THE THEORY OF

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by

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The risk-transfer theory, associated most prominently with the names of J. M. Keynes and J. R. Hicks,[1] is the best-known approach to the problem of speculation. According to this hypothesis speculators are relatively risk-tolerant individuals who accept price risks from more risk-averse hedgers. There is an alternative hypothesis, due to Holbrook Working, which denies any such fundamental difference between the motivations of speculators and hedgers. According to this alternative knowledgeable-forecasting theory, what may look like risk-transfer behavior is only the interaction of traders with more and less optimistic beliefs about approaching developments that will affect prices. On this view, futures markets do not serve mainly to facilitate the transfer of risk; rather, they provide an instrumentality whereby a consensus of informed beliefs about future supply-demand influences can be brought to bear in a timely way upon current production-consumption decisions.[2]

The question of the fundamental explanation of speculation is closely connected to the pattern of price movements over time. Is the normal relation between current prices of futures contracts and the later spot prices such as to reward speculators for bearing price risks? Or is the current futures price merely the mathematical expectation of the spot price ultimately realized,[3] so that there is no reward (on average) for bearing price risk?[4] And, apart from the possible role of risk-transfer, how does the pattern of price movements reflect the relation between current consensus opinion and later revealed actuality emphasized by the knowledgeable-forecasting theory?

A predecessor paper to this one[5] developed a general-equilibrium model of the speculative process upon the following foundations: (1) Speculation

occurs only in an "informative situation." This means that traders anticipate
that additional public information as to exogenous factors influencing supply
or demand will emerge before the close of trading, with consequent impact
upon prices. Accordingly, some or all traders will move not to their final
consumptive choices but to <u>trading positions</u>, intending to revise portfolios
after the price shift has occurred. As a working definition, let us call
those traders who plan for later portfolio revision "speculators."[6] (2) Given
an informative situation, the prospect of price change generates the "price
risk" that the traditional speculation literature emphasizes. But this price
uncertainty is <u>endogenous</u>; it is a resultant of the more fundamental under-
lying uncertainty about the <u>exogenous</u> factors determining demand and supply.
In the simplest case, where these exogenous factors impact only upon individ-
uals' quantitative commodity endowments (e.g., where the weather affects crop
yields),[7] we can speak simply of individuals' speculative behavior as
influenced by the interaction of endogenous <u>price risk</u> with exogenous <u>quantity</u>
<u>risk</u>. In its exclusive preoccupation with price risk the traditional speculat-
ion literature has neglected the even more fundamental influence of quantity
risk upon traders' decisions. (3) In an informative situation, under which
speculation takes place, there are two inter-related market equilibria. The
first is associated with trading prior to, and the second with trading poster-
ior to, the arrival of the anticipated information. Prior-round commitments
are, of course, made under uncertainty. More specifically they depend upon
traders' evaluations of their personal quantitative situations, in comparison
with the behavior of market aggregates influencing prices, over the set of
alternative contingencies (states of the world) that may obtain in the
posterior round of trading. In posterior trading the uncertainty has been
at least partially resolved -- fully so, if the emergent information is
<u>conclusive</u> as to which state obtains.

In the predecessor paper the model employed permitted <u>trading in contingent</u> <u>claims</u> to the risky commodity. With a number of other idealizing assumptions (see below), the results obtained can be summarized in two Propositions: (A) Speculative trading is undertaken <u>only</u> by individuals whose opinions deviate from representative beliefs about the likelihoods of future states of the world (associated with smaller or larger quantitative social endowments of commodities). In this model, contra the Keynes-Hicks theory, <u>differences</u> <u>in risk-tolerance alone never motivate speculative trading</u>. (Given some degree of belief-deviance, however, the individual's risk-tolerance influences the <u>scale</u> of his speculative commitments.) (B) The prior-round price (of a given contingent claim) will be the mathematical expectation of the posterior-round price (i.e., the price revision that takes place between prior and posterior rounds of trading will constitute a "martingale") if the probabilities are calculated in terms of <u>representative beliefs</u> in the market.

The aforesaid model, like all such theoretical constructs, made use of a number of not-entirely-defensible idealizing assumptions. The exact results summarized in Propositions A and B are therefore somewhat too sweeping. Even if the model were "valid," in the sense of providing a usable representation of speculation phenomena, Propositions A and B would only be expected to hold in some more modest sense such as: (A') Speculators are <u>primarily</u> individuals whose probability beliefs (rather than whose risk-tolerances) deviate from those more typical of individuals in the market, and (B') The price-revision relation between prior and posterior rounds of trading will <u>approximate</u> a martingale (calculated in terms of representative beliefs).

In the present paper certain of these idealizing assumptions will be maintained on the ground that, if not literally innocuous, they are

justifiable in terms of a benefit-cost calculation weighing simplicity against inaccuracy. In this group may be listed: (#1) The "standard" theoretical postulates of costless transactions, price-taking behavior, and instantaneous market-clearing. The issues involved in these assumptions are too familiar to warrant discussion here. (#2) For all individuals, an additive utility function (i.e., zero complementarity in preference) as between a single risk-less commodity N and a single risky commodity Z.[8/] (#3) No "real time" between prior and posterior trading. The purpose of this assumption is to isolate the effect of informational emergence upon "before and after" prices from the effect of time-discount. In practice, since information accumulates over real time, Proposition B or B' would have to be adjusted to allow for the temporal element. (#4) Prices reflective of "representative beliefs" of traders in the market. Only under rather special assumptions can the heter-ogeneous beliefs of traders be replaced by "consensus beliefs" (i.e., by a single probability distribution which, if unanimously held, would determine the same market prices.[9/] The special assumption used here to achieve this result is called "representative beliefs," meaning that <u>essentially all the market weight in price determination consists of traders sharing identical probability beliefs</u>. This does not mean that belief-deviant traders are rare or unimportant, but only that their deviating choices can be assumed to cancel one another out insofar as effects on prices are concerned. In practice one would want to interpret "representative" beliefs as <u>average</u> beliefs. Since the key result is that only belief-deviant individuals speculate, the assumption is roughly equivalent to postulating that the markets in which speculation occurs are actually dominated (so far as price determination is concerned) by non-speculative transactions.

The key issue of the present paper is <u>the effect of alternative regimes</u> <u>of markets upon the validity of Propositions A and B</u>. In the predecessor paper the postulated regime was called Semi-Complete Markets (SCM). This permitted trading in contingent state-claims to the risky commodity Z (but not in contingent claims to the riskless commodity N). In a world of two possible states <u>a</u> and <u>b</u>, under SCM the marketable claims are the trio N, Z_a, and Z_b. The seemingly arbitrary limitation preventing trading in distinct state-claims to the <u>riskless</u> commodity is quite essential -- for the convincing reason that under a regime of <u>Fully</u>-Complete Markets (FCM), with trading permitted in all possible claims N_a, N_b, Z_a, and Z_b, speculation in the sense of portfolio revision would not occur at all![10/] Each trader would be able to buy an initial holding covering his desired consumption baskets in the light of the alternative possible information-events as well as over the different state-contingencies; there would be no need for a posterior market to provide for portfolio revisions.[11/]

The purpose in this paper is not to go in the direction of still more ample market regimes <u>precluding</u> speculation, but rather to study models that provide more "realistically" for it. This requires that the permissible scope of trading be <u>reduced</u> rather than enlarged, relative to the regime of Semi-Complete Markets. Specifically, taking account of the frequently-voiced objection that markets in conditional claims are rarely observed, it will be explicitly assumed here (as is already implicitly assumed in the traditional speculation literature) that <u>only certainty claims to commodities may be</u> <u>traded</u>.[12/] This will be called a regime of "Unconditional Markets" (UM). The key question is whether Propositions A and B, arrived at under the seemingly artificial assumption of Semi-Complete Markets, continue to be valid for the "realistic" regime of Unconditional Markets implicitly postulated in previous theoretical and applied studies of speculation.

I. THE NON-INFORMATIVE SOLUTION IN ALTERNATIVE MARKET REGIMES

Let us compare, first, the solutions obtained under alternative market regimes for a _non_-informative situation. Here no speculation takes place, since traders do not anticipate the emergence of information leading to any price change before the close of trading. Each individual moves at once to his optimum consumptive position (in general, a gamble over possible states of the world) in a single trading round.

A trader's endowment, also in general a gamble, can be expressed in "prospect notation" as:

$$(1) \qquad E \equiv [(n_a^e, z_a^e), (n_b^e, z_b^e); p, 1-p]$$

That is, with subjective probability belief p the individual anticipates the particular endowment vector n_a^e, z_a^e associated with the advent of state-$\underline{a}$; he assigns, of course, the complementary probability 1-p to the endowment vector n_b^e, z_b^e associated with state-$\underline{b}$. But since commodity N is riskless, the N-endowment is invariant over states -- i.e., $n_a^e = n_b^e = n^e$. Using this feature, and suppressing the probability parameter p, it will sometimes be convenient to employ the more compact notation:

$$(1') \qquad E \equiv (n^e; z_a^e, z_b^e)$$

The individual's problem is to select among "simple consumptive gambles" C, expressed (in the two alternative notations) as:

$$(2) \qquad C \equiv [(n, z_a), (n, z_b); p, 1-p)] \equiv (n; z_a, z_b)$$

The individual maximizes his expected utility:

$$(3) \qquad U(C) \equiv U(n; z_a, z_b) \equiv pu(n, z_a) + (1-p)u(n, z_b)$$

Under the special assumption of zero complementarity, the preference-scaling function u takes the form:

$$(4) \qquad u(n, z) \equiv f(n) + g(z)$$

The difference between the two market regimes (Semi-Complete Markets SCM versus Unconditional Markets UM) is manifested to the individual in the form of the trading opportunities available. Under SCM his budget constraint is the wealth-value of the endowment combination:

$$(5) \qquad n + P_{Za}z_a + P_{Zb}z_b = n^e + P_{Za}z_a^e + P_{Zb}z_b^e \equiv W^e \qquad \text{Budget equation} \atop \text{(SCM)}$$

That is, he may buy or sell any desired numbers of units of the contingent claims Z_a or Z_b, each having its own price in terms of the numeraire commodity N. Maximizing expected utility subject to this constraint leads to:

$$(6) \qquad p \frac{\partial u/\partial z_a}{\partial u/\partial n} = P_{Za} \quad \text{and} \quad (1-p) \frac{\partial u/\partial z_b}{\partial u/\partial n} = P_{Zb} \qquad \text{Optimality} \atop \text{conditions} \atop \text{(SCM)}$$

Finally, the market-clearing conditions serve to determine the equilibrium prices P_{Za} and P_{Zb}:

$$(7) \qquad \Sigma n = \Sigma n^e, \quad \Sigma z_a = \Sigma z_a^e, \quad \Sigma z_b = \Sigma z_b^e \qquad \text{Equilibrium} \atop \text{conditions} \atop \text{(SCM)}$$

(Of course, one of these conditions is implied by the other two.)

Now, let π symbolize the _representative belief_ parameter attached to state-_a_ in this economy. Then apart from belief-deviant individuals of negligible social weight[13]/ the following variation of (6) holds:

$$(6') \qquad \pi \frac{\partial u/\partial z_a}{\partial u/\partial n} = P_{Za} \quad \text{and} \quad (1-\pi) \frac{\partial u/\partial z_b}{\partial u/\partial n} = P_{Zb}$$

Under the more restricted regime of Unconditional Markets, however, the situation is quite different. Already, under SCM, trading in conditional claims to the _riskless_ commodity N was ruled out. Under UM there is no trading in conditional claims even to the risky commodity Z; only _unconditional_ rights to either N or Z may be exchanged. A unit unconditional right to Z can be regarded as a package consisting of unit conditional claims over all possible states. The unit price of the package can evidently be interpreted

as the price of Z in this market regime, and will be denoted P_Z. In general, of course:

(8) $P_Z \neq P_{Za} + P_{Zb}$

That is, the market price of Z under the UM regime will <u>not</u> generally equal the sum of the separate prices for conditional claims to Z that would have been established under SCM. It also follows that, in general, individuals would not attain the same optimal consumptive gambles C* as in SCM. In particular, under UM individuals with inconvenient endowment compositions will generally find it impossible to move to their SCM-preferred gambles when Z_a, Z_b claims can be bought and sold only in a 1:1 ratio.

This difficulty expresses itself in the form of the budget constraint. Endowed wealth, interpreted as the <u>market value of</u> endowment, is no longer the effective bound on attainable combinations under UM. The restrictions on trading make some endowment compositions partially (or even wholly) unmarketable. The individual must therefore account separately for all the commodities:

(9)
$$n + P_Z \zeta = n^e$$
$$z_a - \zeta = z_a^e$$
$$z_b - \zeta = z_b^e$$

Budget constraints (UM)

And in consequence, the optimality conditions reduce to the single equation:

(10) $$p \frac{\partial u / \partial z_a}{\partial u / \partial n} + (1-p) \frac{\partial u / \partial z_b}{\partial u / \partial n} = P_Z$$

Optimality condition (UM)

The market-clearing conditions (either implying the other) can here be expressed as:

(11) $\Sigma n = \Sigma n^e$ and $\Sigma \zeta = 0$

Equilibrium condition (UM)

For traders of representative beliefs, (10) becomes:

(10') $$\pi \frac{\partial u / \partial z_a}{\partial u / \partial n} + (1-\pi) \frac{\partial u / \partial z_b}{\partial u / \partial n} = P_Z$$

NUMERICAL EXAMPLE 1

Assume that the economy consists of two classes of individuals, differing only in endowment composition. Specifically, suppose that there are "representative suppliers" of the risky commodity Z whose typical endowment is $E = (n^e; z_a^e, z_b^e) = (0; 400, 160)$ and an equal number of "representative demanders" of Z with typical endowment $E = (200; 0, 0)$. Suppose in addition that everyone has the identical utility function $u(n,z) = \log_e nz = \log_e n + \log_e z$, and identical (and therefore _representative_) beliefs assigning the probability parameter $p = \pi = .6$ to state-$\underline{a}$ and $1-p = 1-\pi = .4$ to state-$\underline{b}$.

Under Semi-Complete Markets SCM it can be verified that the equilibrium contingent-claim prices (in terms of N as numeraire) are $P_{Za} = .3$ and $P_{Zb} = .5$. (Here, even though state-$\underline{a}$ is more probable, the price of Z_a-claims is lower than the price of Z_b-claims. The reason is that the smaller marginal utility $\partial u / \partial z_a$, associated with the larger social aggregate of Z_a-claims, overbalances the probability factor operating in the opposite direction.) At the solution prices everyone has equal endowed wealth: $W^e = 200$. Consequently, all end up with the same optimal consumptive gamble $C^* = (n^*; z_a^*, z_b^*) = (100; 200, 80)$.

In the regime of Unconditional Markets UM the result is quite different. The market-clearing price for the package ζ (i.e., for an unconditional claim to Z) works out to $P_Z \doteq .9439$. This price is higher than the sum of the contingent-claim prices under SCM obtained above ($P_Z = .9439 > P_{Za} + P_{Zb} = .3 + .5 = .8$), as may be explained as follows. The representative suppliers of Z wish to sell off some Z-claims for the N-claims they lack. But their endowed state-contingent holdings of Z ($z_a^e = 400, z_b^e = 160$) are highly unbalanced, which is an undesirable feature from the point of view of risk-aversion. They would have preferred to sell more of Z_a than of Z_b claims, as was possible under SCM. Selling units of ζ (certainty claims to Z) under UM does provide

them with the N they desire, but only by further increasing the _relative_ disproportion of their retained Z-holdings. Since the suppliers are therefore somewhat reluctant to sell Z-claims in the 1:1 ratio dictated by UM, whereas the demanders have no such reluctance in buying (since their endowment situation is already balanced at $z_a^e = z_b^e = 0$), the consequence is a relatively high price P_Z.

At the equilibrium $P_Z \doteq .9439$ the suppliers each sell about 105.94 units of Z (whereas, under SCM they would have sold 200 units of Z_a at $P_{Za} = .3$ and 80 units of Z_b at $P_{Zb} = .5$). The demanders in either case pay just 100 units of N. The two classes of traders do not reach the same consumptive gambles under UM. The suppliers of Z attain $C* = (100;294.06,54.06)$, while for the demanders of Z the consumptive optimum is $(100;105.94,105.94)$. In utility terms the Z-suppliers end up somewhat better off, the Z-demanders somewhat worse off. It can be shown that the loss of the latter is greater than the gain to the former -- in the sense that, if the alternative of Semi-Complete Markets were available, the latter group would be happy to return to that alternative while still being able to compensate the beneficiaries of Unconditional Markets.

(END OF NUMERICAL EXAMPLE)

The Numerical Example above indicated a loss in efficiency under the regime of Unconditional Markets. This loss, due to the restriction imposed upon the consumption baskets attainable by individuals, is quite a general consequence of impaired trading opportunities. As compared with UM, a _costless_ shift to a regime of Semi-Complete Markets SCM will (with appropriate compensation) generally be Pareto-preferred. Of course, providing the additional markets will in general be costly. Before coming to absolute conclusions as to efficiency the added expense would have to be weighed against the inefficiency of impaired trading.

Let us briefly consider the impact of divergences in endowment position, in probability beliefs, and in risk-aversion upon the non-informative individual solutions obtained in the two market regimes.

(i) As to endowment position, the budget equation (5) shows that under SCM only the individual's endowed wealth W^e -- and not the detailed composition thereof -- will affect his achieved consumptive gamble. Under UM, in contrast, the significance of the specific commodity-state composition of endowments is revealed by the necessity of accounting separately as in (9) for all the goods entering into the utility function.

(ii) As to probability beliefs, the form of (6) shows that under SCM a belief-deviant individual assigning relatively high belief p_s to any state-$\underline{s}$ will accept a proportionately lower $\partial u / \partial z_s$ for that state -- implying a correspondingly larger purchase of contingent Z_s-claims to that state. Inability to trade separately in Z_s-claims means that under UM traders can no longer achieve an exact inverse proportionality between p_s and $\partial u / \partial z_s$.

(iii) Finally, as to risk-aversion, a greater degree of risk-tolerance means a smaller change in $\partial u / \partial z$ for a given quantitative difference in the amount of Z held in the different states. Then (6) implies that, under SCM, a more risk-tolerant individual would be willing to hold relatively more of the more plentiful state-claims and relatively less of the less plentiful -- in short, the perfectly reasonable result that a more risk-tolerant trader would accept a wider risk than the typical individual. This need not hold under Unconditional Markets! The UM regime does not permit a relatively risk-tolerant trader to widen (or a relatively risk-averse trader to narrow) his absolute risk in terms of Z. Whatever his endowed discrepancy between z_a^e and z_b^e may be, this discrepancy is preserved when Z-claims can only be traded on 1:1 basis. Equation (10) shows that the individual will adjust a kind of weighted average of his

marginal utilities to the market price, and the weighted average may be much the same regardless of whether the individual is relatively risk-averse or risk-tolerant. Another way of looking at this is the following. Suppose a risk-tolerant individual tried to increase his risk exposure by moving more heavily into Z, thus holding less of the riskless commodity N. By doing so he would then necessarily increase the absolute scale of his Z-holdings, and therefore reduce the _relative_ disproportion thereof. The upshot is that, under UM, risk-tolerant individuals need not in general be more heavily committed to the risky commodity!

NUMERICAL EXAMPLE 2

In the situation of Numerical Example 1, consider an individual with the same probability beliefs $p = \pi = .6$, with an endowment composition $(100;200,80)$ that is exactly a cross-section of the social totals, but with the more risk-tolerant utility function $u = n^{\frac{1}{2}} + z^{\frac{1}{2}}$. (The ratio of the logarithmic marginal utilities at the endowment position is $\frac{1/200}{1/80} = 2/5$, while the ratio of the square-root marginal utilities is $\frac{.5(200)^{-.5}}{.5(80)^{-.5}} = (2/5)^{1/2}$ -- evidently closer to unity.) Under SCM this individual will trade to a simple consumptive optimum $C^* = (79.4;317.5,50.8)$. His consumptive gamble in Z is very wide compared to his endowment position, or compared to the choices of individuals with logarithmic utility functions; note that he even draws upon his N-endowment in order to increase the scope of his Z-gamble, in accordance with equation (6).

Under UM, however, this individual's weighted-average ratio of marginal utilities -- the left-hand-side of (10) -- at the endowment position can be shown to equal .871. Since this is less than the equilibrium price $P_Z = .9439$, he will not wish to purchase more of the risky good Z. Indeed, under UM this

highly risk-tolerant individual will sell off some of his Z-claims in order to buy more of the riskless good N!

(END OF NUMERICAL EXAMPLE)

II. THE INFORMATIVE SOLUTION IN ALTERNATIVE MARKET REGIMES: EMERGENCE OF SPECULATION

In order to introduce speculative behavior, let us postulate now an _informative_ situation. Here the anticipated emergence of new information affecting prices divides trading into a prior round and a posterior round. Then some or all individuals may be induced to speculate, i.e., to adopt _trading positions_ -- portfolio holdings in the prior round that do not correspond to consumptive desires but rather to hopes for potential profit consequent upon anticipated price revisions. Of course, speculators will ultimately (in the posterior round) make trades permitting them to end up with desired consumptive gambles at the enhanced or diminished levels of wealth stemming from their degree of speculative success.

We continue to assume, for simplicity, that the emergent information will be _conclusive_ as to which state is going to obtain. Then, under Semi-Complete Markets (SCM), a trader's optimizing problem can be formulated as:

(12a) $\text{Max } U = pu(n',z_a') + (1-p)u(n'',z_b'')$ subject to

(12b)
$$n' + P_{Za}'z_a' = n^t + P_{Za}'z_a^t \equiv W'$$
$$n'' + P_{Zb}''z_b'' = n^t + P_{Zb}''z_b^t \equiv W''$$
$$n^t + P_{Za}^o z_a^t + P_{Zb}^o z_b^t = n^e + P_{Za}^o z_a^e + P_{Zb}^o z_b^e \equiv W^e$$

The trading position $T = (n^t; z_a^t, z_b^t)$ is arrived at by the individual subject to the prior-round market prices denoted P_{Za}^o and P_{Zb}^o. The effective constraint in this round, represented by the third equation of (12b), is the level of endowed

wealth W^e. As for the posterior round, one or the other of the constraints in the first two equations of (12b) will be effective. If state-$\underline{a}$ obtains the single-primed symbols of the first equation represent the effective wealth (W', the posterior market value of the trading position), the decision variables (n' and z_a'), and the posterior prices (only P_{Za}' need be considered, since the price of numeraire N remains unity and Z_b-claims have become valueless). Similarly, the double-primed symbols of the second equation represent the variables conditional upon the advent of state-$\underline{b}$, with only the price P_{Zb}'' appearing.

Taking the two rounds of trading together, in an informative situation the individual can be regarded as selecting a "compound consumptive gamble" that may be denoted in prospect form as $D = [(n',z_a'),(n'',z_b'');p,1-p]$. In the compound gamble permitted by the two rounds of trading, n' and n'' may differ -- something that could not be achieved in a single round of trading under Semi-Complete Markets.[14/]

The posterior-round optimality conditions under SCM have the simple form:

$$(13) \qquad \frac{\partial u/\partial z_a'}{\partial u/\partial n'} = P_{Za}' \quad \text{and} \quad \frac{\partial u/\partial z_b''}{\partial u/\partial n''} = P_{Zb}'' \qquad \text{Posterior-round optimality conditions (SCM)}$$

Since there is posterior certainty, the probability parameter no longer plays any role.

For the prior-round optimality conditions, on the other hand, we must face the awkward problem that the choice of optimal trading position T* depends not only upon the prior-round prices P_{Za}^o and P_{Zb}^o but also upon the posterior prices P_{Za}' and P_{Zb}''. But these latter have not yet been determined; indeed, one of them will never actually be realized, since only one state is going to be observed! What information will traders have as to posterior prices to guide them in their prior-round decisions? In general, conditional posterior prices

are not actually <u>computable</u> from public prior data.[15/] But there is one set

of anticipations which, if universally held, will be self-fulfilling and thus

consistent with equilibrium. Specifically, suppose that every trader -- whether

belief-deviant or not -- anticipates that <u>prices will move in proportion to</u>

<u>changes in representative beliefs</u>:

$$(14) \qquad \frac{P'_{Za}}{P^O_{Za}} = \frac{1}{\pi} \quad \text{and} \quad \frac{P''_{Zb}}{P^O_{Zb}} = \frac{1}{1-\pi}$$

(The numerator of unity on the right-hand-side of each equation corresponds to

posterior certainty, i.e., the information is to be conclusive.) This set of

anticipations concerning price revisions corresponds to the martingale property,

calculated in terms of representative beliefs. (The same anticipations would

not be a martingale calculated in terms of any other set of beliefs.)

If (14) holds true the third constraint of (12b) can be reformulated as a

relation among the endowed and conditional wealths:

$$(15) \qquad \pi W' + (1-\pi)W'' = W^e$$

This leads to a simple condition for optimal <u>prior</u> trading:

$$(16) \qquad \frac{p}{1-p} \frac{du'/dW'}{du''/dW''} = \frac{\pi}{1-\pi} \qquad \begin{array}{l} \text{Prior-round} \\ \text{optimality} \\ \text{condition (SCM)} \end{array}$$

Thus, optimal prior trading involves determining the conditional posterior

wealths W', W''. With W' and W'' the trader can enter the first two equations of

(12b) to find the elements n^t, z^t_a, z^t_b of his optimal trading position T^*. But,

as a point that will take on considerable significance below, note that thw <u>two</u>

wealths are insufficient to uniquely determine the <u>three</u> elements of T^*. In

prior-round trading under SCM there is a degree of freedom; any one element of

T^* can be arbitrarily selected.

It remains to be shown just what the equilibrium prior-round prices P^O_{Za}

and P^O_{Zb} will be. Under the assumption of "representative beliefs" the result

is very neat:

(17) $P^O_{Za} = P_{Za}$ and $P^O_{Zb} = P_{Zb}$

That is, the prior-round prices in an informative situation are simply equal to
the state-claim prices that would have ruled had the situation been non-
informative! We can see immediately that substituting from (14) into (13) yields
a pair of equations exactly like (6') -- if $z_a = z'_a$, $z_b = z''_b$, and $n' = n'' = n$.
The interpretation is as follows: The prior and posterior markets will be in
equilibrium if prices are such that individuals of representative beliefs (who
account collectively for essentially all the social weight determining prices)
find it optimal to achieve the same optimal consumptive baskets D* under an
informative situation as the C* baskets they would have chosen had the situation
been non-informative.

It follows that an individual of representative beliefs will not speculate
in the SCM regime. He can employ the ruling prior-round prices P^O_{Za} and P^O_{Zb}
in an informative situation to move directly to his consumptive optimum position --
i.e., he can choose a trading position T* identical with his desired consumpt-
ive gamble D* = C*. That he is not speculating is evident from the fact that
this choice of T* frees him of any need to revise his portfolio in the posterior
round of trading[16/] after revelation of the information as to which state will
obtain.[17/] The next implication, of course, is that individuals of deviant
beliefs will speculate.

NUMERICAL EXAMPLE 3

Returning to the conditions of Numerical Example 1, in an informative
situation under SCM the prior-round prices are still $P^O_{Za} = P_{Za} = .3$ and
$P^O_{Zb} = P_{Zb} = .5$. Then both the representative suppliers of Z and the representat-
ive demanders of Z have wealth $W^e = 200$. Thus they can all move immediately to

consumptive optimum positions $D^* = C^* = (100;200,80)$. Furthermore, if they have representative beliefs $(p = \pi = .6)$ they can do no better. The conditional posterior prices are (from (14)) equal to $P'_{Za} = P^o_{Za}/\pi = .5$ and $P''_{Zb} = P^o_{Zb}/(1-\pi) = 1.25$. If the former is applicable (state-$\underline{a}$ obtains) the consumptive combination $(n',z'_a)=(100,200)$ meets condition (13); similarly for the combination $(n'',z''_b)=(100,80)$ if state-$\underline{b}$ obtains. This verifies that the prior-round choice of $T^* = D^* = (100;200,80)$ is optimal, precluding any need for trade in the posterior round.

Consider now a belief-deviant individual, for whom the subjective probability $p = .7$ attached to state-$\underline{a}$ diverges from the representative belief parameter $\pi = .6$. Suppose this individual possesses an endowment $E = (100;200,80)$ that is a cross-section of the social totals (and is identical to the preferred consumptive positions of those holding representative beliefs). Had the situation been $\underline{\text{non}}$-informative it can be shown that, at the ruling prices $P_{Za} = .3$ and $P_{Zb} = .5$, condition (6) would have led this individual to the simple consumptive gamble $C^* = (100;233\frac{1}{3},60)$. Placing higher-than-average belief in the advent of state-$\underline{a}$, he is willing to accept a wider gamble that this state will in fact obtain.

What about the deviant individual in an informative situation? With $u = \log_e nz$, it follows that $du/dW = 2/W$. Then condition (16) governing his prior-round choices can be expressed as $\frac{.7}{.3}\frac{W''}{W'} = \frac{.6}{.4}$. From (16) and (15), and since $W^e = 200$, his optimal conditional posterior wealths are $W' = 233\frac{1}{3}$ and $W'' = 150$. Using the degree of freedom to fix $n^t = 100$, his optimal trading position becomes $T^* = (n^t;z^t_a,z^t_b) = (100;266\frac{2}{3},40)$. Note that the deviant individual has used the prior-round trading opportunity to widen his gamble even beyond what his chosen simple consumptive gamble would have been. The reason, of course, is that he here is motivated by <u>anticipations of profit from price revision</u> over and above his immediate consumptive goals.

From (13), it follows that this individual's posterior optimum if state-$\underline{a}$ holds is $C' \equiv (n', z_a') = (116\frac{2}{3}, 233\frac{1}{3})$, and if state-$\underline{b}$ holds is $C'' \equiv (n'', z_b'') = (75, 60)$. In prospect notation his optimal compound consumptive gamble is $D^* \equiv [C', C''; p, 1-p] = [(116\frac{2}{3}, 233\frac{1}{3}), (75, 60); .7, .3]$.

(END OF NUMERICAL EXAMPLE)

So much for the regime of Semi-Complete Markets. How different are results for the regime of Unconditional Markets, where only certainty claims may be traded?

Maintaining the same assumptions as before except for the change in market regime, the decision problem of an individual with belief parameter p can be expressed by equations analogous to (12a) and (12b):

(18a) $\quad$ Max $U = pu(n', z_a') + (1-p)u(n'', z_b'')$ $\qquad$ subject to

(18b)
$$\begin{cases} n' + P_{Za}' z_a' = n^t + P_{Za}' z_a^t \equiv W' \\ n'' + P_{Zb}'' z_b'' = n^t + P_{Zb}'' z_b^t \equiv W'' \end{cases}$$

We do not yet have the expression for the prior-round constraint, the analog of the third equation of (12b). It is more convenient to find an analog to the relation (15) among the wealths. In fact, using (14), (18b), and the budget accounting identities (9), we obtain:[18/]

(18c) $\quad \pi W' + (1-\pi)W'' = W^e + (P_{Za}^o + P_{Zb}^o - P_Z^o)\zeta$

The symbol W^e here does not represent the _actual_ market value of the endowment under Unconditional Markets UM. Rather, it is the _hypothetical_ value that the same endowment would have under Semi-Complete Markets SCM. Similarly, P_{Za}^o and P_{Zb}^o are the corresponding hypothetical prior-round state-claim prices under SCM. The point of this development is that _if_ the prior-round price of the unconditional Z-claim, P_Z^o, satisfies $P_Z^o = P_{Za}^o + P_{Zb}^o$, then conditions (18)

become literally identical with the conditions (12) governing choice of trading position T* under SCM.

The upshot is that markets <u>must</u> clear if the unconditional prior-round price of the risky commodity Z is simply:

$$(19) \qquad P_Z^O = P_{Za}^O + P_{Zb}^O$$

This price must be consistent with equilibrium, as it leads back to the very same equilibrium achieved under Semi-Complete Markets![19/] If (19) holds, each individual will want and be able in the initial round to attain the same posterior conditional wealth-pair W', W" that he could achieve in the prior round under SCM. With the desired balance between W' and W" achieved in selecting a trading position T*, he will of course later engage in posterior-round transactions so as to reach the same compound consumptive gamble D* that was optimal under SCM.

We may also note that martingale-type anticipations are involved in this equilibrium. For, it follows immediately from (14), (17), and (19) that:

$$(20) \qquad P_Z^O = \pi P_{Za}' + (1-\pi)P_{Zb}''$$

That is, if all traders believe that the prior-round price of the unconditional claim to Z is the mathematical expectation (calculated in terms of representative probabilities) of the posterior-round price, this belief will be self-fulfilling in market equilibrium. (Note that in the posterior round when it is known which state obtains, the contingency that has been realized becomes a certainty. Hence, in the posterior round, the symbols P_{Za}' and P_{Zb}'' represent prices of certainty claims that could equally well be written P_Z' and P_Z''.)

NUMERICAL EXAMPLE 4

Consider once again the "representative suppliers" of Z with endowment E = (0;400,160) and the "representative demanders" with endowment E = (200;0,0) -- both having the representative belief parameter π. In a <u>non-informative</u>

situation under Unconditional Markets UM, Numerical Example 1 showed that the equilibrium price is $P_Z \doteq .9439$. The traders could not all attain the ideal SCM solution $C^* = (100;200,80)$ in a non-informative situation under UM. The suppliers actually did somewhat better, but the demanders were even more worse off -- so that the overall outcome was inefficient as compared with the SCM regime.

In an _informative_ situation under UM, the equilibrium conditional posterior-round prices remain $P'_{Za} = .5$ and $P''_{Zb} = 1.25$ (as in Numerical Example 3). The prior-round price implied by the martingale anticipations is then $P^o_Z = P^o_{Za} + P^o_{Zb}$ $= .3 + .5 = .8$. What prior-round trading takes place in this case? The curious result is, none at all! Optimality condition (16) here takes the simple form $\frac{du'}{dW'} = \frac{du''}{dW''}$, so that $W' = W'' = W^e = 200$. Then, if each trader chooses $T^* = E$, posterior trading will guarantee that his overall compound consumptive gamble is the ideal $D^* = (100;200,80)$.

This very special result is dependent upon the particular numerical assumptions made about endowments, however. In general, even individuals of representative beliefs _would_ have to make some prior-round transactions under UM to achieve their desired D^* gambles. In any case, they would be planning for portfolio revisions in the _posterior_ round.
(END OF NUMERICAL EXAMPLE)

To sum up, the development in this Section indicates that the opportunity afforded by the anticipated emergence of conclusive information, to engage in both prior-round and posterior-round trading, remedies the deficiency of the regime of Unconditional Markets that was observed for a _non-informative_ situation. Specifically, in our model above _every trader -- whether belief-deviant or not -- was able to achieve in two trading rounds under UM the exact same consumptive gamble that was optimal for him under the more ample regime of Semi-Complete_

Markets SCM. The next Section is intended to evaluate the generality of this conclusion.

III. ADEQUACY OF MARKETS AS RELATED TO NUMBERS OF STATES AND GOODS

The key result derived for the model of the previous Section was that the two rounds of trading provided by an informative situation allow every transactor, even under the incomplete Unconditional Markets regime, to achieve the same optimal consumptive gamble as was attainable under Semi-Complete Markets. If this is true, the implications for speculative behavior and equilibrium derived for the SCM regime also follow under UM: (A) Only belief-deviant individuals speculate; individuals of representative beliefs do not engage in speculative "risk transfers," regardless of possible differences in risk-tolerances. (B) The mutual equilibriation of prior-round and posterior-round prices requires that price movements have the martingale property (calculated in terms of representative probability beliefs). The two key simplifying assumptions leading to these results -- zero complementarity in preference, and the existence of representative beliefs dominating prices -- have already been discussed, and will be maintained here. These assumptions are, at least arguably, reasonable and useful idealizing approximations. But the preceding analysis also specified a world of just two alternative states ($\underline{a}$ and $\underline{b}$) and just two commodities ($\underline{N}$ and $\underline{Z}$). This is too artificial to be acceptable without further consideration.

More generally, let S denote the number of states and G the number of goods. Then, in a non-informative situation -- or in prior trading in an informative situation -- under Semi-Complete Markets there will be dealing in $S(G-1)+1$ distinct claims or "contracts." (Contingent claims to G-1 different risky commodities will be traded, but only the single unconditional claim to the riskless numeraire commodity N.) In the posterior round, after the emergence

of conclusive information, there is no more uncertainty. At that point trading takes place once again, but only in G contracts -- one for each good. Under Unconditional Markets, in contrast, there will be trading in only the G unconditional contracts in the prior round, followed by a second posterior round involving the same G contracts.

The key feature of the development in Section II is that, <u>for each individual in an informative situation to achieve his optimal gamble, all that he need determine in the prior trading round are his S conditional posterior wealths -- one for each possible state of the world</u>. If $G = S = 2$, as specified in Section II, trading under Semi-Complete Markets SCM in $S(G-1) + 1 = 3$ contracts in the prior round provided an extra degree of freedom in choice of T^*. Since the restriction under Unconditional Markets UM to certainty trading in the risky commodity Z (the fact that Z_a and Z_b claims must be traded only in a 1:1 ratio) used up just this one degree of freedom, the UM regime could achieve the same result as the SCM regime.

More generally, under SCM the excess number of contracts (the degrees of freedom d.f. in choice of T^*) will be:

$$(21) \qquad \text{d.f.} = S(G-1) + 1 - S = S(G-2) + 1$$

Under SCM there will <u>always</u> be such an excess for $G \geq 2$.[20,21] Under UM, the excess number of contracts is simply:

$$(22) \qquad \text{d.f.} = G - S$$

Since the number of potentially distinguishable states of the world S is infinite, it might be thought that under the more realistic UM regime there would generally be a deficiency of markets. To this there are several possible replies: (1) Given our limited mental capacities, the number of <u>actually distinguished</u> states entering into people's subjective calculations is likely to be a rather small number. (2) While Unconditional Markets may be a closer

approximation of real world actuality than Semi-Complete Markets, there is in fact _some_ trading in conditional claims (e.g., in insurance markets). Such markets are likely to be provided just where it is most important to do so in terms of meeting perceived gaps in market adequacy. (3) Most important of all is the consideration that information will not ordinarily be arriving in one single injection. If _successive_ informational inputs are anticipated, all but the last being less than conclusive, repeated rounds of trading become available for rebalancing of portfolios.[22/] Multiple trading rounds thus compensate for inadequacy of markets in any given round. Since the arrival of information is often an essentially continuous process over time, the number of degrees of freedom available tends to rise without limit, even if there are only Unconditional Markets at any point in time. (An explicit analysis of the complex problem of sequential information inputs will not be provided here, however.)

The conclusion, therefore, is that the applicability of the key theoretical results obtained -- the invalidity of the risk-transfer theory, and the martingale proposition -- does not depend in any essential way upon the existence of the regime of Semi-Complete Markets, and certainly not upon the illustrative assumptions of $S = 2$ states and $G = 2$ goods.

IV. BEHAVIORAL IMPLICATIONS -- SPECULATORS VS. HEDGERS

On the other hand, there is one significant difference in implications for trader behavior as between Semi-Complete Markets and Unconditional Markets. Under SCM anyone with representative beliefs can, in the prior round of trading, choose a trading position T^* identical with his optimal consumptive gamble $D^* = C^*$. That is, he could (and, given any transaction costs at all, he _would_) move directly to his consumptive optimum as soon as markets open, foregoing any opportunity for posterior trading at the changed prices to rule after emergence of the anticipated new information. On the other hand, anyone with deviant

beliefs would plan to deal in both the prior and the posterior markets; his prior-round choice of T* would necessarily diverge from his ultimate consumptive choices. This distinction provided a simple identification of speculative behavior under the SCM regime: a speculator is one whose prior-round trading position is linked to <u>prospective portfolio revision</u> in the posterior round, whereas a nonspeculator is one who does not plan for portfolio revision.

This implication is no longer valid under the more constrained UM market regime. Here the limitations upon marketing are such that posterior trading is in general necessary for <u>everyone</u>, whether of representative beliefs or of deviant beliefs. Since under UM everyone will be revising portfolios, the working definition of speculation employed above for the SCM regime (planning for posterior portfolio revision) is not ultimately satisfactory. A more general definition, applicable to both SCM and UM market regimes, is as follows: A speculator is a trader who, in an informative situation, plans to deal in the prior and posterior rounds in such a way as to achieve a compound consumptive gamble D* that differs from the simple consumptive gamble C* he would have chosen in a non-informative situation. Or, putting it less technically, a speculator is one who <u>plans to profit from emergent information</u>; for a non-speculator, on the other hand, planned contingent consumption is identical over information-events (but not, in general, over states of the world).$\underline{23/}$

What of the conventional view in the speculation literature that distinguishes between (a) risk-averse individuals who use the prior round of trading to <u>reduce their exposure to price risk</u> ("hedgers") versus (b) risk-tolerant individuals who trade so as to <u>increase their exposure to price risk</u> ("speculators")? Our analysis shows that this distinction is invalid. While relatively risk-tolerant individuals do (other things equal) seek wider gambles, if they hold representative beliefs their ultimate D* gambles will not differ

from the C* gambles they would have chosen in a non-informative situation.
They will therefore <u>not</u> be planning to profit from incoming information. Per-
haps even more convincing a refutation, to increase one's exposure to <u>price</u>
<u>risk</u> -- in a world where <u>quantity risk</u> also exists (and is, indeed, as we have
seen, an ultimate determinant of price risk) -- is not necessarily to widen
consumptive risk. Putting this the other way, a highly risk-averse trader will
not, in general, accept an opportunity to "transfer" <u>price risks</u> even at fair
odds! (Whereas a risk-averse individual would, by definition, always be willing
to convert a <u>quantity risk</u> into a certainty at fair odds.) This is shown most
clearly by a numerical illustration.

NUMERICAL EXAMPLE 5

Numerical Example 4 illustrated an informative situation under a regime of
Unconditional Markets UM. "Representative suppliers" of Z with endowment
$E \equiv (n^e; z_a^e, z_b^e) = (200; 0,0)$ face "representative demanders" with endowment
$E = (0; 400, 160)$. All have identical beliefs $(p = \pi = .6)$ and identical risk-
averse utility functions $(u = \log_e nz)$. In equilibrium the <u>posterior</u> prices
were $P'_{Za} = .5$ and $P''_{Zb} = 1.25$, and the <u>prior</u> price $P_Z^o = .8$. Calculating in terms
of the representative belief parameter $\pi = .6$, the prior price is the mathemati-
cal expectation of the posterior price (the martingale property holds). In
this particular situation, at the equilibrium prior price there would then be
no actual trading in the prior round. Each individual will do as well as possible
by engaging in posterior trading only.

But this behavior would be regarded, in the conventional view, as failure
to use the prior market to divest price risk! The martingale property shows
that such divestment could be attained at fair odds -- the prior-round price
is the mathematical expectation of the unknown posterior price. This means that
the representative demanders and representative suppliers here could, in the

conventional view, mutually "hedge" without having to pay any premium at all to speculators. Why do they then not do so? Because the risk-aversion argument is plausibly yet incorrectly applied to the divesting of <u>price risk</u>. It is <u>quantity risk</u>, not price risk, that enters into the utility function; the divesting of price risk may actually increase the riskiness (and therefore reduce the desirability) of the overall consumptive gamble attainable. And, in particular, in this case any trading by an individual in the prior round -- whether conventionally regarded as "hedging" or "speculating" -- will preclude his ever attaining a consumptive gamble as desirable as the $D^* = (100;200,80)$ achievable by <u>not</u> trading in the prior round.

The prior-round optimality condition (16), for traders with representative beliefs, here reduces to the simple form $du'/dW' = du''/dW''$ -- implying, with (15), that $W' = W'' = 200$. But it is easy to see that, in this market regime, each individual can assure himself a conditional posterior wealth of 200 only by remaining at his endowment position.

Suppose that one of the representative demanders was somehow induced to make a "long-hedging" purchase of 1 unit of ζ in the initial round. His implied posterior conditional wealths become $W' = 199.7$ and $W'' = 200.45$. Then posterior trading would lead him to the overall gamble, in prospect notation, $D = [(99.85,199.7),(100.225,80.18);.6,.4]$. It can be verified that this is inferior to $D^* = (100;200,80)$ -- which is, in prospect notation $[(100,200),(100,80);.6,.4]$. If one of the representative suppliers were similarly to "short-hedge" by sale of 1 unit of ζ in the prior round, his implied posterior conditional wealths would become $W' = 200.3$ and $W'' = 199.55$. Again, it can be verified that this must lead to an inferior outcome as compared with the "no-hedging" behavior that leads to conditional wealth $W' = W'' = 200$.

(END OF NUMERICAL EXAMPLE)

Does the hedger/speculator distinction then retain any meaning or applicability whatsoever? There is one natural reinterpretation that follows from the general definition of speculation arrived at here. In the spirit of this underlying concept, we can define hedgers as that sub-class of speculators who trade with a view toward attaining D* gambles that are less risky than their C* gambles under SCM. The "speculators," where the word is used in contrast with "hedgers," are then those seeking more risky D* gambles.

As previously emphasized, the crucial role in speculative behavior is played by belief-deviance -- by the degree of optimism p-π (since the probabilities here refer to the better-endowed state-a). We would anticipate that an optimistic individual, one for whom p exceeds the representative belief parameter π, would tend to adopt wider gambles -- stake more of his resources upon state-a obtaining. That this is true, up to a point, is illustrated by the final numerical example below.

NUMERICAL EXAMPLE 6

Table 1 shows the bearing of the belief parameter p upon the consumptive risks accepted by individuals. An informative situation under a regime of Unconditional Markets is assumed (where, following the above, all traders achieve the same solutions as could be attained under Semi-Complete Markets). The numerical data are the same as in the preceding Examples, and specifically: (1) The representative belief parameter is π = .6; (2) The prior-round equilibrium price is P_Z^O = .8, and the posterior-round conditional prices are P'_{Za} = .5 and P''_{Zb} = 1.25; (3) All traders have utility function u = $\log_e$ nz; (4) All traders have endowment of W^e = 200 (calculated in terms of the prices that would have ruled under Semi-Complete Markets, not in terms of the actual prices under UM). But five different endowment compositions are shown.

The main results can be interpreted as follows. Looking at the Simple Consumptive Gambles achievable under SCM (recall that these will not in general be achievable in a single trading round under UM), we see that the "optimist" with $p = .7$ widens his gamble in the direction of the more prosperous state-$\underline{a}$ -- in comparison with the individual of representative beliefs for whom $p = \pi = .6$. The "moderate pessimist" ($p = .5$) narrows his gamble, as one would expect. But note that the "extreme pessimist" ($p = .2$) is so confident about "bad news" (that state-$\underline{b}$ will obtain) as to widen his gamble in the direction of the latter state.

Consider now the Compound Consumptive Gambles achievable in two rounds of trading. Note, first, that for any given value of the belief parameter p the differently-endowed traders all end up with the same D^*. But to achieve this identity of result the prior-round trading ζ (and also the trading positions T^* not shown in the Table) must in general differ.

Note that the initial-round purchases ζ do not vary, for any given p, as among the first three endowments tabulated (types 1,2, and 3). The reason is that these individuals are already holding representative endowed proportions of the risky state-claims Z_a and Z_b. The other two cases, individuals with asymmetrical compositions of Z-claims over states (types 4 and 5) must arrange initial-round exchanges that balance Z_a and Z_b in the course of achieving the desired contingent wealth-pair W',W''. Subject to this proviso, the optimists ($p = .7$) tend to be sellers of Z in the prior round. That is, they are optimistic about the social quantity of Z to be available for consumption, and so they expect on average a low price of Z in the posterior round.[24/] Individuals of representative beliefs ($p = \pi = .6$) all end up with the optimal gamble $D^* = C^*$ (100;200,80); here only endowment types 4 and 5 actually trade in the prior round (for the purpose of balancing their asymmetrical state-endowments).

The moderate pessimists (p = .5), and even more so the extreme pessimists (p = .2), tend to be buyers of Z in the initial round; they attach greater belief to a Z-scarcity and hence to a higher posterior price.

(END OF NUMERICAL EXAMPLE)

TABLE 1: Risks Accepted, by Degree of Optimism/Pessimism

BELIEF PARAMETER $p =$	.7 (Optimist)	.6 (Representative)	.5 (Moderate Pessimist)	.2 (Extreme Pessimist)
SIMPLE CONSUMPTIVE GAMBLE (SCM)				
$C^* = (n^*; z_a^*, z_b^*) =$	$(100; 233\frac{1}{3}, 60)$	$(100; 200, 80)$	$(100; 166\frac{2}{3}, 100)$	$(100; 66\frac{2}{3}, 160)$
DESIRED POSTERIOR WEALTHS				
$W', W'' \quad =$	$233\frac{1}{3}, 150$	$200, 200$	$166\frac{2}{3}, 250$	$66\frac{2}{3}, 400$
PRIOR-ROUND TRADING (UM)				
Endowments				
$E = (n^e; z_a^e, z_b^e)$				
1. $(100; 200, 80)$. $\zeta =$	$-111\frac{1}{9}$	0	$111\frac{1}{9}$	$444\frac{4}{9}$
2. $(200; 0, 0)$. $\zeta =$	$-111\frac{1}{9}$	0	$111\frac{1}{9}$	$444\frac{4}{9}$
3. $(0; 400, 160)$. $\zeta =$	$-111\frac{1}{9}$	0	$111\frac{1}{9}$	$444\frac{4}{9}$
4. $(80; 400, 0)$. $\zeta =$	$155\frac{5}{9}$	$266\frac{6}{9}$	$377\frac{7}{9}$	$711\frac{1}{9}$
5. $(120; 0, 160)$. $\zeta =$	$-377\frac{7}{9}$	$-266\frac{6}{9}$	$-155\frac{5}{9}$	$177\frac{7}{9}$
ELEMENTS OF COMPOUND CONSUMPTIVE GAMBLES				
$D^* = [C', C''; p, 1-p]$				
$C' = (n', z_a')$ $=$	$(116\frac{2}{3}, 233\frac{1}{3})$	$(100, 200)$	$(83\frac{1}{3}, 166\frac{2}{3})$	$(33\frac{1}{3}, 66\frac{2}{3})$
$C'' = (n'', z_b'')$ $=$	$(75, 60)$	$(100, 80)$	$(125, 100)$	$(200, 160)$

V. SUMMARY

Hedging and speculation are conventionally defined as the avoidance in the one case, and the acceptance in the other case, of price risk. We have found this definition unacceptable, in view of the fact that it is the <u>interaction between price risk and quantity risk</u> that governs the overall hazard accepted or avoided by individuals. Failure to appreciate the significance of the quantitative uncertainty that underlies and is the main cause of stochastic variation of prices is the key failing of the traditional speculation literature. According to our modified definition, speculation and hedging consist of <u>trading in the prior and posterior markets</u> (in an informative situation, since only with anticipated emergence of new information can differences between posterior and prior prices be anticipated) <u>in such a way as to achieve compound consumptive gambles D* that differ from the simple consumptive gambles C* that would have been adopted in a non-informative situation</u>. The hedgers are those for whom the net effect of the prior trading activity is to make D* less risky than C* -- with the reverse holding for speculators. Alternatively, it is sometimes terminologically convenient to think of speculation as a wider category of activity (including all prior-market trading leading to divergences between C* and D*), with hedging as a special risk-reducing subclass of speculative behavior.

Among the factors studied here as possibly involved in the speculative decision are: (1) The individual's <u>beliefs</u> about the emergence and content of new information -- which must logically bear a definite relation to his own prior estimates of the likelihood of alternative states of the world. (2) His <u>utility function</u>, involving both preferences as among different commodities and his degree of risk-tolerance (willingness to hold prospects yielding differential outcomes over the various states of the world). (3) The scale and composition of his <u>endowment</u>, as distributed over commodities and states of the world.

(4) The extent of the _markets_ available, and in particular whether or not conditional state-claims to commodities can be bought and sold separately.

The key results can be summarized in three propositions:

(A) Speculative trading is undertaken _only_ by individuals whose opinions, as to the likelihood of future states of the world, diverge from representative beliefs in the market.

(B) Mutual equilibriation of prior-round and posterior-round prices requires that the price-revision relation be a martingale, calculated in terms of representative beliefs.

(C) And, these results are applicable not only for "unrealistic" market regimes permitting trading in contingent claims to risky commodities (Semi-Complete Markets) but also, subject to certain limitations, to regimes in which only certainty claims can be traded (Unconditional Markets) as assumed in the traditional speculation literature.

As is always the case for theoretical models, of course, these propositions depend upon a number of idealizing assumptions and hence could not be expected to be exactly applicable to actual behavior. But the main thrust of the results is to support the observations of Holbrook Working that speculative/hedging behavior is governed primarily by _differences of belief_, rather than by _difference of risk-tolerance_ as postulated by the Keynes-Hicks risk-transfer theory.

FOOTNOTES

1. Keynes (1930), v. 2, Ch. 29; Hicks (1946), pp. 137-39. Also Houthakker (1957, 1968); Cootner (1968).

2. Working (1953, 1962); see also Rockwell (1967).

3. Fama (1970), p. 384; Samuelson (1965).

4. If hedgers are mostly long the physical good (if they are suppliers or warehousers of the commodity) they must be predominantly short in the futures market. Then the speculators must be net long in futures. This premise had led to the inference of "normal backwardation" -- that prices of contracts tend to rise as delivery date approaches, thus rewarding the speculators for making early purchase commitments and thereby bearing the price risk. The evidence does not conclusively support normal backwardation, though the issue remains in debate (compare Houthakker [1968], Rockwell [1967], Telser [1967]). Two main explanations for the supposed failure of normal backwardation have been proposed that are consistent with an underlying risk-transfer theory: (1) If hedgers were predominantly demanders rather than suppliers of the commodity, normal compensation for speculators would dictate a falling rather than a rising price trend over the life of the contract (Cootner [1968], p. 119); or, (2) If speculators are not risk-averse on balance, no compensation is required at all (Friedman [1960]). An alternative explanation, of course, would be that the risk-transfer theory is simply incorrect.

5. Hirshleifer (1975). An earlier version was Hirshleifer (1972).

6. A more fundamental definition of speculation will be provided in Section IV below.

7. There are other exogenous influences whose probabilistic variation may induce a corresponding stochastic distribution of prices -- among them are possible changes in tastes, technology, social institutions, etc. These factors will not be considered here.

8. It has been shown that Propositions A and B do not hold exactly, absent this assumption. But since it can plausibly be maintained that complementarity effects are second-order in importance, as compared with the direct marginal utilities, the weaker Propositions A' and B' remain defensible. See Salant (1976) and Hirshleifer (1976).

9. Rubinstein (1973).

10. See Feiger (1976).

11. For a comparable result in a one-commodity model, see Hirshleifer and Rubinstein (1975).

12. This assumption was explored earlier in Hirshleifer (1972), Section IV.

13. Recall that these belief-deviants are not necessarily numerically unimportant or uninfluential -- only that they cancel each other out so far as price determination is concerned.

14. Recall, however, that under Fully Complete Markets (FCM) an individual can achieve a difference between n_a and n_b consumptions, which is why only one round of trading suffices under that more ample regime of markets.

15. See Radner (1968).

16. Recall, however, that defining speculative behavior as planning for posterior portfolio revision is not ultimately satisfactory. A more fundamental definition is provided in Section IV.

17. Because of the degree of freedom in choice of elements of T^*, the individual might move to a trading position having the same associated posterior wealths W' and W'' as $T^* = D^*$ yet requiring posterior trading to actually achieve D^*. But there is no advantage in not moving to D^* directly in the prior round; any transaction costs, however minute, would make this direct movement strictly preferred.

18. Multiplying the first equation of (18b) by π and the second by $1-\pi$ and summing leads to:
$$\pi W' + (1-\pi)W'' = \pi(n^t + P'_{Za}z_a^t) + (1-\pi)(n^t + P''_{Zb}z_b^t).$$
$$= n^t + P^o_{Za}z_a + P^o_{Zb}z_b^t$$
$$= (n^e - P^o_Z\zeta) + P^o_{Za}(z_a^e + \zeta) + P^o_{Zb}(z_b^e + \zeta)$$
Equation (18c) then follows.

19. As before, we will not attempt to show uniqueness.

20. This result assumes linear independence among the G goods with regard to posterior price distributions. If, for example, two goods had exactly proportional posterior prices over all possible states of the world, more contracts would be needed in the prior round. This qualification will henceforth be ignored.

21. If $G = 1$ the number of contracts exactly suffices if $S = 1$, and is deficient for larger S. This is not of practical significance, of course, but it suggests that single-commodity models of speculation (for which $G = 1$) should be viewed with reserve.

22. This point was made in a private communication by Jacques H. Drèze.

23. Compare Hirshleifer and Rubinstein (1975), p. 799.

24. A low price of Z is, of course, "bad news" for suppliers, other things equal. But other things are not equal, since the price is low precisely when the quantity available is great. Socially speaking, of course, a large quantity available is "good news."

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