the marginal cost of education is an

increasing function of education and investment in education yields a strictly diminishing marginal return, the numerator is positive. In addition the denominator is positive on the assumption that education and underlying ability are complementary factors in the production of skill on the job.

Combining these assumptions yields a positively sloped boundary curve as in Figure 1. It is easy to check that dn/dy is positive to the right and negative to the left.

$$\text{From (5)} \quad \frac{\partial n}{\partial k} = \frac{\partial n^*}{\partial w} \cdot \frac{\partial w}{\partial k}$$

By assumption $\partial w/\partial k$ is positive. Moreover $\partial n^*/\partial w = 1/s_n$ is also positive. Therefore higher curves correspond to higher values of k . For each curve there is minimum education level $y^0(k)$ corresponding to the education of the lowest skill level n^0 , i.e. $n(y^0(k), k) = n^0$. As drawn in Figure 1 there is a k^P such that $y^0(k) > 0$ for all $k < k^P$. Furthermore for $k > k^P$ there is an interval $[n^0, n]$ over which no education is purchased. But such pooling is inconsistent with the notion of a completely separating signalling equilibrium. Therefore k is bounded above by k^P . Since the wage profiles $w(y, k)$ are ranked by the Pareto criterion according to the value of k , the Pareto-supreme profile is indeed $w(y, k^P)$.

The case just discussed is that suggested by Spence in an appendix to [2]. It requires that the boundary curve $c_y - s_y = 0$, cut the n -axis before reaching the minimum skill level. For this to occur, the marginal productivity $s_y(n^0, 0)$ of a first unit of education is, for the lowest skill level, less than $c_y(n^0, 0)$, the marginal cost of this unit.

More plausibly however, net marginal productivity is positive at sufficiently low education levels, as depicted in figure 2, and the

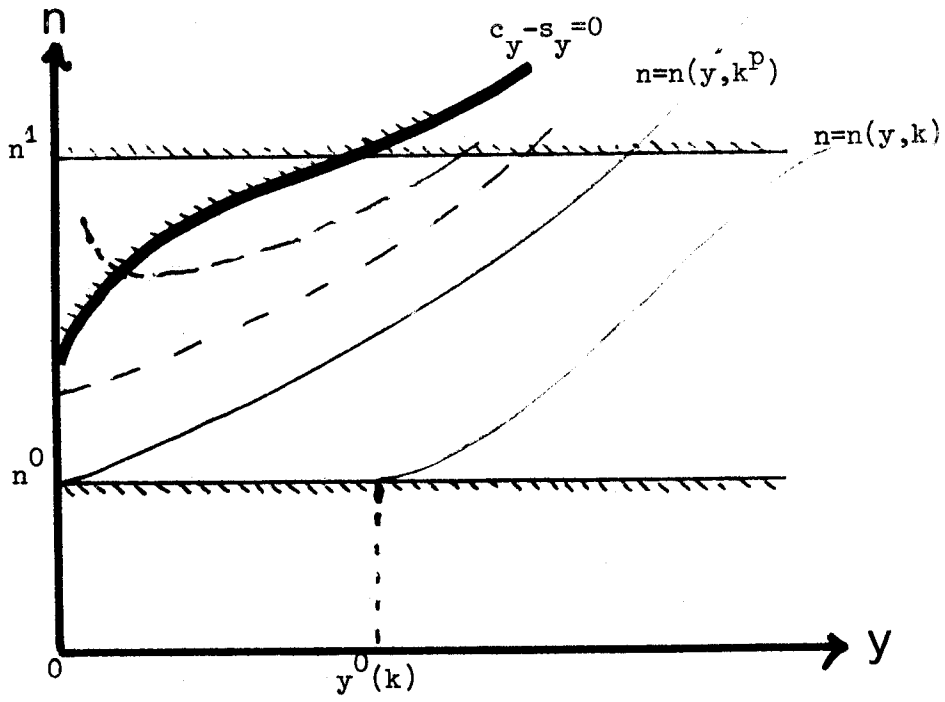


Figure 1

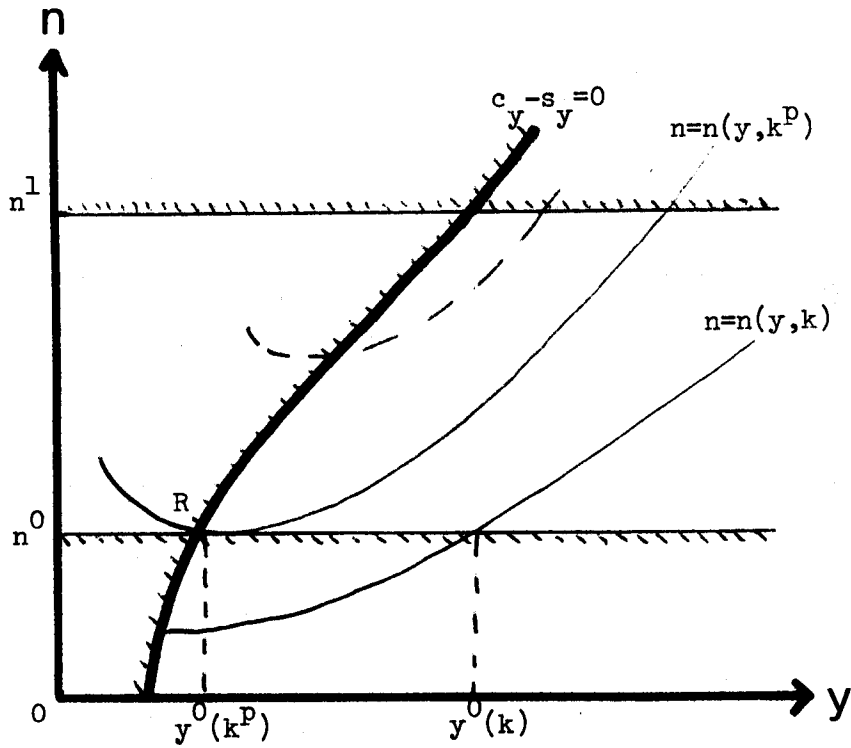


Figure 2

Signalling Equilibria

boundary curve, $c_y - s_y = 0$, intersects $n = n^0$ at R strictly within the positive quadrant.

From our discussion of the necessary conditions each equilibrium schedule has a unique turning point on the boundary curve. All schedules lying below R satisfy the necessary condition over $[n^0, n^1]$. On the other hand, $y = y(n)$ is undefined at low ability levels for any of the schedules lying above R. But as before, higher schedules correspond to higher values of k . Hence the Pareto-supreme schedule is indeed that which passes through R as depicted in Figure 2.

By examining the figures it is clear that in both cases the schedule $n = n(y, k)$ has a strictly positive slope over $[n^0, n^1]$ for all Pareto inferior schedules ($k < k^P$). Then the first and second order necessary conditions are together sufficient for the existence of a full signalling equilibrium.⁵

Moreover, totally differentiating the ex-post equilibrium condition yields

$$s_n \frac{dn}{dy} + s_y = w_y.$$

Substituting from (2) this can be rewritten as

$$s_y - c_y = -s_n \frac{dn}{dy} < 0; k < k^P \quad (9)$$

Equation (9) implies that for every worker the private marginal cost of education c_y exceeds the marginal social product s_y . In particular this must be true for the lowest skill group. It is important to note also that for all Pareto-inferior schedules, the least skilled purchase a non-zero quantity of education ($y^0(k) > 0$).

These two results will be crucial in analyzing the implications of experimentation by firms. The key to understanding the nature of the competitive process is to visualize the wage profile not as a schedule offered by a firm, but as the envelope of a set of offers of the type

$$\tilde{w}(\hat{y}) = \begin{cases} 0, & y < \hat{y} \\ w(\hat{y}, k), & y \geq \hat{y} \end{cases}$$

Then there are a subset of such offers for which there are no takers i.e.

$$\{\tilde{w}(\hat{y}) | \hat{y} < y^0(k)\}$$

To see that there are profitable new offers in this range consider Figure 3. Individuals of underlying ability n^0 maximize net income by choosing an education of $y^0(k)$ where the difference between the two solid curves $w = w(y, k)$ and $c = c(n^0, y)$ is greatest. The dashed curve is the cost curve shifted upwards by the maximized net income i^* . It therefore touches $w = w(y, k)$ at y^0 . All points above this curve, P , yield a higher net income and hence form a set of wage offers which will be preferred by individuals of type n^0 . Wage offers along the crossed, productivity curve $s(n^0, y)$ just break even therefore the set of profitable offers, Π are all the points below, as depicted.

But we have seen that in a signalling equilibrium with $k < k^p$, $s_y(n^0, y^0) < c_y(n^0, y^0)$ hence the two sets must have an intersection, $P \cap \Pi$ which is non empty. Therefore any offer of the form

$$w(\hat{y}) = \begin{cases} 0, & y < \hat{y} \\ w, & y \geq y, w \in P \cap \Pi \end{cases}$$

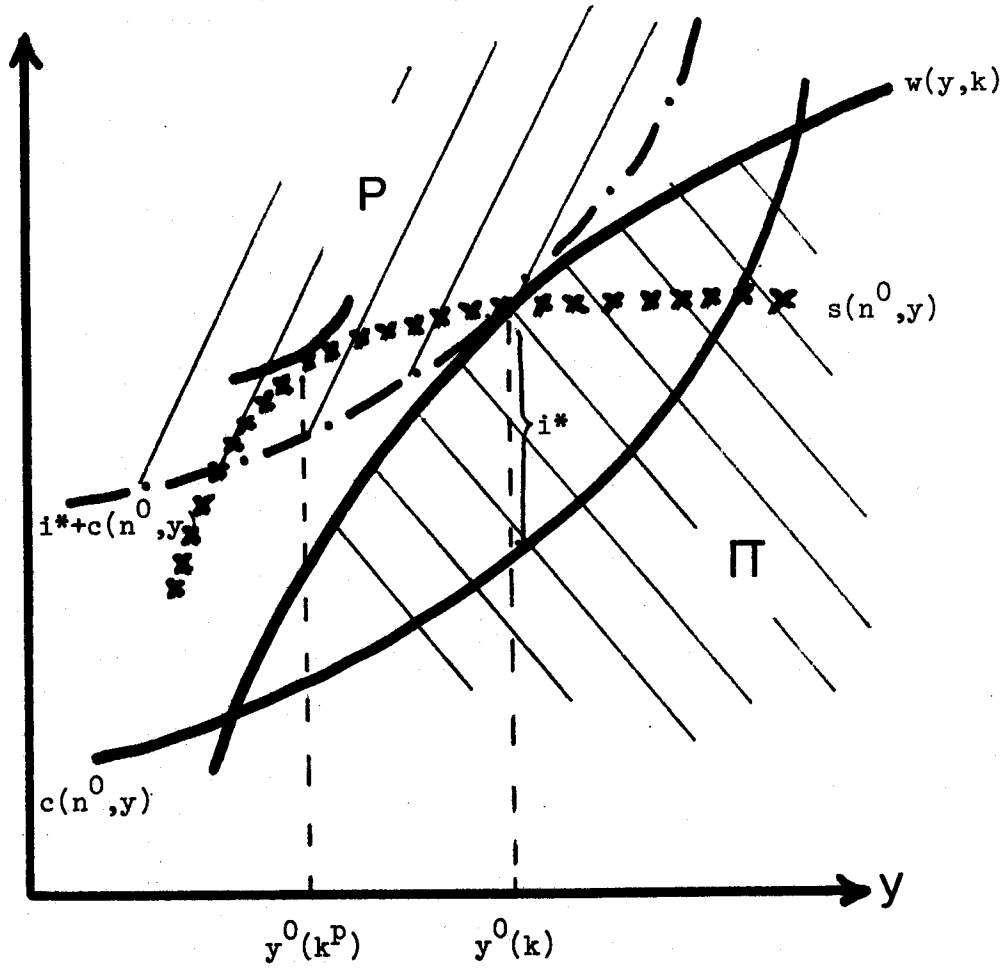


Figure 3

Experimentation in the Region of No Acceptances

is profitable to a firm and preferred by workers. While the first firm to introduce the new offer may appropriate all the gains, by choosing a wage offer on the boundary of the preferred set, competition will eventually bid up the offer, bringing finite gains to the least skilled.

Given our assumptions that the skill and cost functions are differentiable and that there is a continuum of individuals, net incomes form a continuous function⁶

$$i^*(n) = i(n, y(n, k)).$$

Therefore those close to the minimum skill level will also prefer the new offer making it even more profitable, and the initial signalling equilibrium will begin to unravel. But this result holds for any $k < k^P$ therefore no Pareto-inferior signalling equilibrium can be sustained under the postulated experimentation by firms. With $k = k^P$ there are two cases to consider. Either $y^0(k^P) = 0$, as in Figure 1, and the region of no acceptances disappears, or, as in Figures 2 and 3, $y^0(k^P) > 0$. But in the latter case, $s_y(n^0, y^0(k^P)) - c_y(n^0, y^0(k^P)) = 0$, and the offers are no longer profitable for firms. Hence our conclusion, that of the family of signalling equilibria there is only one, the Pareto supreme member, which is a fully confirmed equilibrium.

III. The Instability of a Competitive Equilibrium

Given that any real market adjustment is a complex dynamic process, it might be argued that a sophisticated understanding of the signalling phenomenon and of its implications would be unlikely.⁷ For example if the time lag before market experiments can be tested is great, the 'noise' from the

system in the intervening period might preclude drawing any clearcut conclusions. However it seems desirable, as a preliminary step in understanding markets for which information is imperfect, to explore the characteristics of a world in which such noise is negligible.

Our approach will parallel that of section II. We suppose that initially some signalling is occurring. That is, for at least one segment $[n^a, n^b]$ of the complete range of underlying ability, the necessary conditions are satisfied. Then corresponding to ability levels n^a and n^b are education levels $y^a = y(n^a, k)$ and $y^b = y(n^b, k)$ and over the interval $[y^a, y^b]$ there is an upward sloping wage profile $w = w(y, k)$. This is depicted in figure 4.

To demonstrate instability it is necessary to show that there is some new wage offer

$$\tilde{w}(\hat{y}) = \begin{cases} 0, & y < \hat{y} \\ w, & y \geq \hat{y} \end{cases}$$

which is both profitable to firms and preferred by those seeking jobs. For any $n' \in (n^a, n^b)$ the chosen education level y' is the solution of

$$\max_y \{w(y) - c(n', y)\}.$$

Maximized net income will be written as $i' = (w(y') - c(n', y'))$.

Then all offers yielding a gross income exceeding $i' + c(n', y)$ are preferred by individuals of ability n' . This preferred set $P(n')$ is depicted in Figure 4. Its boundary must touch the wage profile at y' since the latter satisfies the first order condition $w_y - c_y = 0$.

Moreover since c_{ny} is negative, any worker with a lower ability n'' , must have a preferred set $P(n'')$ whose boundary cuts the boundary of $P(n')$

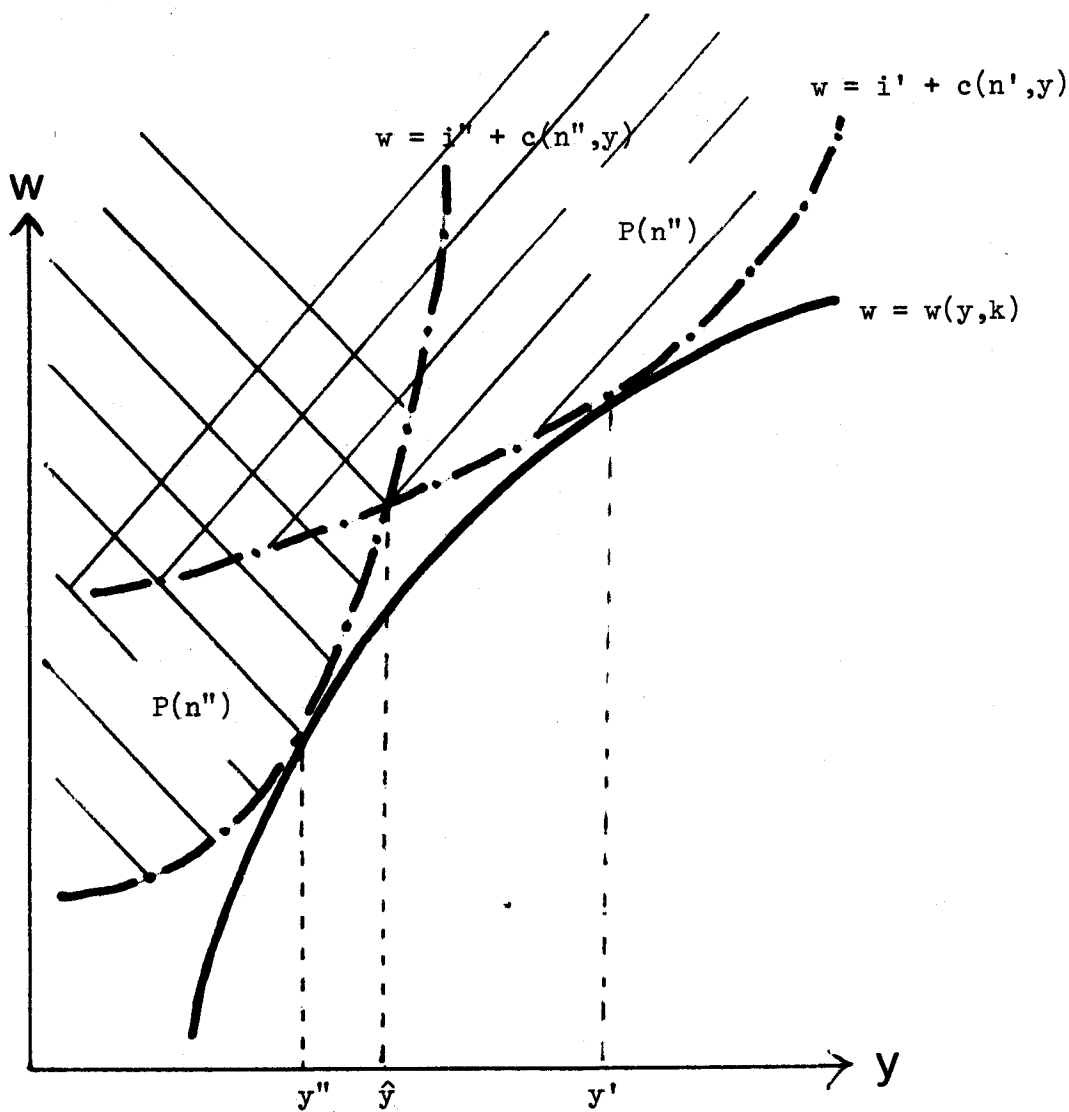


Figure 4

Wage Competition

at a unique point \hat{y} between y'' and y' . This same argument implies that for any $n \in (n'', n')$ the boundary of the preferred set $P(n)$ cuts the boundary of $P(n')$ to the right of \hat{y} .

Now suppose a firm makes an offer of the form⁸

$$\tilde{w}(\hat{y}) = \begin{cases} 0 & y < \hat{y} \\ i' + c(n', \hat{y}) & y \geq \hat{y} \end{cases}$$

All those with ability levels in (n'', n') will find the new offer strictly preferable. Then the average productivity of those accepting is

$$\tilde{s}(y) = \int_{n''}^{n'} s(n, \hat{y}) F_n dn / (F(n') - F(n'')) \quad (10)$$

Both $\tilde{w}(\hat{y})$ and $\tilde{s}(\hat{y})$ are upward sloping differentiable curves which intersect at y' . The former has a slope at y' of

$$\tilde{w}_{\hat{y}}(y') = c_y(n', y'). \quad (11)$$

The latter, applying l'Hôpital's rule, has a slope at y' of

$$\tilde{s}_{\hat{y}}(y') = s_y(n', y') + \frac{1}{2} s_n(n', y') \frac{dn''}{d\hat{y}} \Big|_{\hat{y}=y'}. \quad (12)$$

But \hat{y} is the intersection of the boundaries of the preferred sets $P(n')$ and $P(n'')$ (see Figure 4). Therefore n'' and \hat{y} must also satisfy the condition

$$i'' + c(n'', \hat{y}) = i' + c(n', \hat{y}).$$

Differentiating this expression, totally with respect to \hat{y} and then taking the limit yields the result

$$c_n(n', y') \frac{dn''}{d\hat{y}} \Big|_{\hat{y}=y'} = 0$$

Assuming, as is natural, $c_n < 0$, (12) becomes simply

$$\tilde{s}_y(y') = s_y(n', y'). \quad (13)$$

But from (9) we know that in any signalling equilibrium the marginal social product of education is less than the marginal cost (except possibly for the least able among those signalling). It follows that the slope of the average productivity curve is lower at y' than the slope of $\tilde{w}(\hat{y})$, and there must be some region to the left of y' for which $\tilde{s}(\hat{y})$ exceeds $\tilde{w}(\hat{y})$.

Therefore by choosing \hat{y} sufficiently close to y' , a firm will be able to hire workers at a price below the mean of their productivities. But y' was chosen arbitrarily in (n^a, n^b) , thus the original signalling equilibrium, given by $w = w(y, k)$, can, through competition among firms, begin to unravel at any education level. In other words no equilibrium, which distinguishes individuals of different ability by use of a proxy, can be maintained in the face of competition by firms, which fully understand the signalling mechanism.

If individuals are not all distinguished by their educational attainment, there must be pools of people of different abilities, choosing the same education level. That such a world is also unstable follows readily. Firms have an incentive to introduce a new offer which attracts only the more able from the pool. This they can do by offering a bonus for additional education that is small enough to be preferred only by those with lower education costs. By assumption these are the more able.

We have therefore shown that for any potential equilibrium there are new wage offers which lead to a breakdown of this equilibrium.

IV. Final Comments

In the above sections doubt has been cast over the survival of signalling equilibria in a competitive economy. Importantly, we have shown that even without a sophisticated understanding of the signalling process, plausible experimentation by firms elads to an unravelling of any 'equilibrium'.

However for the following reason it seems likely that this information needed for such an unravelling would spread only slowly through the system. Usually one would expect product quality to be related to a vector of unobservable characteristics \underline{n} , a vector of signals \underline{y} plus a random component $\underline{\theta}$.

$$\text{i.e. } s = s(\underline{n}, \underline{y}, \underline{\theta})$$

While this complication does not fundamentally alter the Spencian analysis (see Spence [3]) or the conclusions discussed above, it does increase the cost of experimentation. A considerable number of observations would then be necessary before reliable inference could be drawn. Given that information would usually become available sequentially, it would also be necessary to correct for changes in job characteristics over the sample period.

It seems possible then that a market might exhibit a slow drift towards Pareto-superior signalling 'equilibria'. This might be followed by pooling of products of different quality and in turn the re-formation of Pareto-inferior signalling. Whether such a scenario is supportable, either by the empirical evidence or by formal dynamic model building, remains a subject for further research.

One final point should be noted. Throughout the discussion those in the

job market play a passive role, simply choosing the net income maximizing level of education. However if work force entrants have some prior knowledge, at least in a probabilistic sense, about their productivity on the job, they will consider the possibility of accepting some form of contingent contract. This need not be an explicit contract. All that is necessary is the expectation, on the part of job seekers, that a high skill level demonstrated now will be rewarded later with a wage in excess of marginal productivity.

If individuals have correct prior expectations and are risk neutral, they will be willing to shoulder all risks and achieve a higher expected income than in any of the signalling equilibria. Realistically however, risk averseness, limits the contracts that individuals are willing to offer. Then in general the possibility of contingent contracting provides a lower bound to individual losses associated with signalling and potentially constrains dynamic fluctuations.

References

1. M. Rothschild and J.E. Stiglitz, Equilibrium in Competitive Insurance Markets: The Economics of Imperfect Information, mimeo 1973.
2. A.M. Spence, Competitive and Optimal Responses to Signals: Analysis of Efficiency and Distribution, J. Econ. Theory, 7 (1974), 296-332.
3. A.M. Spence, "Market Signalling: Information Transfer in Hiring and Related Processes," forthcoming from the Harvard University Press 1974.
4. A.M. Spence, Competition in Salaries, Credentials, and Signalling Prerequisites for Jobs, mimeo 1973.

Footnotes

1. That is, a world in which actual job skill can be predetermined costlessly by employers, instead of a world in which high cost of obtaining such information precludes any such investment.
2. Their brief allusion to the job market problem has also been taken up for the two class model by Spence [4].
3. In a revision of the Rothschild-Stiglitz paper a proof is provided for the case in which individuals differ only in the probability of an accident. Our more general theorem applies also the case in which felicity is systematically related to the probability of an accident.
4. Spence uses 'n' to denote underlying ability and 'N' to denote income net of education costs. To avoid possible confusion, the latter is denoted here by 'i'.
5. Spence discusses but never fully resolves the existence question. From (7), $w_{yy} - c_{yy}$ is strictly negative except in the second case where, for the lowest skill level, $\frac{dn}{dy} \Big|_{k = k^D} = 0$. But then $w_{yy} - c_{yy}$ is strictly decreasing so the schedules indeed define local maxima for every skill level.
6. Since $n = n(y,k)$ and $n_y > 0$ we can invest and write $y = y(n,k)$.
7. This seems to be Spence's argument, for example in an appendix to [2].
8. Indeed any offer above $w = w(y,k)$ and sufficiently close to it can be expressed in this way.