

**A GENERAL MODEL OF MONEY, INTEREST
AND EMPLOYMENT**

by

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Introduction

This paper is a formalization of an alternative to the existing, dominant theory of money and income developed by Keynes, Patinkin, et al. The paper develops a theory of money and income sufficiently general to encompass several kinds of monetary institutions and then specializes the general theory for particular monetary institutions in order to derive contrasting characteristics of economies with different monetary institutions. Three different monetary institutions are considered: classical, modern, and optimal. A less formal sketch of the proper contrast between modern and classical money economies is found in Thompson [1973a].

Part I specifies a generalization of the Cassel-Patinkin money model and some necessary intertemporal characteristics of a Casselian money economy that, I believe, have not been previously noted. For the special case of this environment representing the Modern Money Model, Part II establishes the invalidity of the Classical Dichotomy, the presence of Pigou Effects and the effectiveness of perfectly anticipated inflation on real magnitudes. These are familiar theorems but are established here in

a general intertemporal model.

Then, for the special case we call the Classical Money Model -- a model which allows a competitive, private production of at least part of the money supply and competitive payment of interest on all money -- Part III establishes the validity of the Classical Dichotomy, the absence of Pigou Effects, and the ineffectiveness of anticipated inflation. These theorems are based on a new model of the nature of a classical money economy and lend new support to classical monetary theory against the attacks of Patinkin and others.

Part IV develops a general model of one-period temporary equilibrium, specifies the model at a Keynesian level of aggregation, and contrasts the characteristics of the temporary equilibrium that results from imposing modern monetary institutions on the economy with those that result from classical monetary institutions. An inconsistency in the standard Keynesian model with neoclassical production theory is pointed out. Correcting it allows us to resolve Gibson's Paradox and to uncover a dangerous instability of a temporary equilibrium with modern monetary institutions. In comparing Modern and Classical Money Economies, Part IV shows that employment in a single-output temporary equilibrium with classical monetary institutions is uniformly less sensitive to exogenous shifts than it is with modern monetary institutions. The result, however, does not hold in an economy with two or more outputs.

Part V outlines a monetary system which results in full competitive equilibria which are Pareto optimal and in temporary equilibria which do not admit any of the severe inefficiencies that are found in temporary equilibrium models with modern and classical monetary institutions. This optimal monetary

system has classical characteristics in that money is (1) privately produced without government interference and (2) convertible into an asset whose relative price is determined by non-monetary phenomena. But it also has a modern characteristic in that some asset, the asset into which money is convertible, is a paper asset (meaning an asset with identically zero production costs) whose supply is controlled by the government. We call this asset currency. The demand for currency is also controlled by the government, say by requiring individuals to pay taxes in terms of currency. We can perhaps view our own economy as tending, albeit fitfully, to this optimal economy as the development of credit institutions and relaxation of restrictions on the banking industry are gradually reducing the demand for currency as a medium of exchange and allowing us to approach a "cashless society."

I. A CASSELIAN MONEY MODEL WITH A FINITE SEQUENCE OF MARKETS

Let X_{jt} be the excess demand for asset jt , that is, for good j during period t , where $j = 1, 2, \dots, M$, and $t = 1, 2, \dots, T$. One asset may be a contract to deliver a good in the future (for example, a short bond, a money future promising one dollar in the next period) while another asset may be the same good traded at the future date (for example, one dollar a year from now). If a good generates joint products, each product is evaluated as a separate asset. Some of the assets may be common stocks; others may be

purely contractual rights such as rights to prevent the transactions of others. And there is no assumption of utility maximization subject to a budget constraint without transaction costs. So the general model is substantially more general than the orthodox, Walras-Arrow-Debreu model. But the model is still competitive in the sense that prices are constant to each individual and the same to all. The accounting price of asset jt is a non-negative scalar given by A_{jt}^P . A "solution" set of accounting prices, A^P , is a set of accounting prices such that $x_{jt}[A^P] = 0$ for jt such that $A_{jt}^P > 0$ and $x_{jt}[A^P] \leq 0$ for jt such that $A_{jt}^P = 0$. There is no restriction on the number of such solutions.

We let the asset Mt be "money" for each t . We assume that the accounting prices of money and some nonmonetary good are both positive in each period. That is,

Ax. 1: for each t : $A_{Mt}^P > 0$, and there is a $j \neq M$ such that $A_{jt}^P > 0$. It follows that the supply of money in the last period, S_{MT} , is zero. For since there is no demand for money when there is no future to spend it in, a positive money supply would lead to a zero solution price of money in period T . And, since nobody in period $T-1$ would accept a commodity generating no real interest (as in the Modern Money Model) in exchange for anything valuable if the commodity were about to become worthless, the zero price of money would be transmitted back to period $T-1$, and so on back to period 1.*/

*/ I was made aware of this zero-price-level problem by my colleague, Joseph Ostroy. A similar solution has been recently, independently, produced by Kurz. However, he concludes that money must be made convertible into a real asset in the last period. This is not precisely correct. All that is required for the possibility of a positive price of money is that an issuer of money commit itself to repurchasing all its money at market prices by the end of the final period. Convertibility, or a governmental requirement that a specified fraction of specified market values be held in the form of money at specified times, is required only when we wish to guarantee the positivity of the price of money.

Hence, if money is a durable good, Ax. 1 implies that money creators purchase the money in period T for either some predetermined amount of a real asset or for the amount of a certain real asset that the money buys in period T.

Exchange in every period is always constrained to follow "Walras' Law."

That is,

$$\text{Ax. 2: } \sum_{j=1}^M A_{jt}^P X_{jt} = 0 \text{ for every } t.$$

These T identities imply that the number of independent excess demand equalities corresponding to positively-priced assets satisfied by a solution set of prices is T less than the number of such equalities satisfied by those prices. Hence there are T fewer such independent excess demand equalities than there are positive prices so we have an "underdetermined" system. As a result, at least T of the prices in the solution will be arbitrary.

But the level of accounting prices in each period is supposed to be arbitrary, as prices are stated in terms, for example, of yen in period 1, marks in period 2, pesos in period 3, etc. If we change the unit of account in any period, say from yen to rupies, we will not alter current behavior even though we lower all prices in that period relative to prices in other periods. The reason is that prices are cheaper only in terms of a correspondingly more "expensive" unit of account. It requires the same amount of commodities to obtain a commodity in the period with the new unit of account even though prices in terms of units of account change. Thus, for every jt,

$$\text{Ax. 3: } X_{jt} = F_{jt} [A^P] = F_{jt} (\lambda_1^A P_1, \lambda_2^A P_2, \dots, \lambda_T^A P_T),$$

where each λ_t^A is an arbitrary positive number and A_t^P is the set of accounting prices for period t. We set, for every t,

$$\lambda_t^A = 1/P_{Mt}$$

thereby using the asset M_t for the numeraire in the t 'th period (which, incidentally, is the most natural way to express prices in the corresponding solution for $T-1$ periods). We thus find that the following relations must be satisfied in any solution:

$$X_{jt} \left(\frac{A_{P M1}^P}{A_{P M1}^P}, \frac{A_{P M2}^P}{A_{P M2}^P}, \dots, \frac{A_{P M_{T-1}}^P}{A_{P M_{T-1}}^P}, \frac{A_{P M_T}^P}{A_{P M_T}^P} \right) = X_{jt}(P) \left. \begin{array}{l} = 0 \text{ for } A_{jt}^P > 0 \\ \leq 0 \text{ for } A_{jt}^P = 0 \end{array} \right\}$$

This is a system of at most $(M-1)T$ independent equalities in at most $(M-1)T$ positive prices.

Do solutions exist? A family of sets of excess demand relations does exist -- e.g., any set in which all excess demand relations are continuous functions -- such that the excess demand relations will indeed possess a price solution, P^i . Using a Walras-Cassel rather than a Wald-Arrow-Debreu approach to the "existence of equilibrium," we simply assume that the relevant set of excess demand relations is contained in this family without maintaining some overly restrictive assumptions sufficient for this to occur.

Transcending classical and modern discussions of money economies have been two more assumptions that serve to further distinguish the commodity, "money," from other commodities. One is that there is no "money illusion"; the other is that there is a "determinate" price-level solution for a given nominal money supply.

No "money illusion" means that if everyone's nominal money balances, present and future, and all money prices are increased unexpectedly in the same proportion, λ , then excess demands for nonmonetary assets are unaltered and the excess demand for money increases by λ of its original value. In other words, letting S_M be the set of money supplies through time T ,

$$\text{Ax. 4: } X_{jt}[P; S_M] = X_{jt}[\lambda P; \lambda S_M] \text{ for } j < M.$$

This axiom embodies the standard assumption that individuals are indifferent to the physical composition of the money supply.

"Price level determinacy" implies that if all money prices in any solution are changed in the same proportion and to a sufficient degree, nominal money supplies remaining the same, an excess demand for some asset arises. That is, price level determinacy implies

$$\text{Ax. 5: } X_{jt}[\lambda P^i; S_M^i] > 0 \text{ for some } \lambda > 0, \text{ some } jt \text{ and all } i.$$

This axiom together with Walras' Law have been taken to imply the absence of zero-order-homogeneity with respect to money prices of some excess demand equations for real assets. This is not a correct inference because some non-monetary assets may not be real assets (i.e., they may not generate non-monetary services). The meaning and implications of this will be seen in Part III below.

Setting the λ in Ax. 4 equal to $1/P_{11}$, assuming P_{11} is positive, as we may do by virtue of Ax. 1,

$$(6a) \quad X_{jt}[P; S_M] = X_{jt}\left[\frac{P}{P_{11}}; \frac{S_M}{P_{11}}\right], j \neq M \text{ and}$$

$$(6b) \quad \frac{1}{P_{11}} X_{Mt}[P; S_M] = X_{Mt}\left[\frac{P}{P_{11}}; \frac{S_M}{P_{11}}\right] \text{ for all } t.$$

Hence the supply-demand relations do not yield determinate nominal money values; they can determine price levels only for given money supplies or money supplies only for given price levels. Either S_m^i or P_{ll}^i is determined outside the Casselian Model. This, of course, is a standard property of money models.

II. PROPERTIES OF A MODERN MONEY MODEL

A. The special assumptions of the model

The Modern Money Model assumes that all goods which ever appear in a Casselian Model either generate nonmonetary services or are money. Also, the number of independent equations is assumed to be at the maximum number, $T(M-1)$. And the number of independent variables is also assumed to be at this maximum. We shall see later that these Patinkin-type assumptions are crucial to the results and inconsistent with a Classical Money Model.

B. The invalidity of the Classical Dichotomy

The Classical Dichotomy implies that equilibrium excess demands for assets generating nonmonetary services depend only on ratios of money prices. That is, the Classical Dichotomy implies

$$(7) \quad X_{jt}[\lambda P^i; S_M^i] = X_{jt}[P^i; S_M^i] = 0 \text{ for all } j \neq M, \text{ all } t, \text{ and all } \lambda > 0.$$

From Ax. 2, (7) implies

$$(8) \quad X_{Mt}[\lambda P^i; S_M^i] = X_{Mt}[P^i; S_M^i] = 0 \text{ for all } t \text{ and all } \lambda > 0.$$

Together, (8) and (7) directly contradict Ax. 5. Hence the Classical Dichotomy is inconsistent with a determinate price level in a Modern Money Model.

C. The presence of Pigou Effects

It follows from Ax. 5, determinacy, that, for some j, t , and λ ,

$$X_{jt}[\lambda P^i; S_M^i] \neq X_{jt}[P^i; S_M^i].$$

Hence, by successive application of Ax. 4, for some j, t, and λ,

$$X_{jt} \left[\lambda \frac{P^i}{P_{11}^i}; \frac{S_M^i}{P_{11}^i} \right] \neq X_{jt} \left[\frac{P^i}{P_{11}^i}; \frac{S_M^i}{P_{11}^i} \right] \text{ and,}$$

$$X_{jt} \left[\frac{P^i}{P_{11}^i}; \frac{S_M^i}{\lambda P_{11}^i} \right] \neq X_{jt} \left[\frac{P^i}{P_{11}^i}; \frac{S_M^i}{P_{11}^i} \right].$$

In other words, a sufficient alteration in the price level will alter the excess demand for some real asset by altering the quantity of real balances. This is a Pigou effect -- an effect of price level flexibility on the supply of real balances and excess demand for real assets.

D. The economic effectiveness of anticipated inflation

A change in ratios of money prices in a Casselian Money Model, so long as the change reflects a change in the relative cost of alternative economic activities, implies changes in some excess demands for real assets under certain, admissible forms of the excess demand functions. (The linear form of (6a) of rank (m-1)T is sufficient to establish this generally accepted fact.) It is in this sense that changes in relative prices have "real effects" and in this same sense, we shall soon see, that anticipated monetary inflation has real effects.

An equi-proportionate increase in expected future money supplies and corresponding future money prices induces, at the initial real quantities demanded and supplied, a relative price change. To see the nature of this change, nonmonetary excess demand functions are now written, using (6a)

$$(9) X_{jt} [P; S_M] \equiv X_{jt} [P_1, {}_2^P; S_{M1}, {}_2^S] = Y_{jt} \left[\frac{P_1}{P_{11}}, \frac{{}_2^P}{P_{11}}, \frac{S_{M1}}{P_{11}}, \frac{{}_2^S}{P_{11}} \right]$$

where ${}_2^P = (P_2, P_3, \dots, P_T)$ and ${}_2^S = (S_{M2}, \dots, S_{MT})$.

We are supposing that there is a change from ${}_2^S$ and ${}_2^P$ to $\gamma {}_2^S$ and $\gamma {}_2^P$,

respectively. And suppose that excess demands are unaltered by this change so that

$$(10) \quad X_{jt} [P, S_M] = X_{jt} (P_1, \gamma_2 P; S_{m1}, \gamma_2 S_M), j \neq M.$$

Of course, if the monetary inflation were unexpected in period 1 or behavior in period 1 were unaffected even if the monetary inflation were anticipated, because of Ax. 4, excess demands in periods 2 to T would also remain unaltered. Then equation (10) would hold. But in period 1, with anticipated inflation, individuals see a change in relative prices between current and future assets, namely that each P_{ks}/P_{jl} for all j and k and all s from 2 to T inclusive changes by γ of its initial value. In the Modern Money Model, this implies a change in the real cost in terms of present good 1 of currently holding onto the money that would currently purchase good j in order to purchase good k in future time period s . Substitution of present j for future k is thereby encouraged by an anticipated monetary inflation. Hence, (10) cannot generally hold and the anticipated inflation has real effects. (See Thompson (1973b) for a general equilibrium model describing the particular real effects of anticipated inflation.)

III. THE CLASSICAL MONEY MODEL

A. The assumptions of the model

In the Modern Money Model every asset was either money or a generator of real services. We now drop this artificial assumption. Also, as we are about to see, we necessarily relax the assumptions that the (M-1)T money price variables are independent and the (M-1)T excess demand relations left after applying Walras' Law are independent. To this extra generality, in order to produce a "Classical Money Model", we add the assumption that money bears perfectly competitive interest. This key specification is now described.

Let $M-1$ be the index for the asset serving the function of "backing money" and let the real return, say in the form of real appreciation up to period T , on the corresponding commodity be the return paid for holding money. This asset is not money, but it is not generating nonmonetary services. The entire demand for this asset always comes from the money creators and the entire supply always comes from the money demanders. Let the $M-2^{\text{nd}}$ asset be the same commodity, i.e., the same physical object, used in its nonmonetary function, a function described in conventional value theory. Since commodities $M-1$ and $M-2$ are the same physical object, they have an identical price. Hence, we have at least T fewer independent solution variables than total solution variables in the Classical Money Model. Correspondingly, since the excess supply of commodity $M-1$ is identical to an excess demand for commodity M there are at least T fewer independent equations than equations in the Classical Money Model. We may identify the dependent equations as those expressing a zero excess demand for the $M-1^{\text{st}}$ asset. With the maximum number of independent equations and variables reduced by the same number, no inconsistencies or indeterminacies arise solely because of the existence of an asset to back money.

We are avoiding here the technique of measuring money in terms of the commodity which backs it, thereby removing the money asset and nominal money prices from the economy. Similarly, we could, by evaluating assets at service costs, simply consider both M_t and M_{t-1} as free goods and omit these assets from the discipline of Walras' Law. These techniques, while formally justifiable, tempt us to forget the fact that the money and money-backing markets enter disequilibria in a particular fashion. While both of these alternative approaches get us where we are going, our more explicit approach is hopefully more illuminating.

P_{kt}/P_{11} , for all k and all $t \geq 2$, can never represent a relative cost between alternative economic activities when direct interest is paid on money. The only relative cost it could possibly represent, due to the physical units denoted in the expression, is the relative cost of the k^{th} commodity t periods into the future relative to good 1 in the current period. It may indeed represent this cost in the Modern Money Model, for P_{11} is the gain in money in the future by selling a unit of commodity 1 now and $1/P_{kt}$ is what a unit of this money will buy of the k^{th} commodity t periods in the future. However, if positive interest is earned on the money, the price ratio obviously does not represent this cost because a greater amount of money than P_{11} is available for the purchase of commodity k in period t . Thus, the price ratio enters as only a component of this relative cost when interest is earned on the money. In particular, while the money obtained in currently selling a unit of commodity 1 is P_{11} , the extra money available in period t , keeping other assets at their original levels, will always be $P_{11}P_{M-1,t}/P_{M-1,1}$. Hence, the cost of asset k in time t relative to asset 1 in the initial period is

$$(11) \quad R_{kt} \equiv P_{kt} P_{M-1,1} / P_{11} P_{M-1,t}$$

B. The Validity of the "Invalid" Classical Dichotomy

1. The possibility of an "Invalid" Classical Dichotomy in a Classical Money Economy

We now show that it is possible to determine the equilibrium relative prices described in (11) using only the markets for real assets and then determine equilibrium money prices in the money markets. This Classical Dichotomy was impossible in the Modern Money Model. Consider the first $M-2$ excess demand relations in each period. These represent the markets for the assets generating nonmonetary services, the "real" assets. Since

(12)

$$-P_{M-1,t} X_{M-1,t} \equiv X_{Mt}$$

it follows from Ax. 2, Walras' Law for each period, that "Say's Law" holds for each period. I.e.,

(13)

$$\sum_{j=1}^{M-2} P_{jt} X_{jt} = 0 \text{ for every } t.$$

This implies that there are at most only (M-3)T independent excess demand relations for real assets. Since we have (M-2)T-1 relative prices between real assets described in (11), there seems to be too many independent variables for a generally determinate solution unless we bring in money equations. But inspection of (11), keeping in mind that $P_{M-1,t} \equiv P_{M-2,t}$, reveals that

(14)

$$R_{M-2,t} \equiv R_{M-2,1}$$

Since there are T-1 of these relative price identities, the maximum number of independent relative prices between real assets under the competitive payment of interest on money is only (M-2)T-1-(T-1) = (M-3)T, the maximum number of independent excess demand relations using Say's Law. Specifying certain forms of the corresponding (M-3)T excess demand relations (e.g., forms generating a linear basis) we may now solve equations and determine the (M-3)T relative prices.

Inspection of (11) shows that to find P_{kt} , it is necessary to know $P_{M-1,t}/P_{M-1,1}$ in addition to R_{kt} and P_{11} . Now $P_{M-1,1}$ is determined by $R_{M-1,1}$ and P_{11} ; or, alternatively, P_{11} is determined by $R_{M-1,1}$ and $P_{M-1,1}$. Hence the remaining independent variables are the T money prices, $P_{11}, P_{M-1,2}, \dots, P_{M-1,T}$; or, alternatively, $P_{M-1,1}, P_{M-2,2}, \dots, P_{M-1,T}$. Our remaining equations are T independent money excess demand equations. Using these equations to solve for the price levels gives us a solution set of money prices, P^i . This completes a Classical Dichotomy since relative prices have been determined in the real markets and money price levels determined in the money markets.

The proof of the invalidity of the Classical Dichotomy used in Part IIIa) does not go through because we have introduced here a market for an asset serving as "backing" for money, such an asset being necessary if real interest is to be paid on money. The introduction of this market allowed us to consider disequilibrium money supplies for a given price level, and, finally, a determinate solution set for the real money supplies. Without such a market, the money supply can never be unequal to the money demand due to Walras' Law so that any money supply represents a solution for a given price level, and real cash balances are indeterminate. In other words, any increase in a solution price level for a fixed money supply in a dichotomized Modern Money Model could not disequilibrate any market and therefore is also a solution, thus violating the determinacy axiom (Ax. 5), while in the dichotomized Classical Money Model, the determinacy axiom is not violated because an excess demand for money induced by a higher price level is matched by an excess supply of the asset backing money, with relative prices and excess demands in markets generating non-monetary services remaining unaffected.

2. The necessity of the "Invalid" Classical Dichotomy in the special case characterized by consistency with orthodox value theory.

It is rather trivial to see that this Classical Dichotomy always holds if we restrict the Classical Money Model to be consistent with orthodox value theory. Conventional value theory (e.g., Debreu) determines the set of relative price sets satisfying supply-demand constraints without reference to the price level, money supply, excess demand for money equation or the money-backing services of the assets. That is, it determines the set of sets of relative prices between the (M-2)T assets generating nonmonetary services by having these prices satisfy the corresponding supply-demand relations for assets generating nonmonetary services. Because commodity M-1 is the same phy-

sical object as commodity M-2, the theory also serves to determine the real price of M-1 still without reference to the markets for assets M-1 or M. The excess demand for asset M-1 need not be zero in this theory, as the asset is not part of conventional value theory because it does not generate nonmonetary services. If we then specify sequences of transactions which will achieve a particular relative price solution and the costs (including zero) of each transaction, then we can find amounts of money such that if each individual holds these amounts at each date for a given P_{11} ; $P_{M-1,2}$; $P_{M-1,3}$; ...; $P_{M-1,T}$, then there is no cost of his transactions.*/ Such amounts must exist if the model is to be consistent with orthodox value theory. These amounts represent demands for money. If we now introduce costless supplies of nominal money to each individual from the money creators, we can always find supplies at which demands and supplies are equal. At the quantities that the money creators plan to take at the zero net supply price, there is an equivalent excess supply of assets backing money. Differences between the demand prices and the zero net supply price of money then give rise to an adjustment in supply through a usual Marshallian adjustment mechanism. (Such a mechanism is the appropriate dynamic mechanism in the presence of production decisions, there being a lag in the time of decision and the time of output but no lag in the time of output and the market evaluation of the output.) In this way, a set of solution real cash balances is determined only after the selection of a set of relative prices satisfying the excess demand equalities for the services of nonmonetary assets. Hence, the set of sets of solution real cash balances is determined only after the independent determination of the set of sets of solution relative prices. This is the Classical Dichotomy.

*/ The existence of such sequences and the sufficiency of decentralized individual choices in achieving these sequences via purchases of money, is established in Thompson (1973a).

C. The absence of Pigou Effects

In the Modern Money Model, the money supply does not change when the price level changes. But it does change in the Classical Money Model and this affects the impact of price level flexibility on the system. In particular:

$$(12) \quad X_{jt} [\lambda P^i; S_M(\lambda P^i)] = X_{jt} [\lambda P^i; D_M(\lambda P^i)] \text{ for all } jt$$

defines a money supply function in a Classical Money Model. From Ax. 4,

$$(13) \quad D_M(\lambda P^i) = \lambda D_M(P^i).$$

Therefore, since $D_M(P^i) = S_M(P^i)$,

$$(14) \quad X_{jt} [\lambda P^i; \lambda S_M(P^i)] = X_{jt} [\lambda P^i; S_M(\lambda P^i)], \text{ and, from Ax. 4,}$$

$$(15) \quad X_{jt} [P^i; S_M(P^i)] = X_{jt} [\lambda P^i; S_M(\lambda P^i)], \text{ for all } jt.$$

So real excess demands are not altered by price level changes in a Classical Money Model.

D. The lack of economic effectiveness of anticipated inflation in the Classical Money Model.

The analysis of IID is, of course, sufficiently general to apply to the Classical case. It follows from (9) in that analysis that the effectiveness of anticipated monetary inflation requires the condition, satisfied by the Modern Money Model, that the change in price ratios induced by a proportional anticipated monetary inflation implies a change in the relative cost of alternative economic activities at the original allocation.

In fact, when competitive interest is paid on money, the price ratios that change with a proportional anticipated monetary inflation do not represent relative costs between alternative economic activities. Since proportional monetary inflation converts at original real behavior, both P_{kt}/P_{11} and $P_{M-1,t}/P_{M-1,1}$ to γ times their original values, the identity in (11) shows that R_{kt} is unaffected for all k and t . Hence, the inflation changes no relative costs between alternative real activities. Therefore, anticipated monetary inflation in a Classical Money Model has no real effects.

IV. PROPERTIES OF MODERN AND CLASSICAL TEMPORARY EQUILIBRIA.

A. A Single Period Temporary Equilibrium

A temporary equilibrium set of accounting prices, P_1^* , is a set of current asset prices such that

$$(16) \quad X_{j1}[P_1^*, {}_2P^E(P_1^*)] \leq 0 \text{ for all } j.$$

The set of future prices to all individuals, ${}_2P^E$, is simply the set of expected prices determined by initial expectations and current prices.

Some future excess demands may therefore be positive in a temporary equilibrium.

Individuals may not agree on expected future prices so that Walras' Law (Ax. II) does not hold for future behavior (i.e., it does not follow from the aggregation of individual budget identities). It does, however, hold for the current period. The other four axioms remain unaffected except that in Ax. 5., λP^i is replaced with $\lambda P_{1,2}^E(\lambda P_1^*)$, and the inequality there holds for $t=1$.

Under these conditions, straightforward extensions of the proofs above apply so that (1) a temporary equilibrium exists when they are continuous excess demand relations for given, continuous price-expectations-functions, ${}_2P^E(\cdot)$, proved in (Arrow-Hahn) and (2) the same properties regarding the Classical Dichotomy, the existence of Pigou Effects, and the effectiveness of expected inflation hold in temporary equilibrium as held in full equilibrium for both Classical and Modern Money Models.

B. The Model at a Keynesian Level of Aggregation

1. The General Model

We now consider a single-period temporary equilibrium aggregated to a Keynesian level to facilitate comparisons. Besides money, a Keynes' case model contains a labor aggregate which produces a commodity output under decreasing returns, the latter fact implying the existence of a non-labor input, which, for economy of variables, we take to be the services of capital. Thus, considering a minimum number of goods, there are four goods, money (M), capital goods (Q), labor (L) and capital services (K) in the model. There is a current market and price for each good. The price of current money is set at unity, the price of current labor is W , the price of renting capital is R , and the price of current capital goods, which represent the output of the production process, is P_q . Using Walras' Law to eliminate the market for Q, and assuming that all equilibrium prices are positive, we can use the following three equations to determine competitive prices:

$$(17) \quad X_M(W, P_q, R) = 0$$

$$(18) \quad X_K(W, P_q, R) = 0$$

$$(19) \quad X_L(W, P_q, R) = 0.$$

The above representation differs from standard simple representations (e.g., Patinkin) only in that our market for capital services is absent from the standard formulation, which introduces a market for bonds in order to have a market to remove with Walras' Law. Since there are no compelling a priori restriction on the nature of the excess demand function for bonds,

the usual formulation omits the bond market with Walras' Law retaining the market for capital goods (i.e., homogeneous consumption and investment goods), where restrictions come somewhat more readily. Our approach is unique in that it recognizes the necessity of a capital services market* and uses the familiar a priori restriction on input markets implied by neoclassical production theory to replace the psychological conjectures of Keynes and later writers regarding the market for capital goods. This is elaborated below and shows the inconsistency of the standard Keynesian model of one-period temporary equilibrium with simple neoclassical production theory.

Assuming the markets for the factors of production are perfectly competitive and that the aggregate production function has positive first derivatives and negative second derivatives, we can represent equations (18) and (19) by

*/ Simply adding a market for capital services to the conventional model without removing the bond market and without adding any new variables would obviously yield a direct mathematical inconsistency in the model. To avoid this immediate inconsistency and retain the bond market, one would have to introduce a liquidity difference between our short bonds and rental capital, treating the interest rate on bonds as a different variable than the interest rate on capital services. But then, one would naturally introduce an arbitrage relation between the interest rates, tying the interest rate differential to the transactions cost of selling short bonds to finance purchases of rental capital. This establishes an additional equation so that the general inconsistency would remain.

To avoid these inconsistencies, and still retain an independent spendings relation (as opposed to one which can be removed with Walras' Law, leaving a set of equations whose functions are given a priori independent of spendings propensities), one would have to make the rate of inflation an equilibrating variable rather than an expectations parameter: in which case the rate of change of the money demand -- a component of which are spendings propensities -- and the rate of change of money supply would be equated in describing the equilibrium. This would severely challenge traditional realism claims made in support of Keynesian models and would leave unaltered the qualitative comparative static conclusions developed below for shifts which affect demand or supply levels without altering the immediate future rates of change of these levels (e.g., shifts which increase the current money supply without altering its rate of growth in the immediate future). This latter fact can be easily established in a generalization of the above temporary equilibrium to two periods rather than one, shifting a demand or supply in the same proportion in each of the first two periods.

$$(18') \quad R = P_q \frac{\partial Q(K^*, L)}{\partial K}$$

$$(19') \quad W(L) = P_q \frac{\partial Q(K^*, L)}{\partial L}$$

where K^* is the fixed endowment of capital and $\frac{dW(L)}{dL} \geq 0$, which reflects the presence of sticky, and possibly rigid money wages. The one period money rate of interest is given by

$$(20) \quad r = \frac{R}{P_q} + \frac{P_q^e - P_q}{P_q},$$

where P_q^e is the expected level of prices in the next period. We assume, as is conventional, that P_q^e varies in proportion to P_q so that $\frac{P_q^e - P_q}{P_q}$, the expected rate of inflation, is constant.

2. A Graphical Description of the Modern and Classical Special Cases

a. Equilibrium in the Factor Markets

For a given P_q , (19') will determine an equilibrium level of L , and, given this level of L in addition to P_q , equilibrium R is determined by (18'). In this way equilibrium R is determined for each possible level of P_q . Using (20), equilibrium r is also determined for each possible level of P_q . Hence, we construct the following curve, the F-F curve, describing equilibrium in the factor markets:

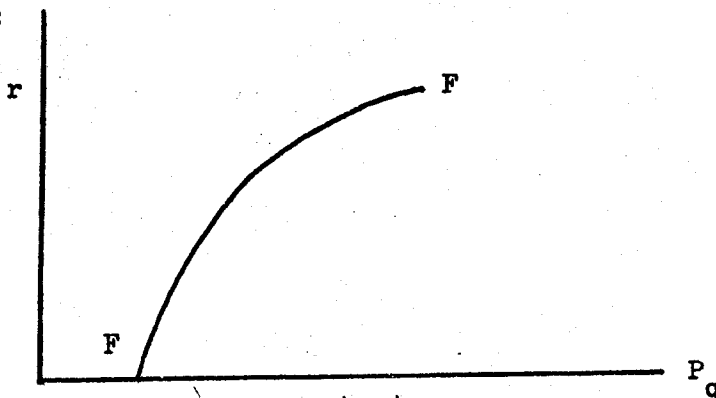


Figure 1. Equilibrium in the factor markets

r rises with P_q because when there are only two factors of production and an aggregate, linearly homogeneous, production function with diminishing marginal products ($\frac{\partial^2 Q}{\partial L^2} < 0$, $\frac{\partial^2 Q}{\partial K^2} < 0$), the factors of production must be complementary. Under these conditions the increase in L induced by an increase in P_q will, by increasing $\frac{\partial Q(K^*, L)}{\partial K}$, increase R by more than proportion to the increase in P_q . Because it is plausible that there is a rising supply price of labor and an elasticity of substitution which is less than unity, this curve is drawn concave from below. Also, because it is plausible that there is some positive money wage at which no labor will be supplied and an upper bound to the marginal product of labor, the FF curve becomes vertical at sufficiently low price levels, indicating that real wages are so high that no production is profitable.

Now, moving to the combination of P_q and r that will produce an equilibrium in the money market, an increase in P_q will increase the demand for money. The effect of this increase in demand for money on the interest rate will depend on the supply of money.

b. Equilibrium in the Modern Money Market

In a Modern Money Model, where the supply of money can be treated as an exogenously determined constant, an increase in P_q creates an excess demand for money and therefore a higher interest rate is required to reduce the demand for money sufficiently to restore equilibrium in the money market. In this case, the curve showing the levels of r and P_q consistent with equilibrium in the money market, the standard LM curve, is drawn as:

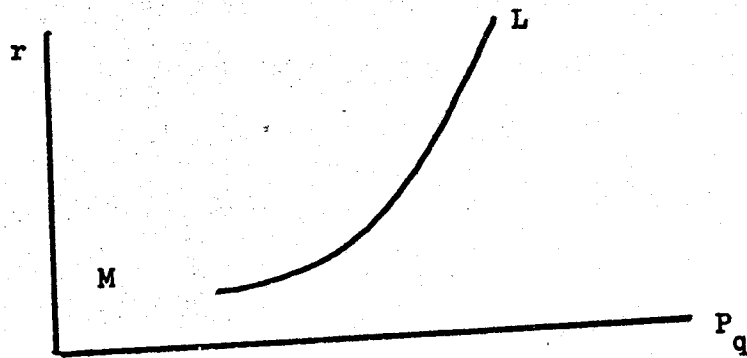


Figure 2. Equation in the Modern Money Market

These display the values of P_q and r satisfying equations (17) and (20). We assume, for graphical convenience, that W has no effect on X_M . The curve is drawn convex from below because of the plausibility of a liquidity trap and of the convex-from-below slope of the standard liquidity preference curve (see Thompson 1973b for a theoretical rationale.)

c. Equilibrium in the Classical Money Market

In a classical money economy, where the supply of money varies with the demand for money so as to keep prices constant at a given conversion rate of money into commodities, the LM curve is drawn as follows:

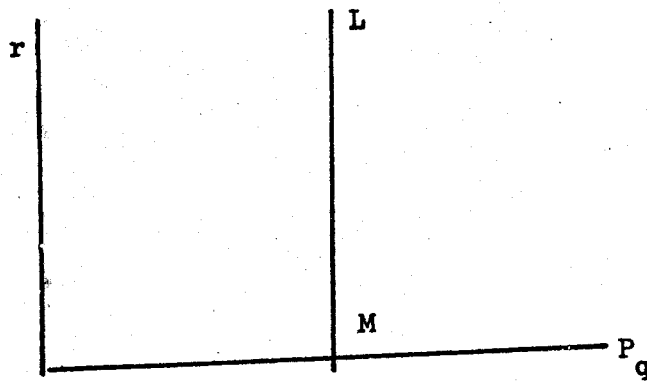


Figure 3. Equilibrium in the Classical Money Economy

d. The Modern and Classical Solutions

The LM and FF curves are put together in Figures 4 and 5 to determine the pairs of r and P_q that are equilibrium in all markets for the Modern and Classical Money Economies, respectively.

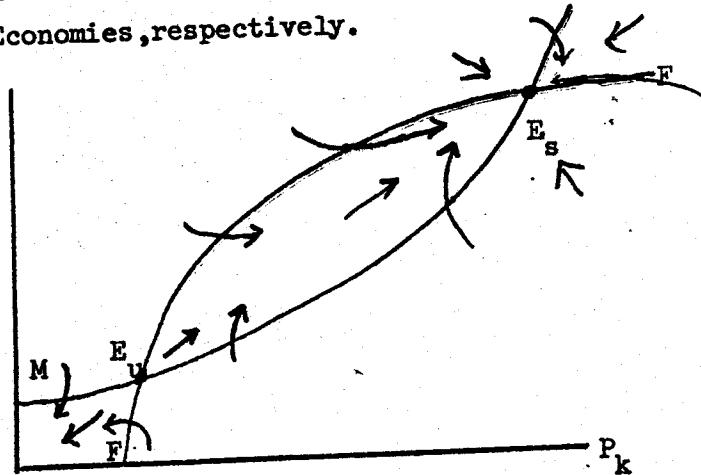


Figure 4. Temporary Equilibrium Points for a Modern Money Economy

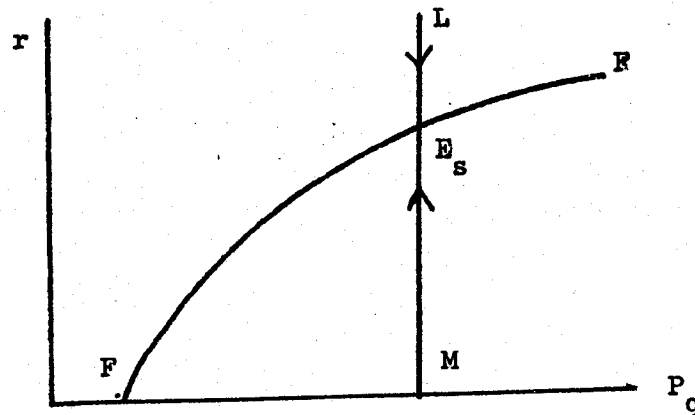


Figure 5. The Temporary Equilibrium in a Classical Money Economy

e. The stability of the Modern and Classical Solutions

E_s denotes a "stable" equilibrium and E_u denotes an "unstable" equilibrium. An equilibrium is "stable" when a small change in prices away from the equilibrium will produce forces returning the economy to the equilibrium. Market adjustments in the classical model are simple. Convertibility insures

a given price level P_q^* ; this P_q^* is taken over to the real markets where it is used to determine employment and then the interest rate. There is no possibility of an instability.

But in a modern money economy, market adjustments are not so simple. The dynamic adjustment conditions are

$$(21) \quad \frac{dP_q}{dt} = f_1(X_m) \quad f_1' < 0, f_1(0) = 0$$

$$(22) \quad \frac{dr}{dt} = f_2(X_K) \quad f_2' > 0, f_2(0) = 0,$$

where the labor market is assumed to remain in temporary equilibrium, as necessarily occurs when the supply price of labor is constant, indicating rigid rather than just sticky money wages. The possible dynamic paths are indicated by the arrows in Figure 4. There is an unstable equilibrium at low price levels and interest rates, admitting the dangerous possibility of a vicious decline toward zero production in a Modern Money Economy. When prices fall, capital rentals fall, which induces increases in the demand for money, which makes prices fall even faster, etc.

f. Comparing a Temporary Equilibrium in a Modern Money Economy to a Conventional Keynesian Equilibrium.

Figure 6 shows the equilibrium points in the commodity market, the IS curve, as implied by Walras' Law.

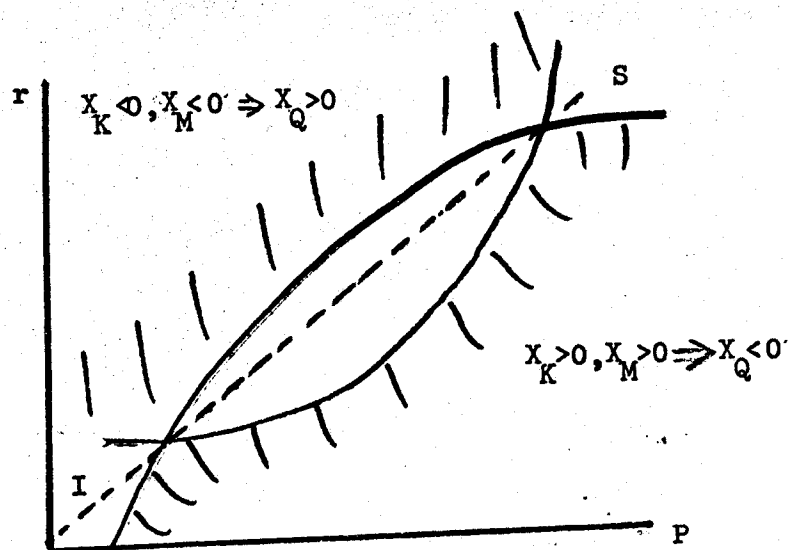


Figure 6. The IS Curve in a Modern Money Economy

As is indicated in the Figure, the curve must have a positive slope, connecting the equilibrium points and staying between the FF and LM curves.

Since the familiar IS curve in conventional macrotheory has a negative slope, some explanation is in order. The conventional theory develops the IS curve from an independent, a priori relation between the interest rate and aggregate spending. If this aggregate spending is to be interpreted as that which determines current income for a given expectation of the inflation rate, then an independent IS curve is inconsistent with neoclassical production theory. The reason is simply that an interest rate, price level pair determines income in neoclassical production theory independent of spending propensities. And since demand conditions are completely described by the LM curve, there is simply no room for an independent determination of aggregate spending given the LM curve, neoclassical production, and a given, expected rate of inflation. The effect of a higher interest rate on aggregate demand and the price level is determined completely by the reduced demand for money and consequent higher demand for commodities.

Conventional macrotheory, with its independent spendings effects given the LM curve and neoclassical production theory, is internally consistent only if we drop the assumption of a given expectation of a rate of inflation. In particular, the expected rate of inflation may depend parametrically upon the observed or expected rate of spending (or, more generally, the rate of decrease in the demand for money). Then an increase in the observed or expected rate of consumption or investment spending (or, more generally, a decrease in the expected future demand for money) would, by increasing r for a given R , shift up the FF curve. In a modern money economy, this shift induces a movement out of money in the current market (a movement along the LM curve) and a higher price level, while a classical money economy admits no such adjustment because of its vertical LM curve. While this procedure is probably the most useful with which to view spendings variables from the standpoint of business cycle policy, it does not capture the Keynesian concept of an equilibrium rate of expenditures.

To have an equilibrium rate of expenditures -- or, most generally, an equilibrium rate of change in money holdings -- the corresponding price, i.e., the rate of inflation, just be an independently equilibrating variable rather than an expectations parameter determined by other variables in the system. Once the expected rate of inflation is replaced with an equilibrium rate of inflation, and a corresponding rate of change of the money supply is added -- which is essentially moving to a 2-period temporary equilibrium (in which it is only markets in periods 3 and later that may have incorrectly expected prices) -- the Keynesian expenditures

conditions can be relevant. However, the spendings functions in such a model would have to be only part of a general function describing the rate of change of the demand for money, the rate of change of the money supply would be necessarily relevant, and it is easy to show that the familiar, Keynesian comparative static results that are based upon a negatively sloped IS curve fail to hold in such a model just as they failed in the above, single period model.

g. Contrasting the Comparative Statics of the Modern and Classical Temporary Equilibrium Models

An increase in the demand (or decrease in the supply) of money in a Modern Money Economy, which shifts the LM curve to the left, lowers the temporary equilibrium price level and interest rate. (The process is that a greater demand for money lowers the demand for commodities and hence lowers the price level, which in turn lowers the demand for labor and employment, which in turn lowers the rental rate on capital and hence the interest rate.) Thus, "Gibson's Paradox," the observation that interest rates are unusually low during periods of unusually low prices while the fluctuations are due to unusual monetary shifts, can be explained by our model of temporary equilibrium in a Modern Money economy while it is inconsistent with conventional Keynesian Models. In a Classical Model, there is no real effect of

a change in the demand for money as the increase in demand for money merely causes an increase in supply, with no resulting change in the excess demand for money or the LM curve.

A reduction in the marginal product of capital shifts down the FF curve and thereby lowers the stable equilibrium price and employment levels in a modern money economy. (The process is that a lower rental rate on capital increases the demand for money and therefore lowers the demand for commodities, which in turn lowers the price level and employment and further lowers the interest rate.) And again there is no effect of the shift in a classical money economy (as is obvious from Figure 5) because whatever the induced change in demand for money, there is a corresponding change in the competitive supply of money and no pressure on the price level.

Finally, a reduction in the marginal physical product of labor (or increase in the supply of labor) will increase unemployment in both economies. The magnitude of the effect is larger, however in a Modern Money Economy. The reason is simply that the induced decrease in employment reduces the marginal physical product of capital, thus shifting down the FF curve and creating an induced reduction in the price level and employment. The same induced reduction in the F-F curve occurs in the Classical Money Economy, but, as we have seen, such a reduction has no effect upon employment in a Classical Money Economy.

h. The Several Output Case

The above analysis shows the dynamical superiority of Classical over Modern Money Economies in a single-output, single-period temporary equilibrium model. That is, the Classical temporary equilibrium has stability

characteristics and employment responses to exogenous shifts which are uniformly superior to those in a Modern Money Economy. However, when another output sector is added, employment in a Classical Money Economy may easily be more affected by exogenous shifts, those generating shifts in relative prices between the output backing money and other outputs. This was shown in Thompson (1973a), where it was argued that such a shift was responsible for the Great Depression and the abandonment of classical-type monetary institutions.

V. AN OPTIMAL MONETARY SYSTEM

There is a monetary system which maintains the desirable static and dynamic characteristics of a Classical Money Model but is not subject to changes in employment due to changes in relative output prices. In such a system, the government creates a paper asset, which we may call currency, and forces privately, but competitively, supplied money to be convertible into this paper asset. The government then controls the value of currency by altering its supply or demand (and not the supply for a demand of money). A positive demand and price for the asset, which is just paper (meaning that it is costless to produce and of zero consumptive or productive value), can be achieved by requiring individuals to pay taxes, say income taxes, in terms of the paper. Then the demand is controlled by controlling the tax rate and the supply is controlled by controlling the expenditure rate. The price levels at some tax payment dates are determined by the equation,

(23)

$$C = tP_q Q,$$

where C is the supply of government currency immediately before tax collections at those dates and t is the fraction of total income over the tax period, P_Q , that represents governmental tax revenue. (The equality must hold on the last tax day of the world because there is no real use for currency beyond that date.) The real value of money is thereby determined by the private value of currency as an asset required for paying taxes. This value is "non-monetary"; it is not a reflection of the value of money as a medium of exchange as it is in standard monetary theory. Money in our model receives no premium for its liquidity because sufficient quantities are always costlessly, competitively supplied.

Equation (23) should not be taken to represent a "currency-quantity theory" because: (1) It only holds at certain dates so that the "velocity" term does not represent a rate of turnover of a given stock of money over time (currency supplies between tax collection dates are irrelevant), (2) the "velocity" term is set by the government as the real effective income tax rate rather than determined by the complex workings of the private economy, and (3) quantity theories -- whether represented by price-specie-flow mechanisms, the English Currency School, or the monetary theories of Marshall, Fisher, or Friedman -- all assume away any independent, non-monetary determination of the price level or aggregate income; our causal relevance is from the independently determined price level to the total money supply rather than vice versa.

Although fiat money bears no direct interest, the presence of perfectly substitutable, competitively supplied paper monies implies that holders of currency must gain through the steady real appreciation of their currency

(at the corresponding own real rate of interest) as the upcoming date of tax payment satisfying the above equality approaches. At such dates, the government will hold all of the currency. Therefore, a jump in the price level immediately following such dates implies no cost to any individual since no one is a net owner of currency when the price jumps. (Competitive suppliers of money convertible into currency at a fixed intertemporal rate must, however, compensate anyone who holds their monies through such periods with an "interest" payment equal to the percentage jump in the price-level.) Such jumps in the price level are, in general, required in order to satisfy the price solution implied by the next date of tax payment satisfying the above equality. Thus, a price level solution to this model is a function over continuous time which falls at the corresponding real rate of interest up to a certain tax payment date, at which time it jumps discontinuously and then again follows the deflationary path to the next tax payment date for which the equality again holds. This continues on until the last date of tax payment in the world, when the equality must hold. (We assume that there is no demand for money after that date.)

The tax payment date at which the equality must hold immediately preceding the last tax payment date is, moving backwards in time, the first date for which the currency supply is less than the taxes which would be payable if we used the price level implied by the deflationary path to the last tax period -- i.e., for which the rate of growth of aggregate real taxes to the last tax date exceeds the sum of the corresponding real rate of interest and the growth rate of the currency supply. If such a tax date exists, then the immediately preceding tax date at which the equality must hold would be the

first tax date, again moving back in time, for which the growth rate of real taxes to this date exceeds the sum of the corresponding real rate of interest and the growth rate of the currency supply. This procedure continues on back to the present period to determine all of the dates in which the equality must hold. For any other date, the supply of currency is less than or equal to the demand for currency to pay current taxes given the price level function derived above.

It is apparent from our equation describing equilibrium in the currency market (not the money market) that equilibrium income is determined solely by the currency (not money) supply and tax rate as long as all taxes are income taxes. This is a direct way of showing that the Keynesian shifts, shifts in the marginal product of capital, in aggregate expenditure and thus the expected rate of inflation, and in liquidity preference, have no effect on equilibrium income in our model.

The variable that determines unemployment is the price of currency relative to labor. If the government controls only the demand or supply of currency so as to keep this relative price constant (or otherwise predictable) from one peak tax payment date to the next, then there is no involuntary unemployment. The potential for superiority of the competitive, currency-standard system over a classical monetary system, which has convertibility into a real commodity, lies in the potentially zero cost of achieving an announced price of the asset that backs money relative to wages. The same potential exists for a Modern Money Model. An announced money price of labor can also be achieved by manipulating the aggregate money supply of that model. However, while the money supply in a Modern Money Model must be manipulated to offset the various Keynesian shifts, such is not the case in the current model. Thus,

while a complete, empirically specified model of the macro economy is necessary to achieve an announced wage level in a Modern Money Economy, no such grandiose empirical model is required to achieve the announced wage level with our competitive money model. In our model, the government need only make the currency supply grow at the growth rate of peak period real taxes evaluated in terms of labor in order to insure full employment.* Furthermore, our competitive, currency standard economy is statically efficient and does not have the stability problems of a Modern Money Model.

*Thus, since $C = tP_q Q = t \cdot \frac{PQ}{W_t} = tsWL$, in order to keep money wages constant, C need only satisfy: $\frac{\dot{C}}{C} = \frac{\dot{t}}{t} + \frac{\dot{s}}{s} + \frac{\dot{L}}{L}$; where the dot over the variable signifies its derivative with respect to time. For a constant price level, which would allow involuntary unemployment, but only that due to shifts in the productivity and supply of labor, the formula is simply: $\frac{\dot{C}}{C} = \frac{\dot{t}}{t} + \frac{\dot{Q}}{Q}$.

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