A THEORY OF GAMES
WITH TRULY PERFECT INFORMATION

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INTRODUCTION

This paper develops a theory of games when the players have more accurate information than that assumed in conventional game theory. Our concept of perfect information, in contrast to the standard game-theoretic concept, is consistent with the notion of institutional information used in standard economic theory. Under the conventional game-theoretic definition of perfect information, each individual knows the actual strategies that the other individuals have selected, and the corresponding solution has each individual taking the known strategies of the others as given. This information is less accurate than that implied in discussions of economic and social institutions in that these discussions have an individual also knowing how the other individuals would respond to his different possible strategies and selecting his strategy accordingly. An example is the familiar situation in which a seller knows how buyers would respond to different prices asked for his product and thereby selects his asking price. Our model thus replaces the conventional game-theoretic assumption that each player expects no reaction to a change in his strategy with the assumption of perfect information regarding player reactions.

If we wish the term, game, to include a description of the behavior described in standard economic models, then the current "theory of games," given the inaccuracies in each player's reaction expectations, should be relabelled, "a theory of interpersonal gambling," where a player can only guess about how the other players will react to changes in his strategy. Our model would then be included in the general theory of games as a theory of rational institutions (i.e., rational reactions), where each individual determines his rational reactions to the strategies of others and communicates these reactions in full knowledge of the rationally-determined responses of the other individuals. To prevent confusion with standard definitions of perfect information, we will refer to our notion of perfect information as "truly perfect information."

In Section I, we specify our general game and show that a necessary consequence of rational strategy selection under truly perfect information is a "hierarchy" of players characterized by a recursive set of rationally-determined reaction functions for all but one player and a player who chooses a simple action and thereby determines the solution set of actions. Such solutions always exist for finite games. In the case of two-person games, this result implies that one player chooses a reaction function and the other player chooses an action. The resulting solution yields a pair of actions.

The solutions developed in Section I differ from those of conventional game theory in that they are derived from the rational behavior of the players given full knowledge of the player interdependence. We shall see that the rational solutions are generally asymmetric with respect to the numbering of the players and are never characterized by mixed or randomized strategies.

In Section II, we show that the solution set to any finite two-person game with truly perfect information always contains a Pareto optimum. The importance of this optimality result is that while numerous authors have conjectured the Pareto optimality of two-party conflicts under sufficiently perfect information, the conjecture has never been derived as the result of the individually rational behavior of the players within the model. However, we also show that the conjecture is not always true, i.e., that the solution set of a 2-person game with truly perfect information may also contain Pareto nonoptimal points. No amount of perfecting of information or communication will remove these inefficient solution points. The only way, to our knowledge, of removing these inefficient solution points is to introduce a higher order game involving competition to determine the order of strategy selection of the two players.[1/]

In Section III, we contrast our theory to others. In contrast to Von Neuman-Morgenstern-Nash, non-cooperative, constant and variable sum games, and to Nash cooperative games, our game always has pure strategy solutions, even for games without saddle-points. Another contrast is that our solution to the "prisoner's dilemma" is an efficient solution even in the absence of two-way communication between the players. A basic feature that distinguishes our game from the Nash cooperative game and the Friedman "supergame," both of which are designed to allow sufficient communication to prevent Pareto nonoptimal solutions in 2-person games, is that we do not <u>assume</u> Pareto optimality to be a characteristic of solutions or of points on rational reaction functions. The fact that Pareto optimality may not characterize all of the solutions to 2-person interactions no matter how perfect the communication indicates the weakness in these other approaches. We illustrate the non-optimality possibility in a possibly empirically relevant game which we call the "slave master's insensitivity."

I. THE GENERAL GAME

A. Description

Let G be a n-person, non-cooperative, finite game. A player of G is denoted i, i = 1,...,n, and we shall let N represent the set of n players of G. Each player has a action possibilities set, A_i, consisting of a finite number of possible pure actions, x_i, for all i.

An outcome of G is an n-dimensional vector of pure actions, and is denoted

$$x = (x_1, x_2, \ldots, x_n), \quad x_i \in A_i, \quad \text{for all } i = 1,\ldots,n. \tag{1}$$

We shall denote a vector of pure actions, one from each player except i, as

$$x_{-i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n). \tag{2}$$

Each player, i, has a payoff function,

$$\pi_i(x) = \pi_i(x_i, x_{-i}), \qquad i = 1,\ldots,n. \tag{3}$$

which describes i's payoff under all possible outcomes of G.

A strategy of a player is a description of what the player will do. It may be an action, which simply states what pure action the player will choose. It may be a reaction strategy, which state the various pure actions the player will choose under the various possible actions of other players. It may be a contingent reaction, which states the various reaction strategies a player will adopt under the various possible reaction strategies of other players. Or it may be a contingent strategy, a generalization of the strategies of reaction and contingent reaction, which states what strategy a player will adopt given the strategies of other players.

A contingent strategy may include, for example, the infinite statement: "I will play x_1' if you say you will react with x_2'' when I play x_1'' if I say I will react with x_1''' when you play x_2''' if you say you will react with x_2'''' when I play x_1'''' if I say I will react with x_1''''' when you play x_2''''' if you say ...."[2/]

We assume all players have truly perfect information regarding the strategies of others and "rationality", i.e., that each player selects the strategy that maximizes his payoff given the responses of the other players to his strategy.

B. The Necessity of Priority in Strategy Selection

If there is to be truly perfect information, there must be one player among the set of n players that determine his strategy prior to the other players, where the response strategies of the other players are known a priori by the prior selector because they are the rational responses to the given strategy of the prior selector.[3/] Given the strategy of the first strategy selector, another player makes a prior strategy selection over the set of the remaining n-2 players in full knowledge of the previous selector's strategy. This strategy selection process continues until all n players have selected a strategy.

A strategy of a prior strategy selector among a set of players describes his response to each of the possible strategies of the other players in the set. Therefore, since each player has an available set of pure actions, each of the possible strategies of a prior strategy selector always contains a reaction strategy.

It follows that the $n^{\underline{th}}$ strategy selector, who faces only prior strategy selectors can achieve maximum payoffs by simply selecting a pure action from A_n, thereby triggering a chain of reactions up through the first strategy selector in the game. This is because the $n^{\underline{th}}$ strategy selector, facing the prior strategies of the other players, sees that the eventual set of actions, or outcome, must be consistent with the chosen reaction strategies of each of the n-1 prior selectors. Hence, if player n responds with a pure action, he will have a free choice over all outcomes consistent with the prior reaction strategies. But if n responds with a contingent strategy, thus giving further choices to the prior strategy selectors, he can only reduce his original choice out of the same set of possible outcomes without expanding the set of possible outcomes as any eventual set of actions must be consistent with the initial n - 1 reaction functions.

Since each prior selector's strategy will be followed by a set of pure actions by the subsequent strategy selectors, each prior selector's strategy set is larger than the set of pure actions contained in the choice set A_i only in that it includes player i's possible reactions to all of the possible actions of the subsequent strategy selectors.

Let us label the n players according to their priority in strategy selection. Then, i's <u>strategy set</u>, S_i, is described by:

$$S_i = \{x_i|(x_{i+1},\dots,x_n), \text{ all } x_i \in A_i, \text{ and } (x_{i+1},\dots,x_n) \in A_{i+1}\times\dots\times A_n\}, \qquad (50)$$

where $x_i|(x_{i+1},\dots,x_n)$ is i's action contingent upon the vector of actions from players i+1 to n. Of course, $A_i \in S_i$. Player i's optimal strategy, $i < m$, is summarized by a reaction function.

$$f_i: \underset{j=i+1}{\overset{n}{\chi}} A_j \rightarrow A_i, \tag{6}$$

which assigns an action to i for each vector of actions from player i+1 to n, given the prior reaction functions of player 1 through i-1.

Any player i who has priority over a non-empty subset of N is called a strategy-maker (or, maker) relative to the players in that subset. Likewise, each player in the subset is called a strategy-taker (or, taker) relative to i. Player 1, who has priority over all others, is called the primary maker. Player n, who is not a maker relative to any other player in N but a taker relative to all others, is called the pure taker. Player i, i $\neq$ 1 or n, is a maker relative to players i+1 through n, and is taker relative to players 1 through i-1.

We thus obtain a hierarchy of makers similar to the order of priority, where the primary maker exhibits a reaction function,

$$x_1 = f_1(x_2, x_3, \ldots, x_n),$$

the secondary maker exhibits,

$$x_2 = f_2(x_3, \ldots, x_n),$$

the third maker exhibits,

$$x_3 = f_3(x_4, \ldots, x_n),$$

and so on up to the n-1st player's simple reaction function,

$$x_{n-1} = f_{n-1}(x_n).$$

C. Solutions

Solution sets for all n players are easily constructed for a given set of reaction functions. The pure taker selects an action that maximizes his payoffs given the set of n-1 reaction functions. The n-1st player then follows his established reaction function, which gives the two actions necessary for the n-2nd player to determine his action. This process continues until the primary maker's action is determined.

With the assumption of truly perfect information and the condition of rationality imposed upon the takers of a given maker, the maker can solve for his solution reaction function.

This determination can be made since each maker, i, knows: (1) the reaction functions previously selected by his makers, (2) the reaction functions that will be chosen by i's takers given i's choice of reaction function and the previously selected functions of i's makers and (3) player n's rational response to these reaction functions.

Thus, a solution to G is directly obtained from a set of n-1 independent reaction functions, $\{f_i^*, i \neq n\}$ where f_i^* is the solution to

$$\max_{f_i \epsilon F_i} \pi_i \quad [f_i, f_{i+1}(f_i), f_{i+2}(f_i, f_{i+1}(f_i)), \ldots, x_n(f_i), f_1^*, f_2^*, \ldots, f_{i-1}^*], \qquad (7$$

where F_i is the set of all possible functions relating $\chi_{j=i+1}^{n} A_j$ to A_i. The set of actions, $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$, implied by the set of solution reaction function is called a <u>solution outcome</u>, or simply a "solution," to G.

D. Determination of Priority and the Role of Committments

Our game G, with its predetermined priority, is not symmetric in that the solution to G will vary with the order of priority. One may think of the priority in strategy making in the above model as being arbitrarily determined by an umpire of the game, or by some random device allocating the positions in the hierarchy.

The priority of strategy selection can also be determined by playing a "higher-level" game, G'. In such a game, the n players interact to determine the order of strategy selection according to the value (payoff differential) of selecting strategy prior to the other players. Since in order for any player to know this value he must determine his solution reaction function given a particular position in the hierarchy, a solution to G' also implies a solution to G.[4/]

Another reason for introducing a higher order game is to assure that a maker can carry out his chosen strategy. That is, a player may have to contract, in a higher-order game with outside parties to pay the outside parties a large sum in case the player does not carry out his announced strategy. For example, a player may adopt a strategy of imposing punishments on other players if they do not adopt certain strategies, punishments which are painful for the player to impose. Without a commitment to an outside party to carry out this strategy or else suffer even greater pain, the announced strategy might be regarded as an empty threat by the other players. This higher order game of contracting with outside parties and the higher order game, G', which determines the order of strategy selection may be combined into a single, higher-order game in which players interact to determine the order in which they make strategy committments with outside players. (See Faith and Thompson (2),)

II. THE NATURE OF SOLUTIONS TO TWO-PERSON GAMES

In this section, we shall first show that the solution set to any finite, two-person game with truly perfect information always contains a Pareto optimal point. The solution set is simply the set of solution outcomes, i.e., the set of all outcomes such that the reaction function for the maker given the rational response of the taker yields maximal payoffs to the maker. The recursive and finite structure of the optimization problems of the maker and taker obviously insures that the solution set is non-empty.

To prove that the solution set must contain at least one Pareto optimal point, suppose it is not the case. Then, for any pair of solution actions, say the pair (s_1,s_2), there is a pair of non-solution actions, say (p_1,p_2), such that

$$(8) \quad \begin{aligned} &\pi_i(p_1,p_2) \geq \pi_i(s_1,s_2) \quad \text{for both } i = 1 \text{ and } i = 2, \text{ and} \\ &\pi_i(p_1,p_2) > \pi_i(s_1,s_2) \quad \text{for } i = 1 \text{ or } i = 2. \end{aligned}$$

With no loss of generality, we let player 1 be the maker. Under the hypothesis, if player 2 chooses action p_2, then player 1 chooses an action other than p_1; otherwise player 2 would, in view of (8), be at least indifferent between (p_1,p_2) and (s_1,s_2) so that (p_1,p_2) would be in the solution set. In particular, player 1's solution reaction to p_2 must generate a lower payoff to player 2 than $\pi_2(s_1,s_2)$. Now if player 1 replaced his solution reaction function with a function which reacted with action p_1 to player 2's action p_2, he would, in view of (8), be no worse off than at the solution point (s_1,s_2). And since, under the replacement, player 2 would be no worse-off by choosing

p_2 than by choosing s_2, player 1's solution reaction function yields him no greater payoff than a non-solution reaction function, which is a contradiction.

The existence of Pareto optimal points in the solution set does not imply that Pareto nonoptimal points cannot also belong to the solution set. In fact, Pareto nonoptimal points do belong to the solution sets to certain games. These occur when there are Pareto nonoptimal points for which the only Pareto superior points have only one party better-off. Then, assuming that the taker is the party who is better-off in moving to the Pareto optimum point, and that the Pareto optimum is in the solution set, it is clear that the Pareto nonoptimum yielding the same payoff to the maker but a lower payoff to the taker is also a solution as long as the choices offered by the maker are even worse for the taker than the Pareto nonoptimum. The latter proviso can obviously hold if the maximal payoff to the maker over the entire set of payoffs occurs at the Pareto optimal point under discussion. A quasi-realistic example is provided in Section III.

III. CONTRAST TO TRADITIONAL THEORY OF GAMES

In this part, we shall contrast the approach and solutions to n-person non-cooperative games with truly perfect information developed above with the approach of conventional game theory. In particular, we contrast our game to the following two-person games: Zero-sum games, general-sum, non-cooperative games, Nash's cooperative games, and Friedman's supergame.

A. Two-person zero-sum games.

The constructive solution concept pioneered by Von Neumann and Morgenstern (VNM) for two-person, zero sum games is the minimax solution, where each player selects a strategy which maximizes the minimum gain (or minimizes the maximum loss) over all possible outcomes to the game. VNM's content theorem states that the pair of minimax pure actions (in those cases where there exists a saddle point in the payoff function) or a pair of randomized actions (in the non-saddle point case) is also the solution to a "perfect information" game, a game in which each individual knows the other's strategy and accepts it as fixed in determining his own strategy.

In contrast, the solutions to zero-sum games using our assumption of truly perfect information - both with and without saddle points - are never characterized by mixed strategy solutions.

To illustrate this contrast, consider a two-person, zero-sum game without a saddle point in pure actions (e.g. figure 1). From our above discussion, there is one and only one maker, a player whose choice set must be expanded beyond the set of simple actions to include that player's possible reactions to the various actions of his opponent. The expanded payoff matrices for players 1 and 2 are shown respectively in figures 2 and 3.

It is easy to see in figure 2 that the rational reaction function for player 1, if maker, is to play action x_1' if player 2 plays x_2', and to play x_1'' if player 2 plays x_2''. Facing this reaction function, player 2's payoff maximizing choice is x_2''. It is similarly easy to see in figure 3 that if player 2 is the maker, his rational reaction function is to play x_2'' if player 1 plays x_1', and to play x_2' if player 1 plays x_1''. Player 1's payoff maximizing choice is x_1''. Hence if player 1 is maker, the solution outcome is the pair (x_1'',x_2'') giving a payoff of 3 to player 1 while if player 2 is the maker, the solution occurs at the pair (x_1'',x_2') giving a payoff of 2 to player 1. Note that each solution pair corresponds to the pair of personal minimax actions for the games in Figures 2 and 3, respectively.

Hence, we have two possible solution outcomes depending upon which player is the maker. The equilibrium solution is attained once the priority of strategy selection is determined.

If there exists a saddle point in pure actions, then the solution outcome in both our model and the VNM model occurs at the saddle point, where each player plays his minimax action. To see this, first note that in any zero sum game, a rational reaction function for the maker is one which consists of those actions yielding the highest payoff to him for each possible choice of the taker. Since, in the presence of a saddle point, a player's minimax action is his payoff maximizing action given that the other player chooses <u>his</u> minimax action, the minimax pair of actions (the saddle point) will be a point on the rational reaction functions of both players. And since each player chooses the same action as a taker as when minimaxing, the solution outcome is

1 \ 2	x'_2	x''_2
x'_1	4	1
x''_1	2	3

Figure 1. Payoff Matrix A - No Saddle Point

1 \ 2	x'_2	x''_2
x'_1	4	1
x''_2	2	3
$x'_1 \mid x'_2,\ x''_1 \mid x''_2$	4	3
$x''_1 \mid x'_2,\ x'_1 \mid x''_2$	2	1

Figure 2. Payoff Matrix A with Player 1 as Maker

1 \ 2	x'_2	x''_2	$x'_2 \mid x'_1,\ x''_2 \mid x''_1$	$x''_2 \mid x'_1,\ x'_2 \mid x''_1$
x'_1	4	1	4	1
x''_1	2	3	3	2

Figure 3. Payoff Matrix A with Player 2 as Maker

the saddle point regardless of the identity of the maker.

Consider the two-person,zero-sum game with a saddle-point illustrated in Figure 4. The minimax solution to the game is the pair (x_1', x_2''). If player 1 is the maker, his expanded payoff matrix is shown in figure 5, with the solution occurring at $(x_1'|x_2'', x_2'')$. If player 2 is the maker, his expanded payoff matrix being shown in Figure 6, his optimal reaction function is x_2'' regardless of the action of player 1, so the solution actions are again (x_1',x_2''). These solutions are therefore the same as the minimax pair of actions in the original game.

Solutions to two-person,zero-sum games under the conventional definition of perfect information are obtained by VNM by considering the majorant and minorant of the original game. When there exist a saddle-point, the majorant/minorant solutions are identical to our solution. The reaction function of the second mover (the maker) is assumed by the first mover (the taker) to consist of the individual payoff-maximizing reactions of the maker to each possible action of the taker. In contrast, our theory does not force such a short-sighted reaction function upon the maker. In our theory, the second mover (the maker) initially commits himself to the payoff-maximizing reaction function given the known rational response of the first mover (the taker). The two theories yield identical solutions only because the zero-sum condition implies that the payoff-maximizing reaction function will consist of the individual payoff-maximizing response of the maker to each action of the taker and because there is a saddle point. When there does not exist a saddle-point, the solution the majorant/minorant games are mixed strategy solutions, contrary to the pure action solutions illustrated in Figures 2 and 3.

1 \ 2	x_2'	x_2''
x_1'	4	3
x_1''	2	1

Figure 4. Payoff Matrix B - Saddle Point

1 \ 2	x_2'	x_2''
x_1'	4	3
x_1''	2	1
$x_1' \mid x_2', \; x_1'' \mid x_2''$	4	1
$x_1'' \mid x_2', \; x_1' \mid x_2''$	2	3

Figure 5. Payoff Matrix B with Player 1 as Maker

1 \ 2	x_2'	x_2''	$x_2' \mid x_1', \; x_2'' \mid x_1''$	$x_2'' \mid x_1', \; x_2' \mid x_1''$
x_1'	4	3	4	3
x_1''	2	1	1	2

Figure 6. Payoff Matrix B with Player 2 as Maker

B. Variable-sum games.

1. Non-cooperative.

The standard approach to non-cooperative, variable-sum games has been to identify pairs of mixed strategies which satisfy a Nash equilibrium.[5/] Nash defines an equilibrium to a 2-person bi-matrix game $[\pi_1, \pi_2]$ as a pair of vectors, $\vec{x}_1^*$ and $\vec{x}_2^*$, consisting of the probabilities of playing each action in A_1 and A_2, respectively, such that:

$$\begin{aligned} \vec{x}_1 \pi_1 \vec{x}_2^* &\leq \vec{x}_1^* \pi_1 \vec{x}_2^* \\ \vec{x}_1^* \pi_2 \vec{x}_2 &\leq \vec{x}_1^* \pi_2 \vec{x}_2^* \end{aligned} \qquad (9)$$

for all $\vec{x}_1, \vec{x}_2$ such that $0 \leq \vec{x}_1^k \leq 1$, $0 \leq \vec{x}_2^j \leq 1$, $\sum_i \vec{x}_1^k = 1$, and $\sum_j \vec{x}_2^j = 1$.

The equilibrium conditions in (9) are not for a game with truly perfect information. Rather, they describe behavior in which each individual assumes the strategy of the other is unaffected by his strategy -- a straight-forward generalization of a Cournot duopoly model. As in the model of Cournot (1), each player is behaving irrationally under truly perfect information because the other player will, by the rule, generally alter his strategy in response to a change in his opponent's strategy.

Let player 1 be the maker. Then 1's problem is to choose a reaction function, a function determining a reaction of player 1 to each possible action of player 2, that yields maximum payoffs to 1 given that 2 will always select his payoff-maximizing action subject to 1's reaction function. Player 1 thus chooses a reaction function, $x_1 = f_1[x_2]$, satisfying:

$$\max_{f_1[x_2]\varepsilon F_1} \pi_1 \left(f_1[\bar{x}_2(f_1[x_2])], \bar{x}_2(f_1[x_2])\right), \tag{10}$$

where $\bar{x}_2(f_1[x_2])$ satisfies, for any given $f_1[x_2] \varepsilon F_1$.

$$\max_{x_2 \varepsilon A_2} \pi_2(f_1[x_2]; x_2)$$

Notice that the equilibrium conditions in (10) differ from the Nash equilibrium conditions (9) in that player 1 plays contingent actions as represented by his reaction function rather than independently choosing a simple action.

To illustrate the difference in the two concepts of equilibrium, we consider the "prisoners' dilemma" game, depicted in normal form below in Figure 7. The Nash equilibrium outcome to this game is the action pair (x_1'', x_2''). Under truly perfect information, this solution is irrational, since either player, if maker, can assure himself a higher payoff (4 rather than 1 in the illustration) by exhibiting a reaction function where he plays his first action if the other player plays his second action and he plays his second action if the other player plays his first, given the payoff-maximizing behavior of the taker as constrained by the maker's rational reaction function. This results in an equilibrium at (x_1', x_2') which is unattainable under the Nash solution concept. With this reaction function, a player, as taker, never has an incentive to play his second action - a result not true in the Nash model. Hence, in spite of the absence of two-way communication, our solution to a prisoners'dilemma game is an efficient solution.

Notice that the maker's reactions are not the payoff maximizing actions of the maker for given actions of the taker, as they were in the zero-sum case.

1 \ 2	don't confess x_2'	confess x_2''
don't confess x_1'	4,4	0,10
confess x_1''	10,0	1,1

Figure 7. Payoff Matrix $[\pi_1, \pi_2]$

1 \ 2	don't confess x_2'	confess x_2''
don't confess x_1'	4,4	0,10
confess x_1''	10,0	1,1
beat up the other prisoner x_1'''	-1,-2	-1,-2

Figure 8

master \ slave	work x_2'	rest x_2''
beat the slave x_1'	10,-10	0,-6
insult the slave x_1''	10,-4	1,0
leave the slave alone x_1'''	10,0	0,4

Figure 9 The Slave Master's Insensitivity

2. Cooperative games and supergames

In Nash's model of cooperative games[6/] "cooperation" refers to the individual players' incentive to reach an outcome mutually superior to the Nash non-cooperative outcome, rather than implying collusion or side payments. In the model, each player initially commits himself to playing a "threat" action if the player is not satisfied with the size of his payoff. These threats are communicated to one another, and then each player independently and secretly chooses his "demand", or minimum acceptable payoff. If the two demands are not mutually attainable, the players are committed to playing their threat actions. Introducing a set of axioms imposing desirable equilibrium characteristics, including Pareto optimality, Nash proves the existence of an equilibrium pair of randomized actions, and derives the form of the optimal threats. These optimal threats consist of the individual players' minimax actions for the game.

The so-called "supergames" of Friedman (3) are infinite sequences of conventional non-cooperative games in which each game in the sequence has one and only one outcome Pareto superior to that game's Nash non-cooperative solution. To achieve a sequence of such Pareto outcomes as a solution, Friedman has each player selecting a reaction strategy where: one plays his Pareto action in game t, if all other players play their Pareto actions in game t-1, and one plays his Nash action in t and all subsequent games in the sequence, given any other actions of his opponents in game t-1. In the initial game (t=1), it is assumed all players choose their Pareto action. Hence, each player is discouraged from choosing an action in any game which maximizes his payoff given the others' Pareto actions.

The motivation behind both models is apparently to achieve solutions which are Pareto optimal - solutions which are generally unattainable under the information assumptions of conventional non-cooperative games. It is easy to see that Nash's model of cooperation differs from our theory in that each player in Nash's model does not know how his opponent will react to his demand choice. Further, Nash requires several plays of the game in order for the players to learn about his opponent's demand strategies, revise his own strategy, and finally settle upon a pair compatible demands. However, there is nothing in the model which permits the attainment of any rational solution other than a non-cooperative one. Because the "demand game" is played under the conventional information assumptions, the only rational solution occurs at the optimal threat outcome, i.e., the Nash non-cooperative solution, which we have already contrasted to our theory.[7/]

In the supergame approach, each player selects a reaction function and an action in response to the expected reaction strategy of the other players. In a solution, a player's response of a reaction strategy to a prior (in this case, expected) reaction strategy must yield actions consistent with the prior reaction strategy, or else his reaction strategy is meaningless. Therefore, a response of a reaction strategy rather than an action merely implies giving additional choice to the other player(s). As pointed out in section IB, allowing additional choice by a prior selector cannot expand the set of possible outcomes from which the subsequent selector may choose and hence is irrational. Therefore the supergame approach is not generally rational. Furthermore, the forms of the reaction functions assumed by Friedman are, in general, irrational.

To see this, consider a version of the prisoners' dilemma with the additional alternative action of "beating up" one's fellow prisoner (figure 8).

Ignoring for the moment the new actions, the supergame solution and the maker/taker solution both occur at (x_1', x_2') for a payoff of (4,4). If we now add the new action, the supergame solution remains at (x_1', x_2'). However, if player 1 is the maker, he can achieve (x_1'', x_2') as a solution by commiting himself to play action x_1''' whenever player 2 plays something other than x_2'. It is no longer rational for player 1 to resort to his Nash action, x_1'', if 2 deviates from some Pareto optimum. Thus, with this ability to punish, the Pareto optimum occuring at (x_1', x_2') is not a rational solution.

While our model does produce Pareto optimal solutions to any prisoners' dilemma game and while Pareto optimal points are always contained in our solution sets for two-person games, our model does not preclude Pareto non-optimal solutions in all two-person games.

Figure 9 contains an example of such a game, which we call the "slave master's insensitivity." The Nash non-cooperative solution has the slave resting while the master insults the slave. This is both non-optimal and empirically unrealistic. Friedman's solution concept cannot apply because there is not a single Pareto optimum superior to the Nash outcome. Making the master the strategy maker, our solution set contains an optimum (10,0) as the master will beat the slave if and only if he rests in order to induce the maximal output from the slave. But the solution set also contains (10,-4) as the master may also insult the slave, lowering the slave's benefit to -4 without altering either the master's payoff or the slave's optimal taker decision. Our solution set thus contains Pareto non-optimal as well as Pareto optimal points. The non-optimality disappears when we allow a competitive bidding process to determine which of the two players is to be the strategy maker (see Faith and Thompson (2)).

FOOTNOTES

1. See Faith and Thompson (2).
2. While a general game theory containing contingent strategies has been produced by Howard (4), he makes the conventional perfect information assumption and therefore winds up with a conventional set of Nash solutions.
3. In case there is more than one response strategy yielding the maximum payoff, we assume, for the sake of determinacy, that the player give the choice among his maximal strategies to the prior strategy selector. We call the resulting strategy the player's "rational response".
4. For an example of this simultaneous solution determination in a model of non-competitive interdependence, see Faith and Thompson (2).
5. See Nash (5).
6. See Nash (6).
7. Although Nash expects his players will eventually agree upon a compatible outcome, there remains the question of the distribution of payoffs over the set of Pareto optimal outcomes. A rational determination of who "backs down" in this bargaining situation is not achieved. This conflict is the same that motivates the Nash non-cooperative equilibrium, and that outcome i.e., the optimal threat outcome, would again appear to be the rational solution to the conflict.

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