Price Indices for Escalating Deferred Payments

by

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Imperfectly anticipated inflation can result in unanticipated redistributions of real wealth if deferred payments are fixed in units of money. Linking deferred payments to some indicator of the purchasing power of money, or price index, is often recommended as a means of reducing the real risks associated with long term contracts. The voluntary use of a "tabular standard" of deferred payment was suggested as early as 1807 by John Wheatley, intensively investigated by the British Association for the Advancement of Science between 1887 and 1889, and supported by many distinguished economists including W. S. Jevons, A. Marshall and F. Y. Edgeworth. Irving Fisher, in his Purchasing Power of Money, stated that "Perhaps the most important purpose of index numbers is to serve as a basis for loan contracts (1922, p. 208)." But, even if borrowers and lenders agree in principle with such arrangements, the problems remain of choosing a particular price index to which payment should be linked and an extent to which payment should be escalated when that index deviates from its anticipated value.

Periods of rapid general inflation most dramatically reveal the risk associated with using money as standard of deferred payment; but its use in non-inflationary periods is not entirely risk-free. The general price level -- some broad average of money prices -- might be quite stable while the prices of individual goods fluctuate in response to changing conditions of supply and demand. The relative value of any particular individual's source of future wealth and anticipated consumption goods can be quite uncertain. The ultimate consumption uncertainty he faces might be reduced if his deferred receipts and payments were denominated in units of appropriate non-money goods. The problems of choosing an index for escalating money payments in inflationary and of explaining the general acceptance of money as standard of payment in non-inflationary times thus appear intimately connected.

The objective of this essay is to explore the choice of index for deferred payments in a tractable pure exchange situation. Within this context the following observations will be made: First, a price index universally appropriate for escalating payments between all pairs of individuals can exist despite significant differences in their consumption preferences. Second, the Paasche price index, recommended by Fisher in his early work, is the appropriate escalating index when consumption preferences are homothetic (Engel curves are straight lines through the origin). More generally, a type of 'marginal price index' which weights the price changes of luxury goods more heavily, and of necessities less heavily, should be used. Finally, full escalation of deferred money payments by the proportionate change in the price index is not desirable if positive money balances are held in the economy.

Such results are in sharp contrast with the substantial traditional literature on economic price indices (Samuelson and Swamy (1974) provide a recent survey). In the context of that literature, the Paasche index is 'biased', little can be said when preferences are not homothetic, and it is presumed that the 'real value' of deferred payments should be maintained by full escalation. The reason for these differences is not hard to find. A traditional economic price index gives the ratio of minimum money expenditures required to achieve some fixed level of welfare in two price situations; we shall look for a price index which, when used to escalate deferred payments, efficiently allocates the risks of future price change among individuals. An index constructed as an aid in determining (or preventing) changes in real welfare is generally not appropriate for purposes of allocating risk, and vice versa. The criteria for judging a price index cannot be divorced from its intended use.

I. Specification of the Problem

Early arguments for linking deferred payments to some measure of the purchasing power of money were based on a concept of equity in economic exchange. If a contract was initially just, leaving each party in relatively the same position, then a later alteration of the terms of exchange due to unforeseen price changes brings inequity and injustice into the relationship. Irving Fisher shifted the emphasis to a reduction of risk that might be achieved:

> In the first place, it should be pointed out that though there is a gain and loss there is not necessarily any "injustice" wrought because of a change in the level of prices.... Each party knew or should have known that the price level might change, and took the risk.
>
> It is, however, sound public policy to lessen in advance the risk element ... so that future contracts may be made by all parties on the most certain basis possible. In the problem of time contracts between borrowers and lenders, the ideal is that neither debtor nor creditor should be worse off from having been deceived by unforeseen changes.
>
> (Fisher, 1922, pp. 209-210)

It was recognized, of course, and previously pointed out by Harry Brown (1909), that this "ideal" was generally not attainable. No specification of a deferred payment can eliminate the risks of relative price changes which are exogenous to a borrower and lender.

But the emphasis on leaving neither debtor nor creditor worse off naturally focused attention on the intended use of the deferred payment. If the payment could both be sufficient to fulfill its intended future use to the lender yet not require the borrower to forgo more future opportunities than anticipated, no matter what prices prevailed, then it appears that the "ideal" is realized. Complications arise, and compromises are necessary, only when the intended future purchase of the lender differs from the intended forgone purchase of the borrower, such as when their consumption preferences differ. For example, if both lender and borrower consume only bread, then one might conjecture that fixing the deferred payment in loaves of bread is optimal.[1]

However this line of reasoning ignores the agents' sources of future wealth. If their future endowments were also fixed in loaves of bread, then specifying the payment in units of money, or any other good whose price relative to bread might change, does expose both individuals to unnecessary risk. But suppose that the lender has no future income other than the deferred payment and the borrower's future income is fixed in units of money. With payment fixed in loaves of bread the lender, on one hand, faces no uncertainty about his future consumption. The borrower, on the other hand, faces considerable uncertainty. If the money price of bread rises, not only will a larger share of his income be absorbed in making the deferred payment, but each remaining dollar buys less bread for his own consumption. Specifying the payment in loaves of bread can have no influence on the combined future consumption of these individuals; it merely shifts all risk to the borrower.

The specification of deferred payments thus influences not only the level but also the allocation of ultimate consumption risks among individuals. The natural criterion to use for judging alternative price indices to which deferred payments might be linked is whether they facilitate a Pareto Optimal allocation of the risks associated with future price uncertainty.[2]

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[1]Such a conjecture appears consistent with Fisher's reasoning (1922, p. 213).

[2]This criterion is also used by Steven Shavell (1976). He considers payment contracts which are contingent on an uncertain 'state-of-the-world' variable.

For purposes of analysis it is useful to abstract from the good (money) in which payment will actually be made. Escalating a money payment in proportion to the change in some price index with fixed weights insures that the payment will be sufficient to purchase a particular collection of goods. The collection purchasable is reflected in the weights attached to various prices in the index. An index-linked deferred payment contract thus represents a transfer of non-contingent claims to future goods, with the weights of the chosen price index representing the proportions in which claims to various goods are transferred. Consequently we define a <u>loan agreement</u> as a bilateral exchange of current money for claims to future goods, with the recipient of current money designated the borrower and the other agent the lender. We shall call the vector of claims, normalized to sum to one, which the borrower transfers to the lender the <u>standard of deferred payment</u> of the agreement. The special case in which a zero quantity of current money is transferred is termed a <u>pure futures contract</u>. For example, the agreement to purchase five loaves of bread at $2 per loaf, or the agreement to pay a baker the difference between $10 and the future price of five loaves, can both be thought of as the receipt of future claims to $10 and -5 loaves by the baker. If two individuals for whom no mutually beneficial pure futures contract exists enter into a loan agreement, we term the contract a <u>pure loan agreement</u>.

Consider now a two period pure exchange economy with n goods, H individuals and a government. Each individual has an initial endowment of current goods and claims to future goods. Let us conceptually distinguish the various types of exchanges that can be made in the current period by supposing them to take place in distinct 'markets.' Exchanges involving only current goods (including money) take place in the current spot market. Exchanges involving only claims to future goods take place in the futures contract market. Exchanges involving current money and future claims take place in the loan market. In the next period all claims to future goods materialize, and there is a further opportunity to trade in the future spot market. The current spot, futures contract and future spot markets are assumed to be perfectly competitive Walrasian markets. However since our ultimate interest is in the proportions in which future claims are transferred in the loan market we must explicitly recognize that loans are bilateral agreements between pairs of individuals.

The government will be supposed to trade only in spot markets. Each period it determines a random--from the point of view of individuals--quantity of various goods to be purchased. The purchases are financed solely by printing and issuing additional money. Since the government's future demands are unknown, the prices which will clear the future spot market are uncertain while current exchanges are being made.[3] All individuals are assumed to have the same subjective beliefs about the possible future prices that might be faced, represented by a probability distribution on future spot market prices.

It is assumed that, when trading in any one current market, each individual has full knowledge of the terms he will face in any other current market.

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[3] A government is not necessary, of course, to generate subjective uncertainty about future prices in the minds of individuals. To the extent that actions in current markets do not completely reveal future preferences, lack of information about even one individual's future demands will leave uncertainty in the minds of others about future market-clearing prices.

In other words individuals plan all their current period trades at one time. Execution of these trades then takes place in the following manner: The current spot market meets and goods are exchanged to fulfill current consumption desires. Correctly anticipating the opportunity to make loans which follows, some individuals spend more than the value of their current goods endowment while others spend less. The demand for money balances to be held over until the future period is treated like the demand for any other good. Such money balances yield unspecified services in the current period in addition to becoming, through storage, claims to future money. The futures contract market then meets and claims to future goods are exchanged until an equilibrium is reached. Finally the loan market meets. Individuals who overspent in the current spot market acquire the excess current money balances (over desired storage) of those who underspent in exchange for claims to future goods, using the proceeds to settle their outstanding spot market accounts. Since the futures contract market is closed, each lender and borrower choose a standard of deferred payment such that no further reallocation of claims between them could make both better off. They make a best possible loan agreement assuming no further exchange can take place before the future spot prices are revealed.

In the next period all claims materialize into goods, and the government determines its demands for non-money goods, prepared to issue sufficient new money to acquire them at whatever prices prevail. Market-clearing prices are established at which the government obtains its demands and all individuals acquire their preferred consumption bundles and money balances subject to the relevant budget constraints.

Our ultimate concern, of course, is only with the standards of deferred payment chosen in the loan market. In general there need be no similarity between the standard chosen by one pair of individuals and that chosen by another. However by requiring a futures market equilibrium before negotiation of the loan agreements we have confined our attention to pure loan agreements, thereby eliminating one cause for differences among standards chosen. In the next section we shall assume individuals' utility functions belong to a restricted class, with the result that the same standard of deferred payment is appropriate for all pairs of individuals, despite differences in their consumption preferences and attitudes toward risk. The use of this universal standard preserves the efficient allocation of risk established in the futures contract market. To this universal standard there corresponds a single price index and degree of escalation appropriate for all deferred money payments.

II. Determination of the Standard of Deferred Payment

1. Some notation:

The n goods are indexed $i = 1,\ldots, n$, and H individuals $h = 1,\ldots, H$. Good 1 is designated money and used as a numeraire when representing prices. Each individual is initially endowed with both current goods and claims to future goods. Suppose that the current spot market has already met. Each individual is committed to some current consumption bundle, including money balances to be stored until the next period, and has some fixed quantity m^h of <u>excess</u> current money balances to be lent (or borrowed if m^h is negative) in the loan market. It is assumed that $\sum_{h=1}^{H} m^h = 0$, or that the consumption-savings decisions are consistent in aggregate. An individual's holding of claims to future goods after making storage decisions and trading in the current spot market is designated by $w^h = (w_1^h,\ldots, w_n^h)'$.[4] His holding after trading in the futures contract market is designated by $x^h = (x_1^h,\ldots, x_n^h)'$, and after making any loan agreements by $y^h = (y_1^h,\ldots, y_n^h)'$. The aggregate privately held claims to future goods are $W = \sum_{h=1}^{H} w^h = \sum_{h=1}^{H} x^h = \sum_{h=1}^{H} y^h$. Since current money balances, through storage, represent claims to future money, W_1 is the current aggregate money stock. Future spot market prices in terms of money are $p = (1, p_2,\ldots, p_n)'$. The futures contract prices of claims are $p^0 = (1, p_2^0,\ldots, p_n^0)'$.

Each individual has a utility function $U^h(c^h)$ defined on his future consumption vector $c^h = (c_1^h,\ldots, c_n^h)$. The future money value of the individual's

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[4] All vectors are column vectors; x' denotes the transpose of x, $x'p$ the inner product of the vectors x and p. If z is a vector and $f(z)$ is a function of z, then $f_z(z)$ denotes the vector of partial derivatives of f with respect to the various components of z.

claims will be $M^h = p'y^h$, and his consumption demands $c^h(p, M^h)$. The indirect utility function $V^h(p, M^h) = U^h(c^h(p, M^h)) = \max_c \{U^h(c)$ such that $p'c \leq M^h\}$ denotes the level of utility he will attain. We interpret $c_1^h(p, M^h)$ as his demand for nominal money balances in the future period.

The government's future demand for non-money goods is a random vector $g = (0, g_2, \ldots, g_n)'$. The government budget constraint $\Delta W_1 = p'g$ requires it to increase the nominal money stock sufficiently to pay for these expenditures. Consequently the equilibrium condition determining future spot market prices is

$$(1) \qquad g + \sum_{h=1}^{H} c^h(p, p'y^h) = W + \Delta W_1 e_1 \qquad (e_1 = (1, 0, \ldots, 0)').$$

From a given individual's point of view, future prices are uncertain for two reasons. First, he may not know the other individuals' claim holdings or future demand functions, although he knows his own. This gives rise to what Radner (1968) termed "strategic uncertainty." Second, even if all individuals pooled their information they may not know g with certainty. This source of uncertainty is exogenous to the private sector. If the probability distribution of g was known, they might deduce from (1) a rational probability distribution on p. But we shall assume only that all individuals have the <u>same</u> subjective probability distribution on p.

When trading in the current period individuals are assumed to maximize their expected future utility $E(V^h(p, M^h))$ with respect to the claims they acquire (i.e., $M^h = p'x^h$ or $p'y^h$). Little can be said about the allocations established in the futures contract or loan markets without further knowledge of consumption preferences, attitudes toward risk, and the distribution of p. For the remainder of the paper we confine our attention to a particular class of utility functions.

2. Restrictions on preferences:

Specifically, we suppose each utility function to have the form, up to a positive linear transformation,

$$U^h(c^h) = \log f^h(c^h - a^h) \tag{2}$$

where $f^h(\cdot)$ is an arbitrary quasi-concave linear homogeneous function and the vector a^h is a non-homotheticity parameter. The consumption demand functions have the form

$$c^h(p, M^h) = a^h + z^h(p, M^h - p'a^h) \tag{3}$$

where z^h is directly proportional to $(M^h - p'a^h)$ for fixed p. This class of utility functions allows a reasonable amount of diversity in individuals' consumption preferences, requiring them only to have linear Engel curves through some fixed point a^h. However it does considerably restrict attitudes toward risk in the large. The class consists of all homothetic utility functions with a constant index of relative risk aversion equal to one, and all translations of such utility functions. Since an individual's utility becomes infinitely negative as c^h approaches a^h, we will refer to a^h as <u>necessities</u> to individual h. The aggregate necessities are $A = \sum_{h=1}^{H} a^h$.

The indirect utility functions have the form

$$V^h(p, M^h) = \log f^h(z^h(p, M^h - p'a^h)) \tag{4}$$

and partial derivatives with respect to money wealth

$$V^h_M(p, M^h) = \frac{(f^h_z)'z^h_M}{f^h} = \frac{(f^h_z)'z^h}{(f^h)(M^h - p'a^h)} = \frac{1}{M^h - p'a^h} \tag{5}$$

$$V^h_{MM}(p, M^h) = \frac{-1}{(M^h - p'a^h)^2} .$$

The first equality in (5) follows from the chain rule, the second from the proportionality of z^h to $(M^h - p'a^h)$, and the third from Euler's theorem since f^h is linear homogeneous in z^h. The Arrow-Pratt index of relative risk aversion is

$$(6) \qquad RRA^h = -M^h V^h_{MM}/V^h_M = \frac{1}{1 - (p'a^h/M^h)} \quad .$$

All individuals are risk averse for levels of income where V^h is defined (i.e., $M^h > p'a^h$). With a positive level of necessities, $a^h > 0$, an individual is infinitely risk averse at income levels near $p'a^h$, but his index of relative risk aversion declines toward one as M^h increases; for $a^h < 0$ his index is 0 at $M^h = 0$ and increases toward one as M^h increases. It should be noted that, through appropriate choice of the function f^h and the parameter a^h, a function of the form (2) can provide a second order approximation at a point to any quasi-concave utility function exhibiting aversion to income risk.

3. Equilibrium in the futures contract market:

We can now examine the equilibrium allocation of claims resulting from trade in the futures contract market. A futures contract is an exchange of future claims only, with no current goods or money involved. This market is assumed perfectly competitive. Each individual begins with some vector of future claims w^h, faces the same prices p^0, and is assumed to demand some vector of future claims x^h which maximizes $E(V^h(p, p'x^h))$ subject to a budget constraint $(p^0)'x^h = (p^0)'w^h$. From the Langrangian expressions

$$(7) \qquad L(x^h, \lambda^h) = E(V^h(p, p'x^h)) + \lambda^h(w^h - x^h)'p^0$$

are obtained the first order conditions for these maxima:

$$(8)\qquad L_{x^h} = E(V_M^h\, p) - \lambda^h p^0 = 0$$

$$L_{\lambda^h} = (w^h - x^h)'p^0 = 0 \qquad h = 1,\dots, H.$$

Notice that possible later alterations of his holdings in the loan market are ignored by the individual. He is assumed to trade as if x^h must be held until the future spot market opens.

Substituting the expressions for V_M^h derived in (5) into (8) yields the following equilibrium conditions for the futures contract market:

$$(9)\qquad E\left[\frac{p}{(x^h - a^h)'p}\right] = \lambda^h p^0 \qquad \text{for } h = 1,\dots, H \text{ (first order conds.)}$$

$$(w^h - x^h)'p^0 = 0 \qquad \text{for } h = 1,\dots, H \text{ (budget constraints)}$$

$$\sum_{h=1}^{H} x^h = \sum_{h=1}^{H} w^h = W \qquad \text{(market clearing).}$$

A solution to this system of equations, which may be verified by substitution, is

$$(10)\qquad x^h = a^h + \left[\frac{(w^h - a^h)'p^0}{(W - A)'p^0}\right](W - A) \qquad h = 1,\dots, H$$

$$\lambda^h = \frac{1}{(w^h - a^h)'p^0} \qquad h = 1,\dots, H$$

$$\frac{p^0}{(W - A)'p^0} = E\left[\frac{p}{(W - A)'p}\right]$$

Each individual leaves the market holding claims to his necessities a^h, plus whatever proportionate share he can purchase at prices p^0 of the remaining aggregate non-necessary future goods $(W - A)$.

The above market equilibrium has several points worth noting. First, the equilibrium prices p^0 in terms of future money claims depend solely on subjective beliefs about the distribution of p and the non-necessary component of aggregate endowments (W - A). Neither the initial distribution of claims embodied in the w^h, nor aspects of future consumption preferences embodied in the functions $f^h(\cdot)$, have any influence on p^0. Consequently little information about future aggregate consumption demand is conveyed through futures contract prices. Second, an individual's claim holding x^h is not directly dependent on either the nature or degree of future price uncertainty. The fact that his particular anticipated consumption bundle may or may not have a very uncertain future price has no influence on his claims purchased. Third, although each individual is risk-averse, his claim holding is independent of all aspects of his future consumption preferences except a^h. This is somewhat surprising since one might have expected a risk-averse individual to acquire a collection of claims more closely resembling his anticipated future consumption bundle if given the opportunity. Finally, notice that if all consumption preferences are homothetic (i.e., $a^h = 0$ for $h = 1,\ldots, H$) then all individuals hold claims to future goods in the same proportions as aggregate private endowments W.

4. The specification of loan agreements:

Let us now turn to the loan market, through which borrowers acquire current money to settle their outstanding current spot market accounts and lenders loan out excess money they chose not to hold for the future period. Loan agreements take the following form: A potential lender and borrower initially meet holding future claims x^L and x^B, and excess current money balances $m^L > 0$ and $m^B < 0$, respectively. The lender gives the borrower some quantity of current money in exchange for a vector $t = (t_1, \ldots, t_n)$ of claims to future goods. After making the agreement, the lender holds claims $y^L = x^L + t$ and the borrower holds $y^B = x^B - t$. We do not attempt to characterize the process by which the agreement is negotiated. There are n + 1 parameters to the contract--t plus the amount of current money transferred. But we shall require the outcome to be <u>efficient</u> in the sense that no alternative choice of t could further increase the expected future utility of one agent without decreasing that of the other. Any strategic aspects of the bargaining process, determining for example the expected nominal interest rate on the loan, are confined to the distribution of expected future wealth between the lender and borrower.

The standard of deferred payment is the proportions in which future claims are transferred, or $t/\sum_{i=1}^{n} t_i$. The efficient standard can readily be found by examining the equilibria of the futures contract market. The allocation x^h (h = 1,..., H) in (10) equated all individuals' marginal rates of substitution between claims in yielding future expected utility, and hence is a Pareto Optimal allocation of non-contingent future claims. The set of all Pareto Optimal allocations is identical to the set of futures contract market equilibria for all feasible initial allocations w^h (h = 1,..., H). Such allocations have the form

$$
(11) \qquad x^h = a^h + \alpha^h(W - A) \qquad\qquad h = 1,\ldots, H
$$

$$
\alpha^h > 0, \ \sum_{h=1}^{H} \alpha^h = 1 \qquad\qquad (V^h \text{ is defined only for } \alpha^h > 0).
$$

Since the trading of futures contracts terminated with an allocation of the above form, the overall Pareto Optimality of the claims allocation will be preserved if and only if a t proportional to (W - A) is used by our borrower and lender. Since their negotiations have no direct effect on the holdings of other agents, a standard of deferred payment proportional to (W - A) is also in their own mutual best interest. The same argument applies to each pair of potential borrowers and lenders. Consequently a standard of deferred payment proportional to (W - A) is universally appropriate for all loan agreements.

III. The Price Index and Coefficient of Escalation

It is usually not practical for deferred payment contracts to require payment of particular quantities of various goods. Unnecessary transaction costs would be incurred by the borrower in acquiring these goods if he was not the original producer, and by the lender in disposing of them if he was not the ultimate consumer. Money, as medium of exchange, is the usual means of payment for loans. Allowance for unanticipated price changes can be accomplished by agreeing to escalate the money payment in proportion to the change in some mutually agreeable price index.[5] The percentage increase in the money payment per one per cent change in the price index is termed the coefficient of escalation (Collier, 1969, p. 51). It remains to find a price index and coefficient of escalation corresponding to the universal standard of deferred payment determined in the previous section.

1. Homogeneous aggregate consumption demands:

Let us look first at a situation with $A = 0$; aggregate consumption demands are linear homogeneous in individuals' discretionary wealths.[6] Efficient loan agreements must transfer claims in the same proportions as aggregate endowments. That is $t = kW$, where the scalar k indicates the

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[5]The agreements to deliver a specified bundle of goods or sufficient money to purchase that bundle are equivalent only if there is no rationing in the future spot market. We are utilizing the assumption that the future spot prices are market-clearing prices.

[6]By discretionary wealth we mean the individual's wealth beyond that required to purchase his necessities, i.e., $M^h - p'a^h$. From (3) it is apparent that is we increase each individual's discretionary wealth by the same factor λ, and if $A = 0$, then the aggregate demand for each good must increase by the factor λ. The requirement that $A = 0$ is slightly less restrictive than requiring all individuals to have homothetic preferences , i.e., $a^h = 0$ for $h = 1,\dots, H$.

magnitude of the repayment. The future money value of t will be $m = k(p'W)$, and, if $\bar{p} = (1, \bar{p}_2, \ldots, \bar{p}_n)'$ denotes current spot market prices, the current value of such a bundle of goods would be $\bar{m} = k(\bar{p}'W) = \bar{p}'t$. The future money value of the payment can be written alternatively as $m = \bar{m}(p'W/\bar{p}'W)$ in which the ratio $p'W/\bar{p}'W$ captures the change in prices between the two periods. This ratio as it stands does not fully reflect changes in the purchasing power of money since the price of money in terms of itself (identically 1 by choice of numeraire) appears both in the numerator and denominator. But if we define the level of future relative to past prices by the Paasche price index

$$(12) \qquad P \equiv \frac{\sum_{i=2}^{n} p_i W_i}{\sum_{i=2}^{n} \bar{p}_i W_i} = 1 + \Delta P = 1 + \left[\frac{\sum_{i=2}^{n} (p_i - \bar{p}_i) W_i}{\sum_{i=2}^{n} \bar{p}_i W_i}\right]$$

then the ratio $p'W/\bar{p}'W$ is expressible as

$$(13) \qquad \frac{p'W}{\bar{p}'W} = \frac{W_1 + \sum_{i=2}^{n} p_i W_i}{W_1 + \sum_{i=2}^{n} \bar{p}_i W_i} = 1 + \Delta P \left[\frac{\sum_{i=2}^{n} \bar{p}_i W_i}{W_1 + \sum_{i=2}^{n} \bar{p}_i W_i}\right]$$

$$= 1 + \Delta P \left[\frac{v}{1 + v}\right] \qquad \text{where } v = \frac{\sum_{i=2}^{n} \bar{p}_i W_i}{W_1} .$$

A deferred payment equivalent to t is thus the money payment $\bar{m}$ escalated by $v/(1 + v)$ times the change in the price index P.

Interestingly, Irving Fisher in his early work recommended this price index for escalating deferred money payments:[7]

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[7]This is not, however, Fisher's "ideal price index"--a geometric mean of Laspeyres and Paasche indices--which he favoured in later work.

> "To cut these Gordian knots, perhaps the best and most practical scheme is that which has been used in the explanation of the P in our equation of exchange, an index number in which every article and service is weighted according to the value of it exchanged at base year prices in the year whose level of prices it is desired to find.... What is repaid in contracts so measured is the same general purchasing power." (1922, p. 217)

But notice that payment should not be escalated by the full change in P since $v/(1 + v) < 1$ if the money stock is positive. This less than full escalation merely reflects the fact that someone must hold money balances and that efficient allocation of risk entails an equiproportionate sharing of any increase or decrease in the real value of such balances. If the economy is endowed with the same quantities $W_2, \ldots, W_n$ of non-money goods each period then, since W_1 is the current money stock, v may be interpreted as the velocity of money. The smaller is this velocity, the smaller is the appropriate coefficient of escalation $v/(1 + v)$.

2. Inhomogeneous aggregate consumption demands:

Turning to the case where $A \neq 0$, or aggregate consumption demands are linear but not homogeneous in individual discretionary wealths, efficient loan agreements transfer claims in the same proportions as the non-necessary part of aggregate future endowments, that is $t = k(W - A)$. The future money value of t is $m = k\, p'(W - A)$, and can be written alternatively as $m = \bar{m}(p'(W - A)/\bar{p}'(W - A))$, where $\bar{m}$ is the current money value of t. The appropriate price index to capture the change in money prices is

$$(14) \qquad p^* = \frac{\sum_{i=2}^{n} p_i(W_i - A_i)}{\sum_{i=2}^{n} \bar{p}_i(W_i - A_i)} = 1 + \Delta P^* = 1 + \left[\frac{\sum_{i=2}^{n} (p_i - \bar{p}_i)(W_i - A_i)}{\sum_{i=1}^{n} \bar{p}_i(W_i - A_i)}\right]$$

and a future money payment equivalent to t can be specified as

$$(15) \qquad m = \bar{m} \left[1 + \Delta P^* \left[\frac{v^*}{1 + v^*} \right] \right] \qquad \text{where } v^* = \frac{\sum_{i=2}^{n} \bar{p}_i (W_i - A_i)}{(W_1 - A_1)} .$$

The index P* is similar to a "marginal price index" suggested by S. N. Afriat (1974) for escalating nominal incomes to preserve the real income distribution when prices change. Compared with the Paasche index, P* weights <u>less</u> heavily the price changes of relatively necessary goods (higher than average A_i/W_i) and <u>more</u> heavily the price changes of remaining luxury goods.

The parameter v* can no longer be interpreted as the velocity of money. But if money balances are a luxury relative to other goods, then v* is less than v, and the appropriate coefficient of escalation is smaller than if aggregate demands are homogeneous: $v^*/(1 + v^*) < v/(1 + v)$.

IV. Concluding Remarks

We have explored how deferred payments might be specified to efficiently allocate risks of price change among individuals. In contrast with early writings on this problem--which focussed on the goods a lender anticipates purchasing with, and a borrower anticipates foregoing to make, future payment--aggregate future endowments, sources of income, play a more important role than consumption preferences in determining the appropriate specification. From a practical viewpoint this is rather encouraging. It suggests that desirable standards of payment might be determined from observable data. In the case where aggregate demands were homogeneous, for example, the appropriate specification depended only on aggregate future endowments of real goods and the velocity of money. No further information about individual endowments, consumption preferences or the true distribution of possible future prices was needed.

The context of these results, of course, is exceedingly limited. There is no production, capital, or uncertainty about future private endowments. Price risks result from demand rather than supply variability. More significantly, the analysis was confined to pure loan agreements. Without an efficient allocation of risk prior to negotiating loans, the 'second best' standard of deferred payment would generally differ for each pair of agents and each size of loan. In the absence of complete futures markets other institutions--such as stock markets, explicit forward purchases and implicit agreements among individuals, combined perhaps with beliefs that many prices move together--would have to establish efficient prior allocations of risk for there to be any universally 'best' standard of payment.

However the important point remains that the measures of price change generated by the conventional theory of economic price indices do not provide desirable standards for deferred payment. Uncertainty about future prices must stem from underlying real uncertainty; and in the presence of real uncertainty it is generally neither feasible nor desirable to completely shield individuals from the associated price changes.

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